**Indian Institute of Technology Kanpur**
**National Programme on Technology Enhanced Learning (NPTEL)**
**Course Title**
**Basics of Finite Element Analysis**

**Lecture – 41**
**Eigenvalue problems**

**by**
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Hello, welcome to basics of finite element analysis, today is the second last day of this week that is the seventh week, and in today's lecture, tomorrows lecture and maybe in the third lecture also we will discuss Eigenvalue problems. Or how what they mean, how they are formulated and how are they analyzed, so that is.

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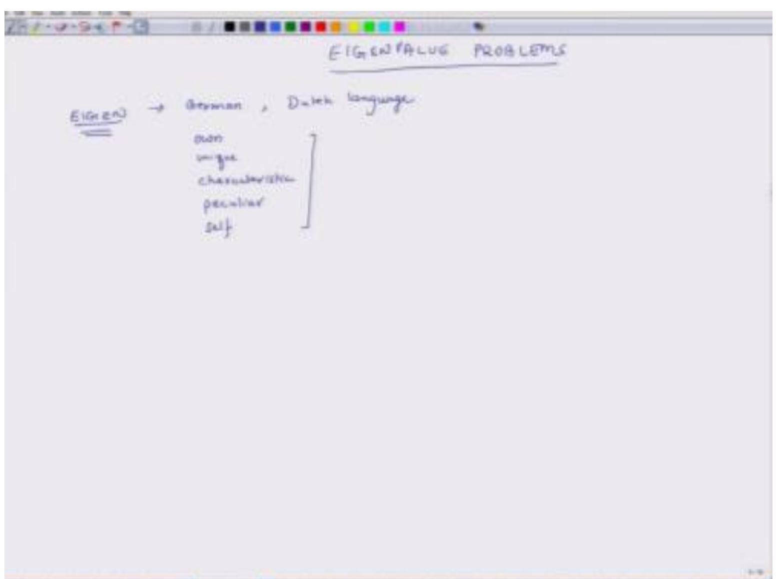


Going to be our focus Eigenvalue problems okay. Now this term Eigen it comes from German, and it is also there in Dutch language and it has several meanings but they are all related, first meaning is own okay, second meaning is unique, another interpretation is characteristic, another interpretation is peculiar or self. So these are the ideas which when a German speaks this term Eigen they come to his mind own, self, unique, characteristic, peculiar. In context of mechanical engineering problems or any problem for the mathematical problems, we say that system has an Eigenvalue. It means that there may be some number which is peculiar or it is a characteristic of that system, so it will not depend on some external force.

But it is, it is independent of some external force because if I am putting some force on the system then that number may change, but there may be some number or some property of that system which is unique about itself which I which is its identity, another interpretation of Eigen is identity.

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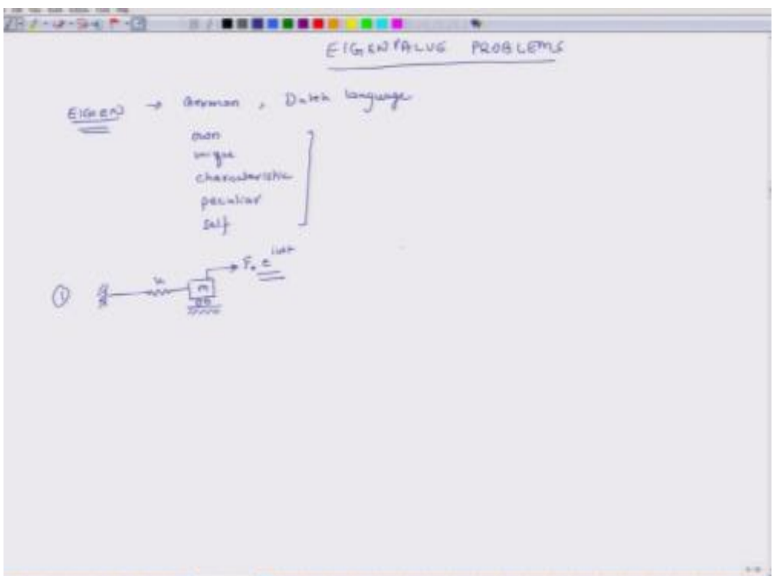


So I will give you an example, first example so I have a spring, mass and I apply a force and let us say I apply a harmonic force $e^{i\omega t}$ okay. Now when I am exciting it with this force, when I am exciting this system with this force whose frequency is $\omega$ then the system will get excited at the same frequency $\omega$ so this $\omega$.

