Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 43
Introduction to time dependent problems

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Hello, welcome to basics of finite element analysis, this is the last week of this particular book course and in this week we will be focusing on how to solve time-dependent problem. So that is going to be the focus for this entire week.

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And in the last lecture we will have a closer comments, so that is what we plan to do in this week. So as I mentioned what we are going to discuss today and the remaining part of this week is time dependent problems. So once again as we have done in this entire course the category of solute, problems which we will address will have a single dimension in space and of course the second dimension for the problems which we are going to consider today will be time. So we will first have a general equation, differential equation and then we will solve some special cases of that general differential equation.

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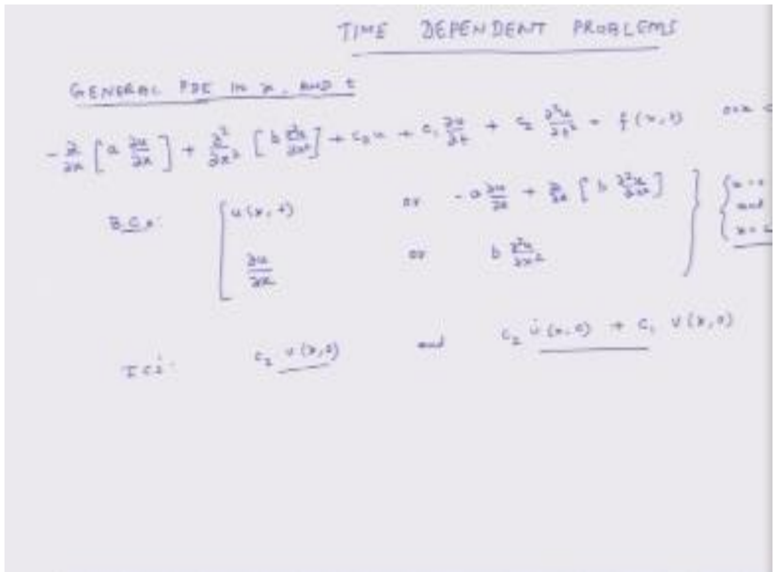

So the general differential equation, general pd in X and t. First we will state that and that is equal to partial with respect to X a $\partial u/\partial x$ plus second partial derivative of b $\partial^2 u/\partial x^2$ plus c1$\partial u/\partial t$ plus c2 ∂t second derivative of U with respect to time is equal to a source term F which depends on X and T, and the associated boundary conditions would be that either at the boundaries u is known or we know the corresponding secondary variable which is – a $\partial u/\partial x$ plus $\partial/\partial x$ b times $\partial^2 u/\partial x^2$.

And the second set of boundary conditions would be $\partial u/\partial x$ that would be the primary variable or the secondary variable associated with it is B times second derivative of U with respect to X. So these are the boundary conditions. And then we have another set of conditions known as initial conditions so the initial conditions are c2 times u(x, 0) is known and c2 u dot at x and time t is equal to 0 plus c1 u at x 0 is known.

So these boundary conditions we have to know so this is a partial differential equation is valid for the range x is between l and 0. So this partial, so these boundary conditions they have to be specified at X is equal to 0 and at x is equal to L, for the initial conditions which are two in number they have to be specified at time T equals zero. So boundary conditions are valid at all times but they are specified only at the boundaries of the system and the initial conditions they are valid at only t is equal to 0.

But they had they have to be applicable to the entire field or at all values of x. So that is one thing I wanted to state, the second thing is that unlike all the questions or the problems which we have addressed in earlier lectures because they were not dependent on time. So we were dealing only with boundary conditions, now in this case this is a partial differential equation.

