Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 39
Finite element formulation shear deformable beams:
Part-II

by
Prof. Nachiketa Tiwari
Dept. of Mechanical Engineering
IIT Kanpur

Hello again, welcome to basics of finite element analysis, today is the third lecture of this week and we will continue our discussion related to shear locking in this lecture but before we discuss any further I wanted to point out two errors, these are important errors which were made in the last lecture so the first error was.

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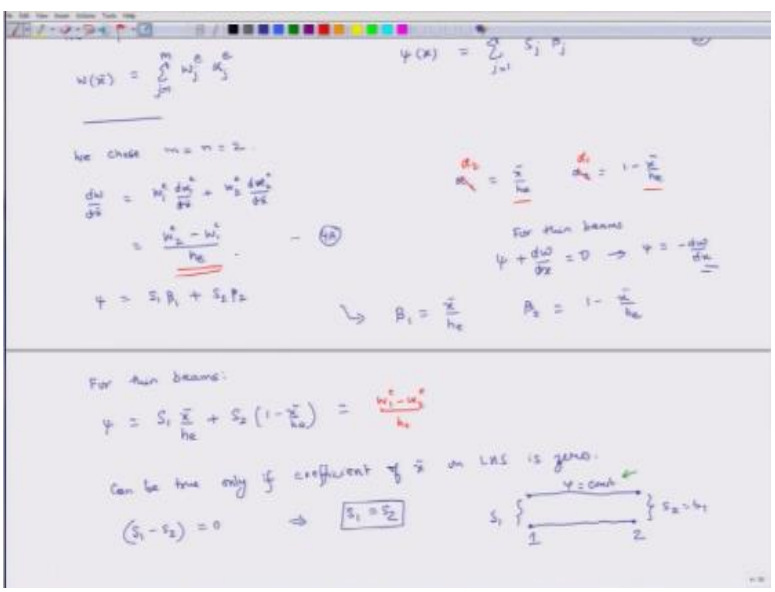

That when I was defining these interpolation functions α_1 and α_2 $\alpha1$ was defined as $\bar{x}$ over h_e actually this should be α_2 and similarly this should have been $\alpha_{2,}$ so α_1 1- $\bar{x}$ over h_e and $\alpha_{2\ is}$ $\bar{x}$ $h_{e,}$ so this was one error and as a consequence of that error.

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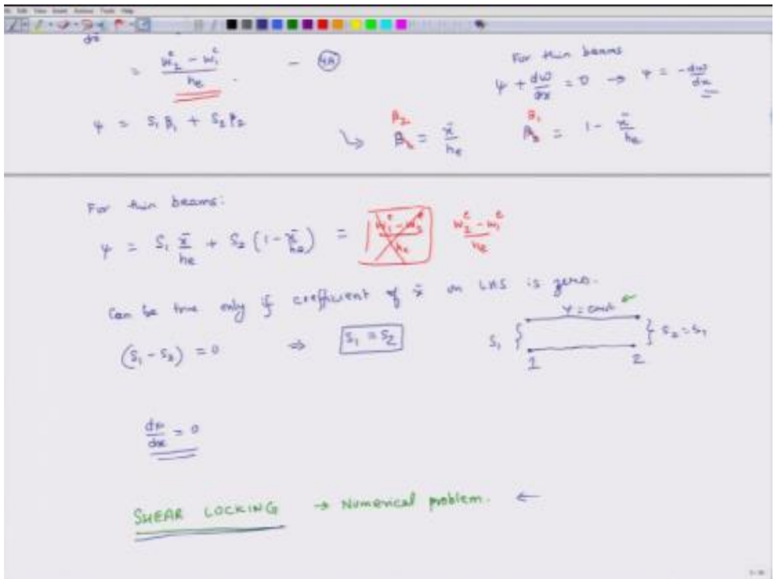

That error got duplicated and here again when I defined β's then β_1 was defined as $\bar{x}$ over h_e actually that should have been β_2 and β_2 which was defined as 1 minus $\bar{x}$ over h_e that should have been defined as $\beta_{1,}$ and the last error which flew out of this was that this term on the right-hand should have been w_2 -w_1 over $h_{e,}$ so the conclusions are still the same but because of these errors you will have a problem of science and that will create problems so but all the conclusions and understanding remains unchanged.