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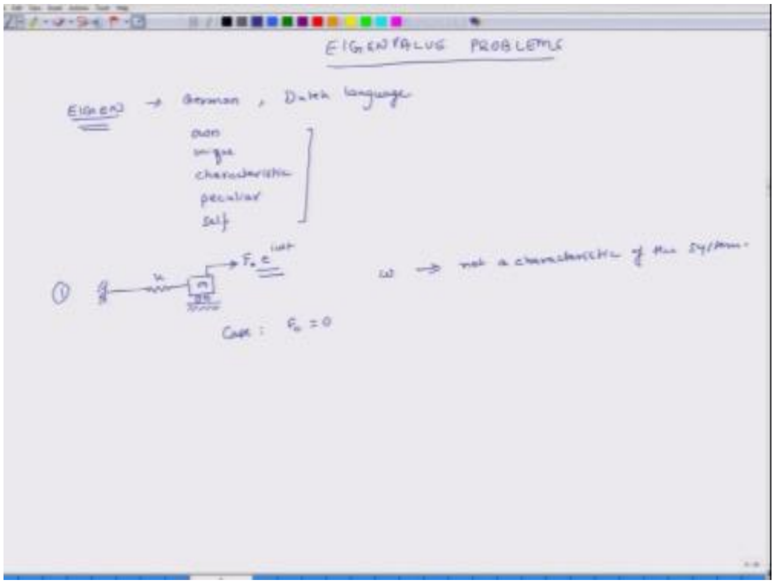


Is not a characteristic of the system okay, but suppose I remove this force, so case when $F_0$ is equal to 0, and then if I disturb this mass by a little amount then the system will vibrate back and forth at a unique frequency. Let us call that unique frequency as ω.

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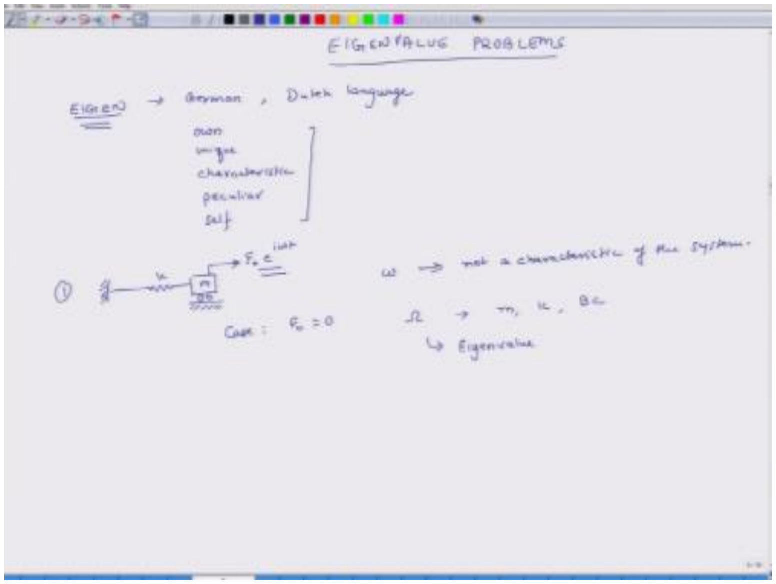


That unique frequency will depend on mass, stiffness and the boundary condition of the system and that unique frequency will be called Eigenvalue, and Eigenvalue is one word of the system. So it does not depend on an external factor, it depends on everything which is internal to the system and what are the things internal system boundary conditions, and in this case a stiffness and the mass. In another system it may be something different but some things which are unique about an internal to the system. So Eigen values are related to something internal things related to system.

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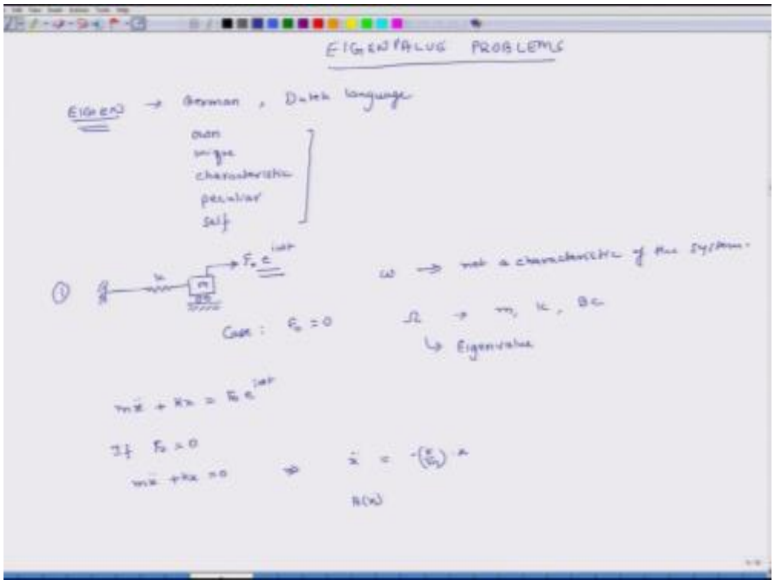


Another example so, so before I go to another example here the governing equation is $m\ddot{x}+kx$ is equal $F_0\ e^{i\omega t}$ okay, and if $F_0$ is equal to 0 if $F_0$ equals 0 then my equation becomes $m\ddot{x}+\ k_x$ equals 0 or I can express it as $\ddot{x}$ equals minus k over m times x or I can write this as a function of x which is the right side equals.