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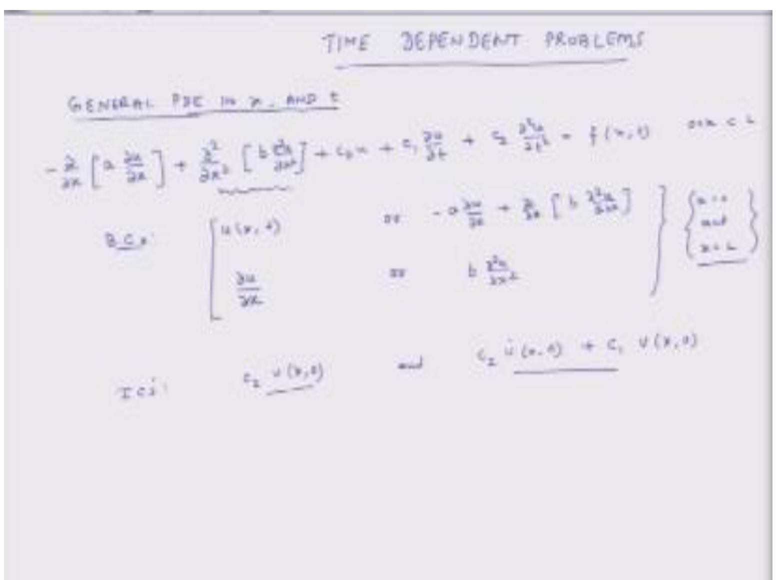

And it is of second order in time and it is also second order in space which is X. So when I integrate this partial differential equation either through exact methods or through finite element method when I integrate it, I will have to integrate it two times in X, right? And then or actually if this term is non zero b times $\partial^2 u / \partial x^2$ then I will have to integrate it four times in x, and because this is a second order differential partial differential equation in time then I will have to integrate it two times in time also.

So when I integrate two times in time I need I will get two integration functions or and those values those things will come if we know the initial conditions because when we are integrating it two more times I have to get two more conditions I have to know two more conditions.

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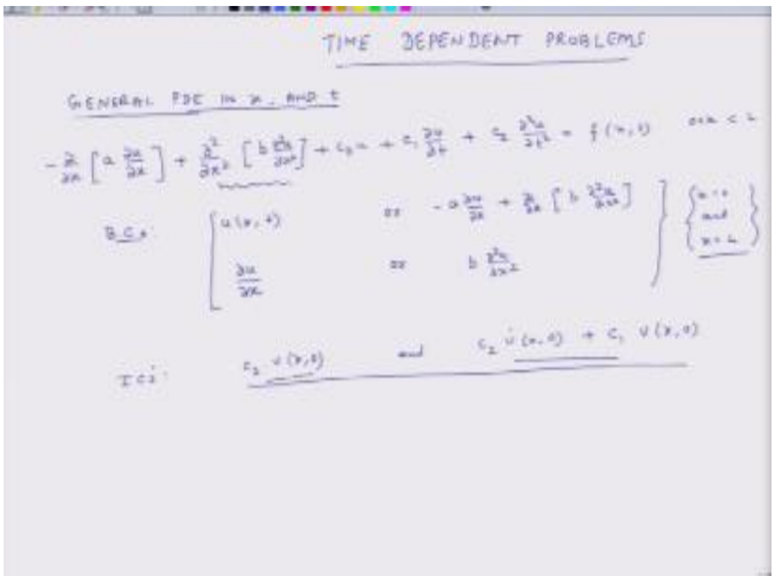

So that is why we need initial conditions when we deal with time-dependent problems because we have to go through extra rounds of integration, in case c2 was 0 then we would have to integrate this entire equation only once in time, okay. Because of this c1 $\partial u/\partial t$, in that case we would need only one set of initial conditions.

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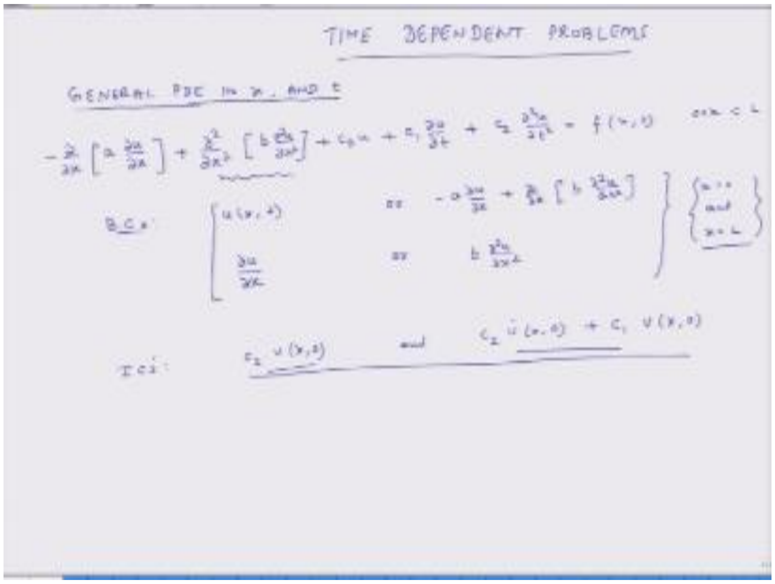

If the PD is having second order derivatives in time then it has to go through two rounds of integration and in that case we meet two sets of initial conditions. So we have to know also u as well as u dot as initial conditions.

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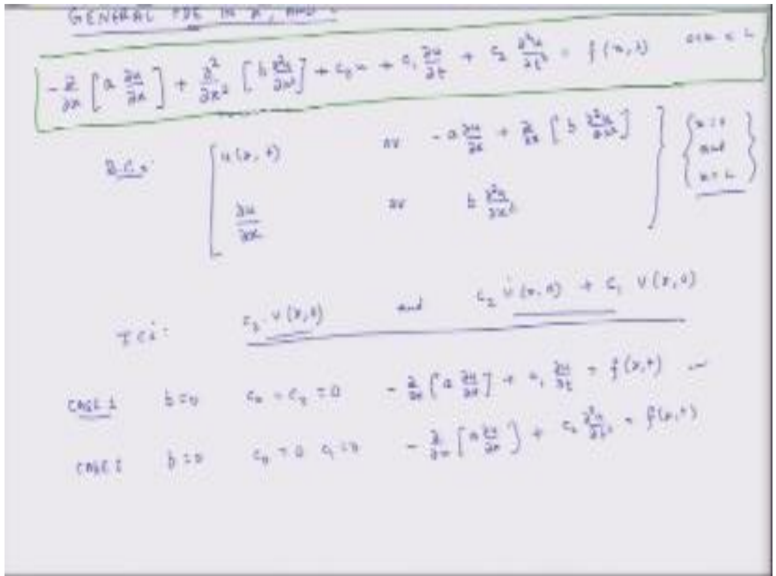

So this is the overall differential equation and this is pretty generic equation, and depending on which terms we omit and which terms we account for this general differential equation it represents a large number of physical problem, physical problems for instance we will just look at a few cases not necessarily all, case one if b is 0 and c0 is equal to c20 then my partial differential equation becomes - $\partial/\partial x$ a $\partial u/\partial x$ plus c2 ∂^2 u over excuse me is equal to f of x and t, okay.

And this equation it represents heat conduction where it is not a steady state situation but also the temperature. So temperature represented in this case by letter u. So this is a long equation the original equation which is now boxed in green it reduces to this equation and it represents heat conduction in a one dimensional heat conduction and which is a in a not a steady state system but in a transient system.

Another case, case two b equals 0, c0 equals 0, c1 equals 0, in that case my partial differential equation becomes plus c2 second derivative of U with respect to time is equal to f (x, t). Now this second equation can represent several physical systems, one physical system it will represent is vibrations or motion in a bar which is under axial compression or tension.

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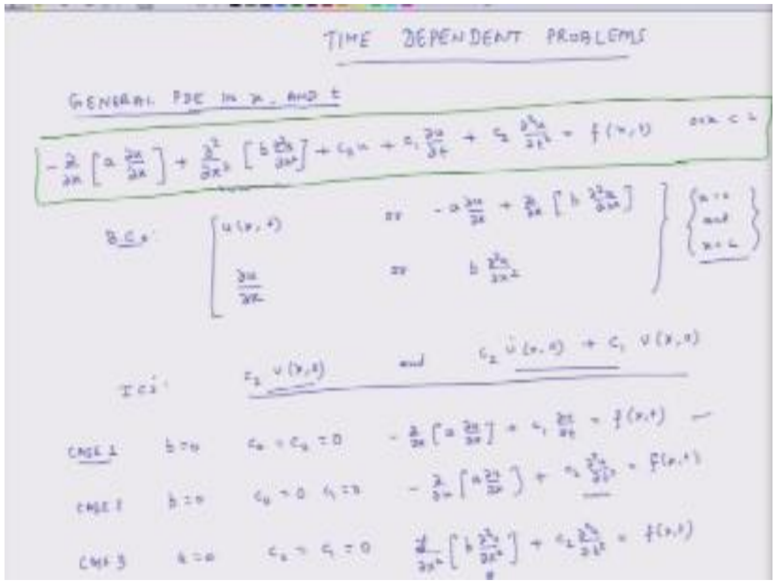