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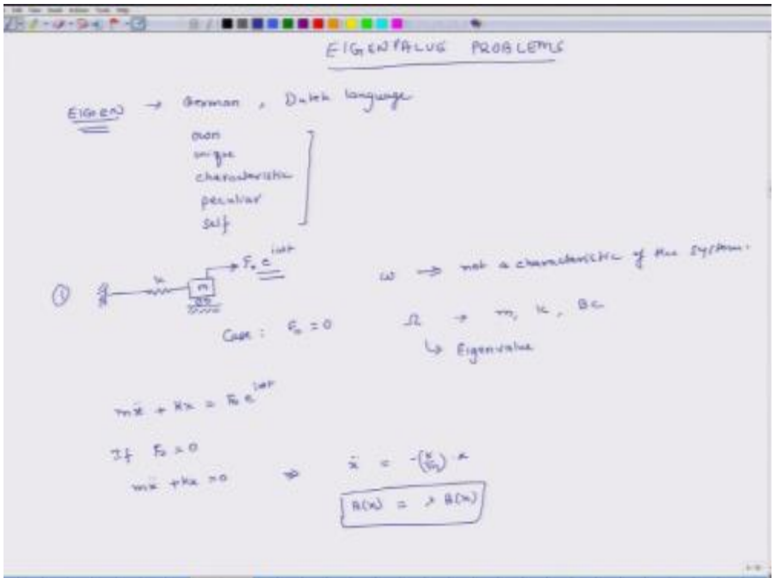


Some constant λ and this λ maybe an Eigenvalue, so if it, so sorry before I use that, times another function of x which is B(x), so if I can express so if there is a system which does not have which is not seeing any external force and if I can represent that system as.

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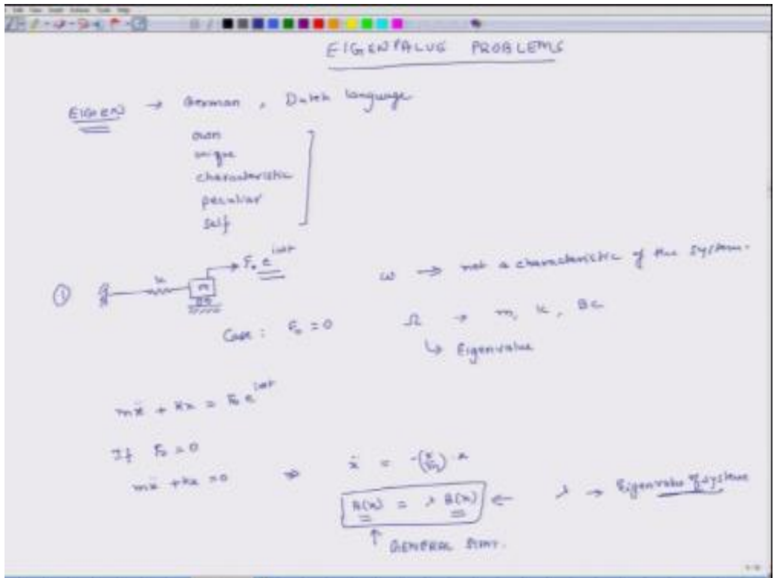


In this form A(x) equal to λ times B(x), then λ so A will depend only on the internals of the system there is no external force in the system, B also depends only on the internals of the system, so and these two sides will be equal if there is appropriate value of λ, so this λ is known as Eigenvalue of the system, so this is Eigen value of the system. So this is a very general statement for one-dimensional Eigen value problems, it is very general statement.

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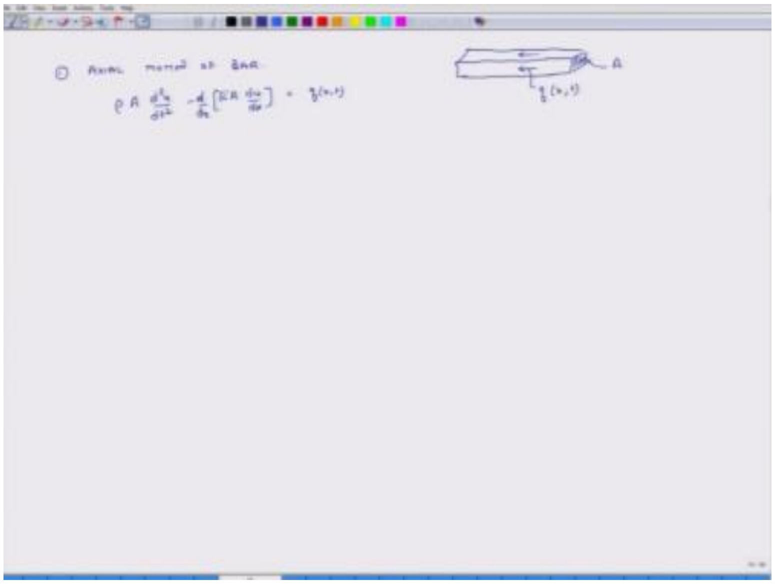

We will see another example, axial motion of bar okay, so you have the governing equation is ρ A, second derivative of displacement so u is the displacement not velocity. So ρ A times acceleration of particle minus EA du over dx and I am differentiating this whole thing with respect to x, and on the right side I have the external force, so how is this, suppose this is a bar this small element of the bar maybe there is some q, so this q could be a function of x and t, this is my area of cross section, E is the Young's modulus, ρ is the density so the axial motion of this bar is governed by.

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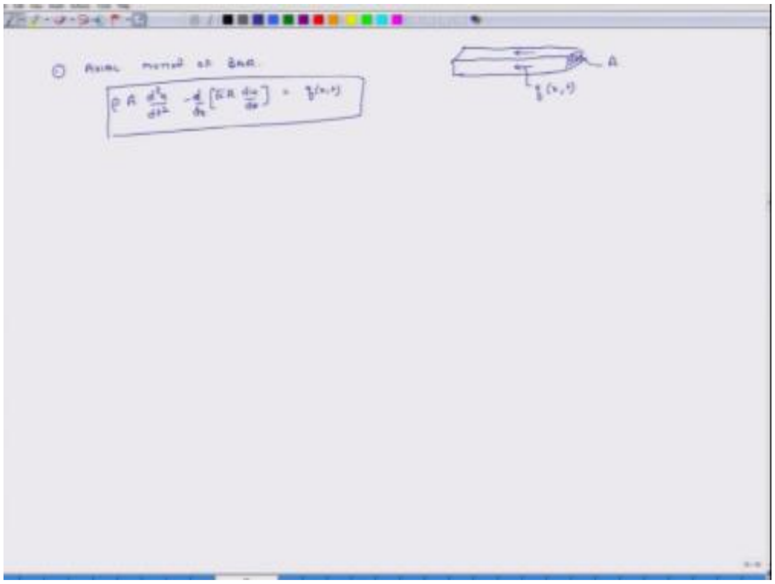

This differential equation, if I have to find the Eigenvalue of the system what should I do? I should make q to be 0 and then solve the equation then the solution will give me the Eigen values of the system, because when I am solving it using q it is not everything is not internal to the system right, so when I am doing.

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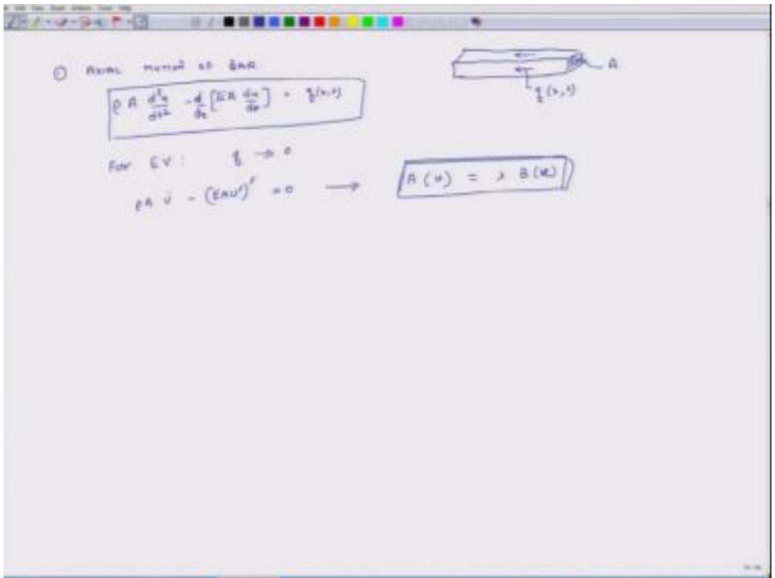


Eigenvalue of this then so far EV, first thing I do is q I remove q, so I said q to be zero, so ρA ü – (EAu') and then again I differentiate this is equal to 0, and if I mathematically process or manipulate this equation I can again express this equation as A, so as a function of, so A which is a function of u is equal to some number times another function of u. I can express this as like this okay. So this is, so I am again able to, so this is my Eigenvalue formulation of the system so this is again same format.

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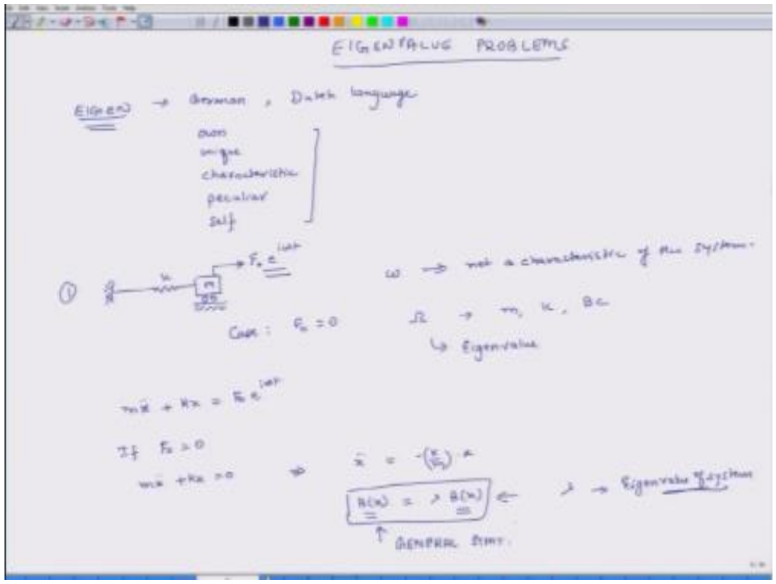


Which we saw earlier.