And that external force which is being applied on it is F and it is vibrating and that is why and because its motion is changing with time that is why we have this c2 times $\partial^2 u/ \partial t^2$. Another example would be when a is equal to 0 and c0 is equal to c1 is equal to 0. So then my overall governing differential equation is second derivative of B $\partial^2 u/ \partial x^2$ plus c2 $\partial^2 u/ \partial t^2$ is equal to f(x, t). And this equation would represent vibrations of a beam because of this fourth order derivative in space.

So the point is that this particular equation which we are going to discuss going forward it is a very general equation not necessarily representing one single system but if we selectively omit out and pick specific terms from it then it can be expressed as a large number of, it captures their sense of several physical systems, okay. So that is there.

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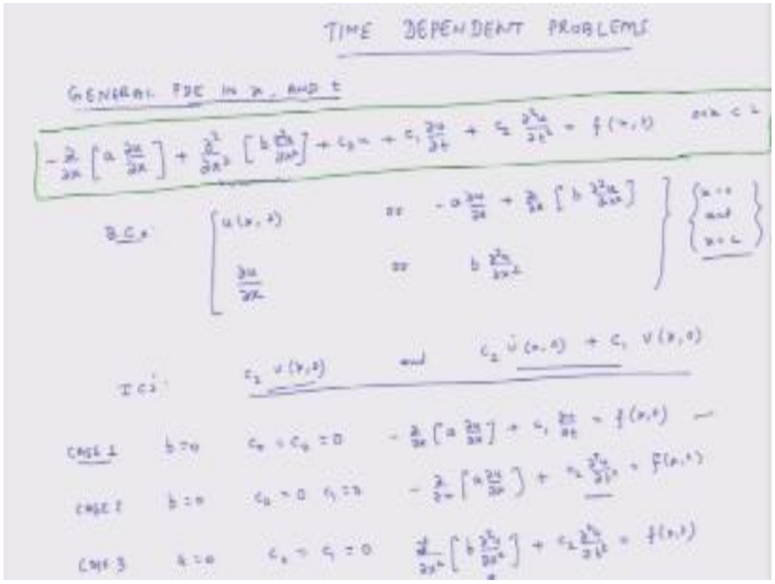

So that is the overall differential equation we are going to handle and then as we walk down in time we will look at specifically at several specific special cases of this partial differential equation. Now what we are going to discuss with you is the overall scheme of how we address time-dependent problems, okay. So first we will get an overview and then we will start working out the details.

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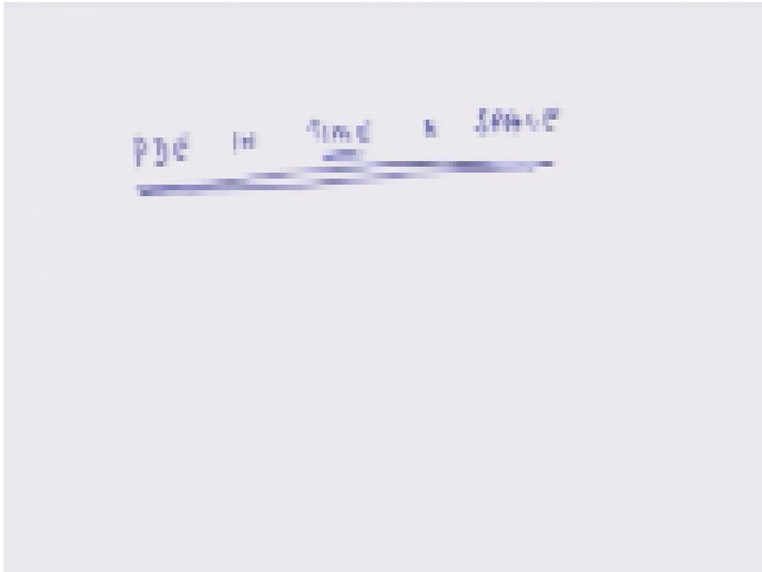

So we note that a partial differential equation in time and space, okay. So there is a partial differential equation in time and space, earlier we did not have the differential equation in spare time. So now so earlier we had ordinary differential equations, now we have partial differential equations and they are partial derivatives with respect to time and partial derivatives with respect to space.