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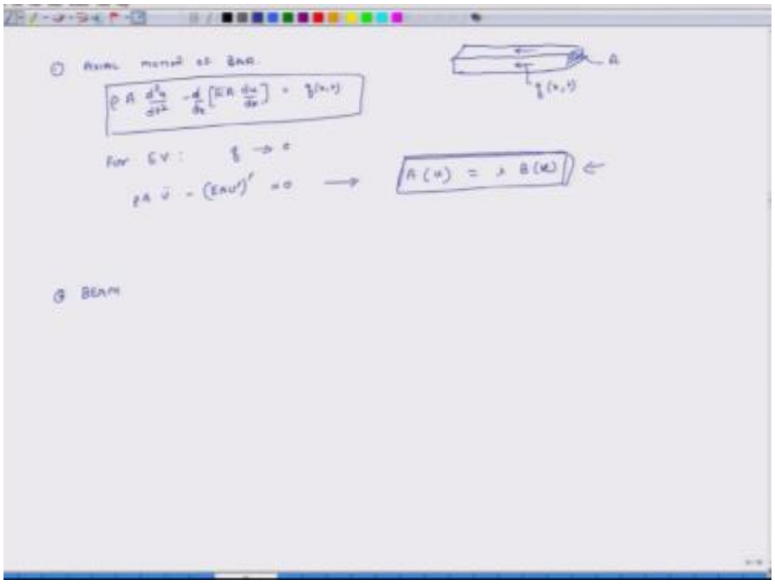

How do we get there we will actually see it, but I can express it in this form, I will do another example, so the other thing I wanted to mention is that Eigen values are not only not always about vibrational frequencies. But they reflect something internal and unique about the system. We will see one example where they are not the frequencies of the system.

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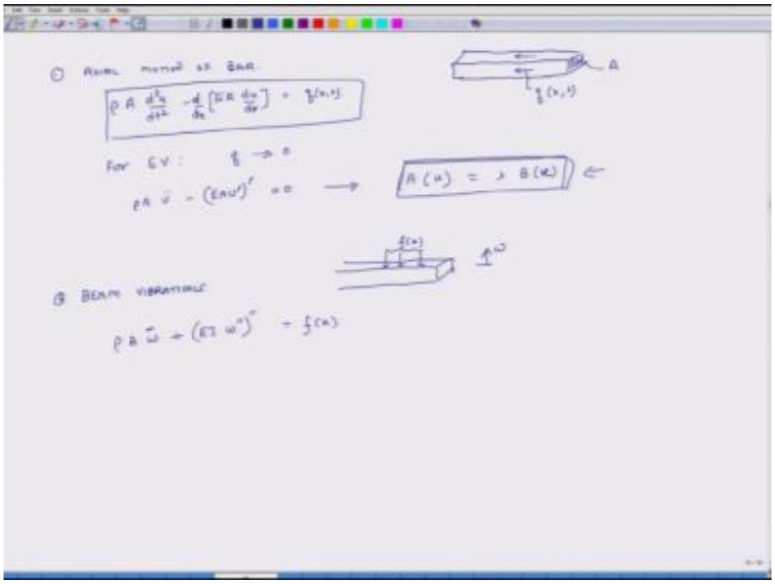

But right now we will discuss beam vibrations okay, so for a beam which is a cross section A and a moment of inertia I and Young's modulus e and density is $\rho$ the governing differential equation, the dynamic dynamical equation is the $\rho A\ \ddot{w}$, second derivative of W, so W is reflection plus $EI(w'')$ equals f, f is the distributed force on the beam, again if I have to find the Eigenvalue of this system the first step I have to do is I have to, I have to equate.

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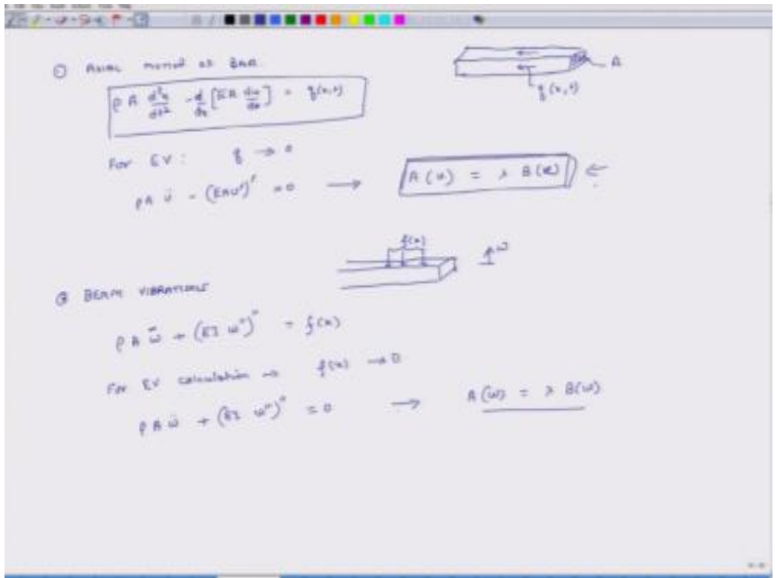

f to be 0 so for Eigen value calculation f(x) we make it 0 so my equation is reduced to $\rho A w''$+ EI w excuse me, $\rho A$ times second derivative of w + EI $w''$ the entire thing differentiated twice in x, this equals zero okay, and again by mathematical manipulation I can express this thing as in this form, how do I do it, I can consider w as a function of t and x, I can separate the variables and I do all the mathematics I can express it in this form by variable separable method. I can use the same variable separable method and I can get it here also okay, so that is not a problem.

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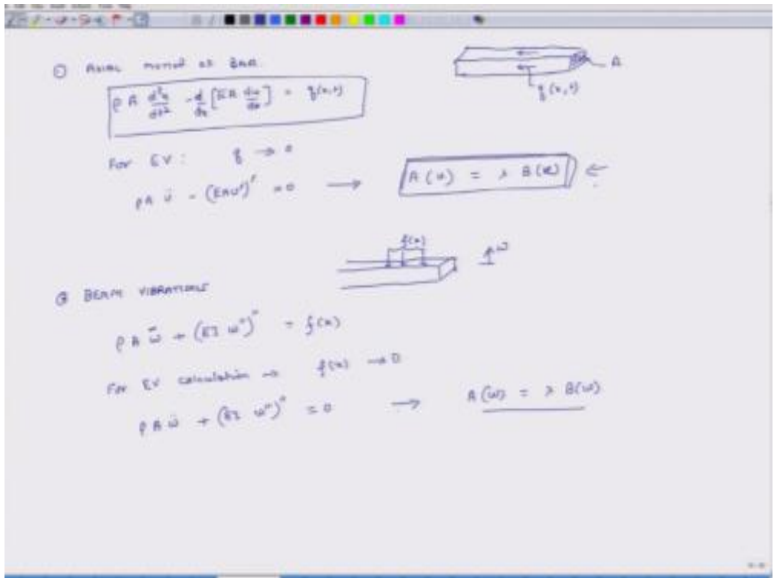

So in all these examples.

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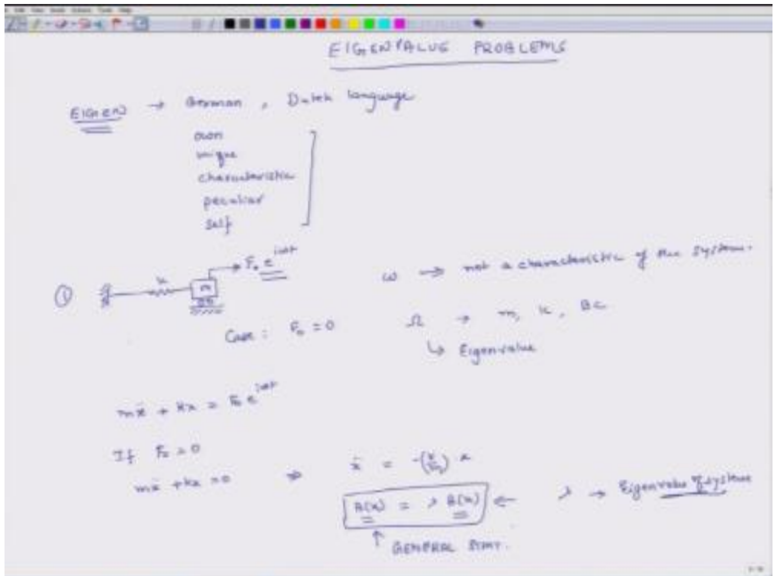


This example spring-mass system.

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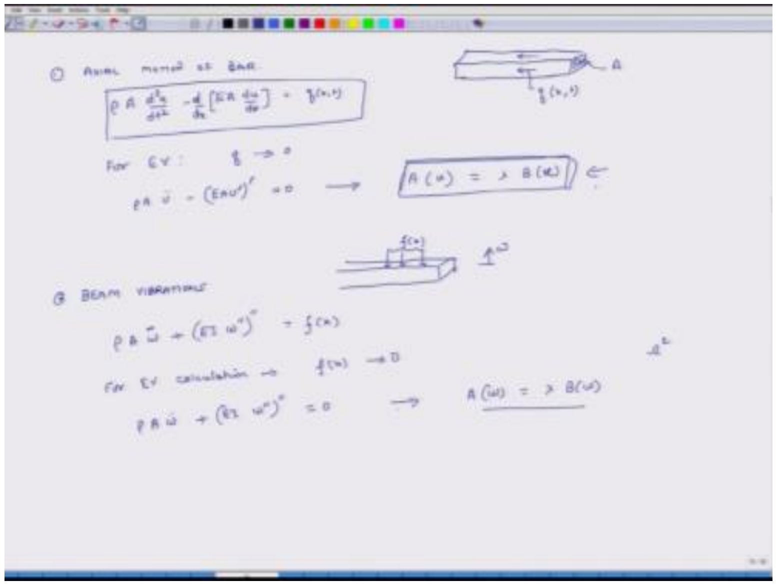

Axial motion of the bar and beam vibrations, it turns out that λ represents the angular frequency square of angular frequency okay, that is what it turns out. But now we will consider case where?