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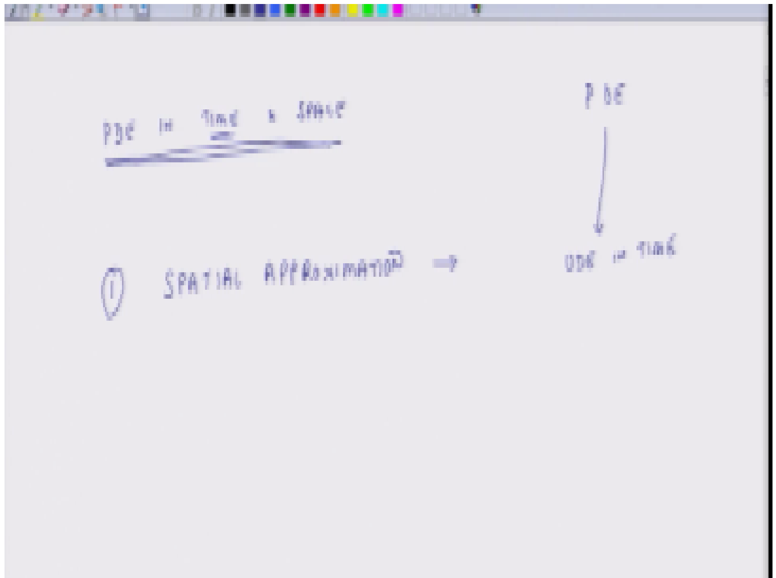

So the overall scheme of things is that the first step you do when you encounter such a problem is that you try to integrate the equation in space, okay. So this step is known as special approximation, step is known as special approximation and at the end of this so you have initially PD and when you do a special approximation you are left with an OD ordinary differential equation in time, okay. And all this activity we are doing at the element level. So the first step we do is special approximation.

And that is that we integrate the equation in space and in general how we do it, essentially the way to do it is we assume some approximation functions put that in the system in the equation, multiply so then we get a residue and then we multiply that residue with the weight function integrate that over space, we integrate.

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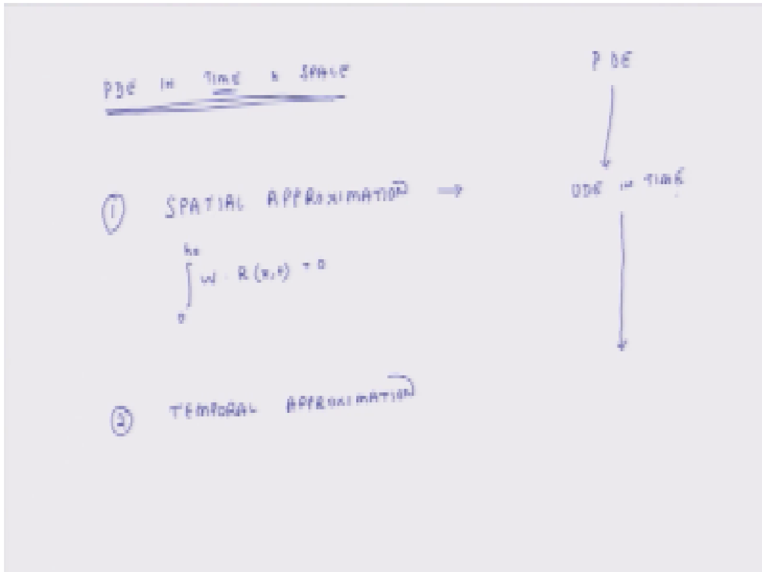

So the overall scheme is that we integrate 0 to h_e, the product of weight times residue of the equation which will depend on X and t and then we equate this integral to 0. So this is known as special approximation and once we do this all the derivatives with respect to X they get eliminated and the only things which are left are stuff which are related to derivatives of time. So that is why now we are left with only derivatives in time and my PD has gone down and has become a much simpler equation.