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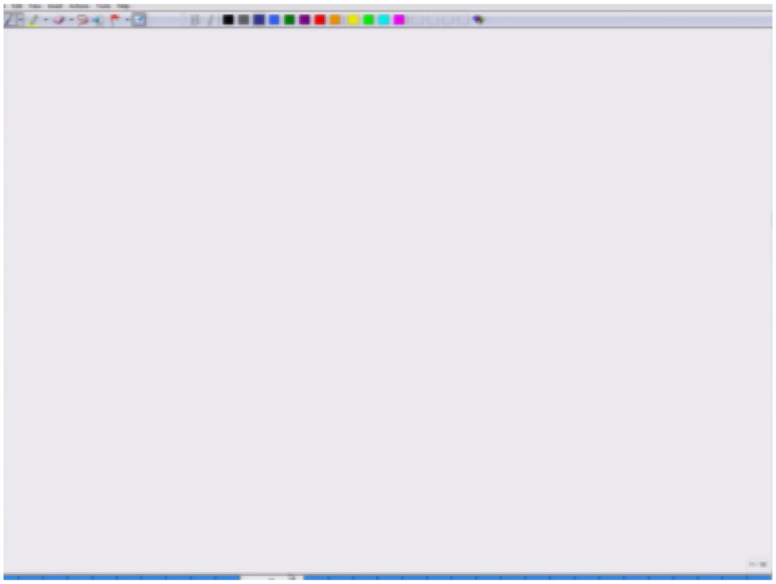

That is not the case where it is different, so here we will discuss.

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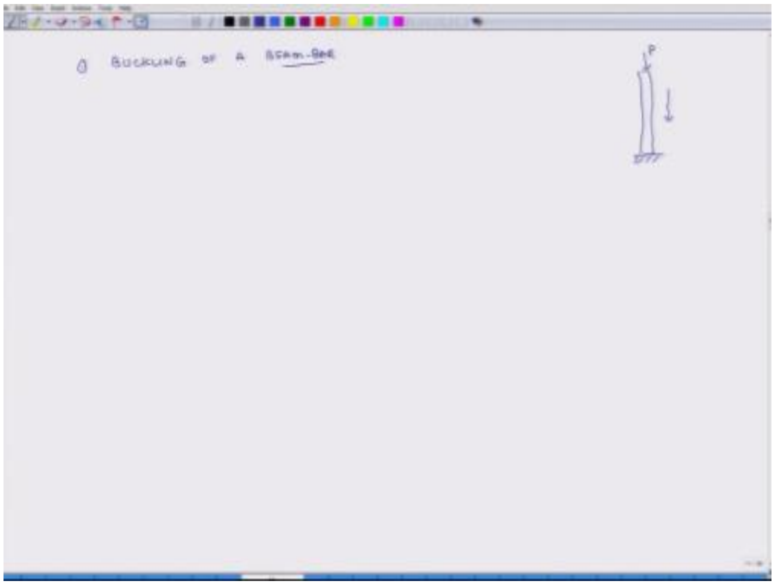


Buckling of a beam bar, what is it? So I have a bar and I am applying some force and if I keep on pressing the bar initially the bar will just get compressed in the axial direction. But once the bar the force exceeds a particular threshold.

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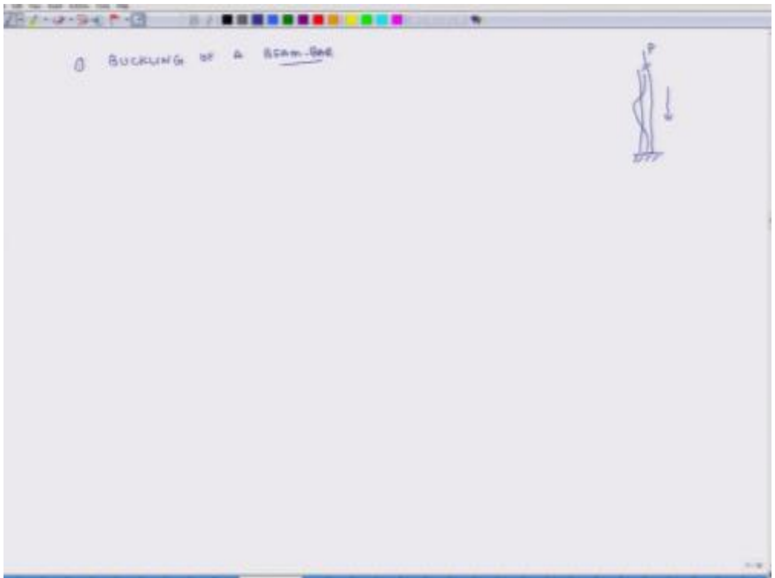

The bar will do this, so it will buckle when the compressive force P exceeds a particular number, that particular number depends on the characteristic of the system it does not depend on P, so this is one case where the Eigenvalue is not related to the.

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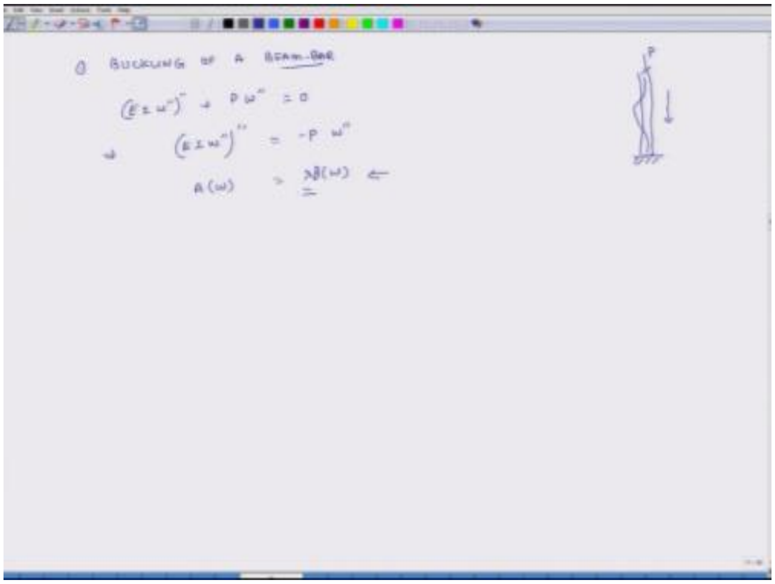

Angular frequency of the system but something different, so for this case the governing equation is (EI w″) + Pw″ 0 or in this case this is very straightforward, EI w″ differentiated twice and the whole thing is differentiated twice is equal to minus P w″. Now here we directly see that this is nothing but of the form A(u), A(w) is equal to λ(w), it is of that form. So this is again an Eigenvalue problem λB(w), so this is one example where the Eigenvalue is not related to the natural frequency of the system it is a different thing, here Eigenvalue corresponds to the buckling load of the system.

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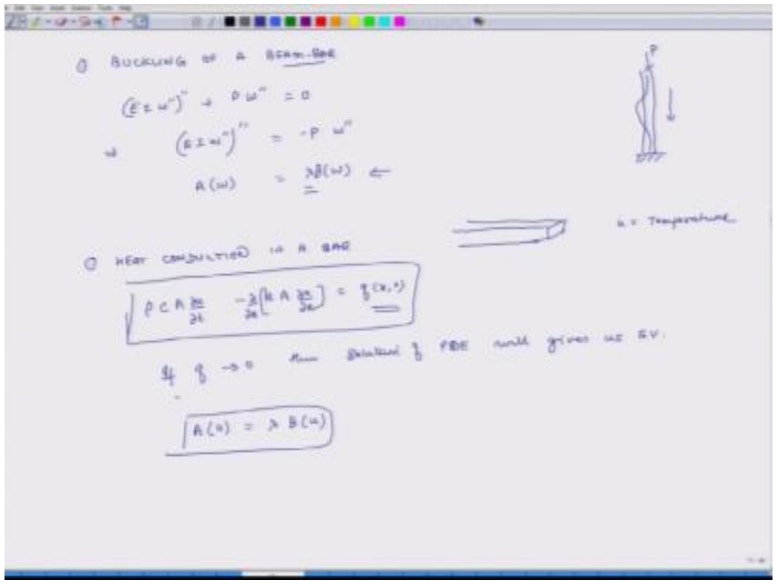

And a last case is for heat conduction in a bar, so here if I have bar with cross sectional area A then the governing equation is ρCA du over dt, so here u is equal to temperature in this case okay. And this minus KA okay so I have to make a small correction it is now partial and same thing here, ∂u over ∂x and the entire thing is differentiated with respect to x, and this equals heat which is generated internally to the system per unit volume so this is the differential equation.

Now in this case also, if I have to compute the Eigenvalue this is the thing which is the q is the term which is external to the system right, so if q is equal to 0 then solution of this differential equation will give us Eigenvalue okay, it will give the Eigenvalue and again in this case also, if we do this variable separable method we will be able to express the overall equation as A(u) equals λ B(u) this form.

So in a sense a quick summary of this thing is that when we are talking about Eigen values we are talking about finding some numbers which are specific to a particular system and they can depend on the material properties of the system, they can depend on the geometry of the system and they will depend on the boundary conditions of the system. What they will not depend is on any other external any external factor, which could be a force or a heat source or an excitation or whatever okay. So that is the overview of Eigen value problems, next what we will do is we will actually solve a particular Eigenvalue problem, and that is for the vibrations in a bar that is what we will solve, and that is going to be the focus of ours for tomorrow's lecture. So today what we have done is we have explained what an Eigenvalue problem is, and tomorrow we will actually solve one of these Eigenvalue problems using the finite element method. So that concludes our discussion for today and we will meet tomorrow. Thanks.


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