And that is an ordinary differential equation. The next step we do is we do temporal approximation. So, so from OD once we are done with this temporal approximation. So special approximation means we are handling terms related to derivatives with respect to space that is $\partial/\partial x$, when we are doing temporal approximation the term temporal refers to time. So here now we are looking and focusing explicitly on time-related derivative and again we are integrating it in time.

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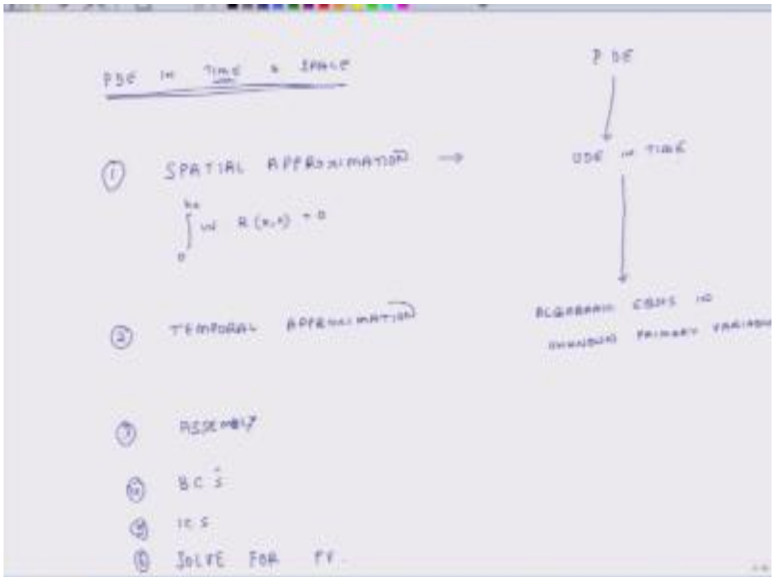

And once we have done that then what we are left with is algebraic, equations in equations in unknown primary variables. So, so what we are left with is algebraic equations and unknown primary variables, okay. And then and at this stage we apply, we do assembly using the same methods which we had discussed earlier then we apply boundary conditions, then we apply initial conditions and then we solve for primary variables at each node. So this is the overall scheme of things. And as we walk down we will specifically look at how we exactly do all these individual steps.

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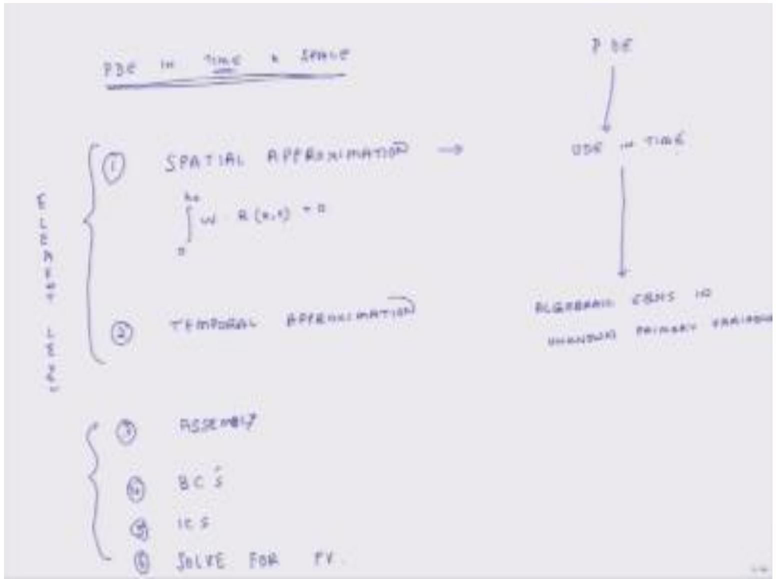

So, so this, the first two steps are at element level and these steps three, four, five, and six are done at assembly level. So this is what I wanted to cover in today's lecture and in the next lecture we will start discussing the details of this method. So thank you very much and we will continue this discussion tomorrow, look forward to seeing all of you tomorrow, bye.

<u>Acknowledgement</u>
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