So we had discussed in the last class the cause of this shear locking problem and the fundamental reason why this shear locking comes into picture is because

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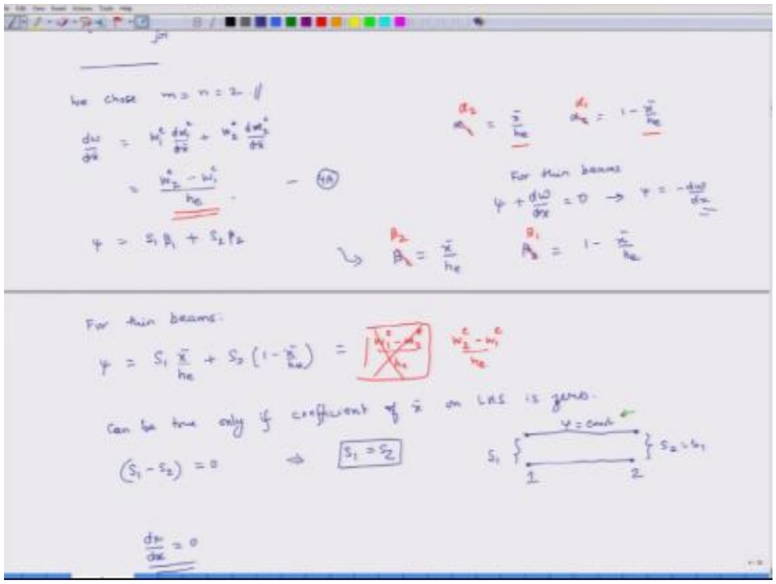

The order of our linear excuse me of the order of the element associated with w and that associated with ψ they are in this case they have been assumed to be the same, if the order of w was one more than that for ψ then this problem would not have happened okay, so the way we avoid this problem is.

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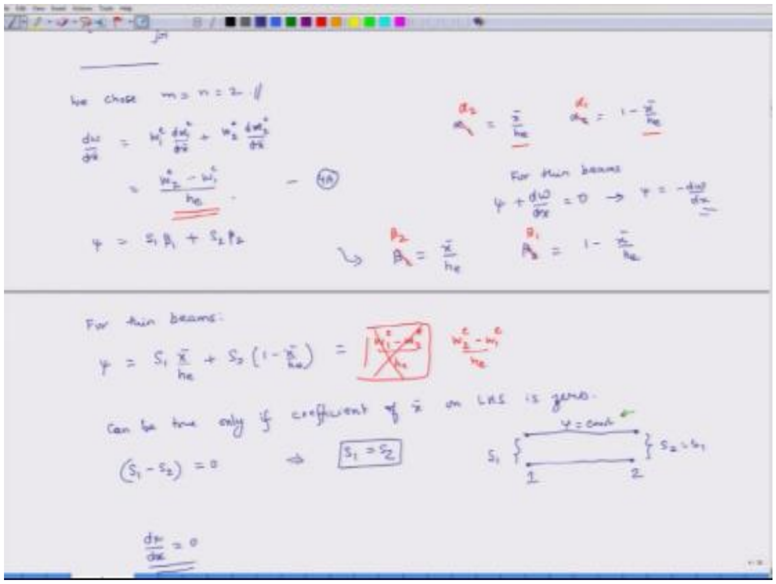

We can avoid this problem into two ways, so we can avoid shear locking in two ways

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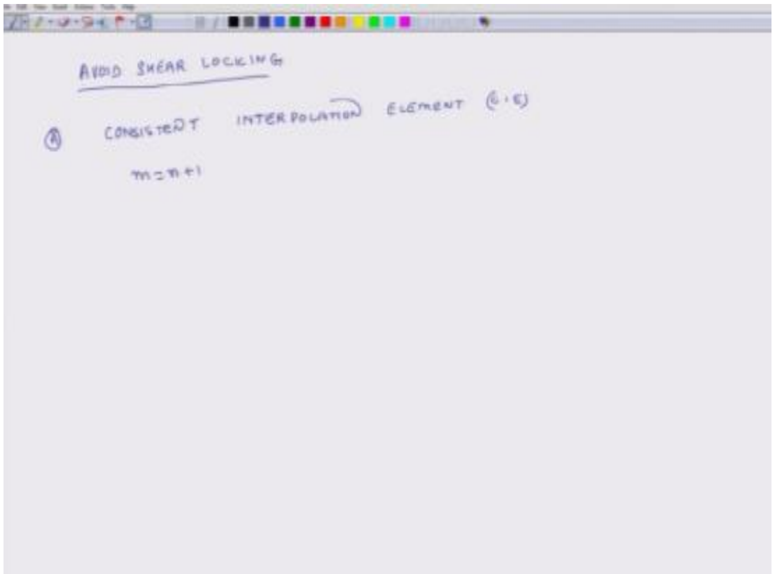

So the first method is that instead of having equal orders that is m and n are same we take m as n + 1 so in this, this method is known as consistent interpolation element C, I, E, so in this case what we do is m equals n +1 so if that is the case then this mathematical locking which we saw earlier.

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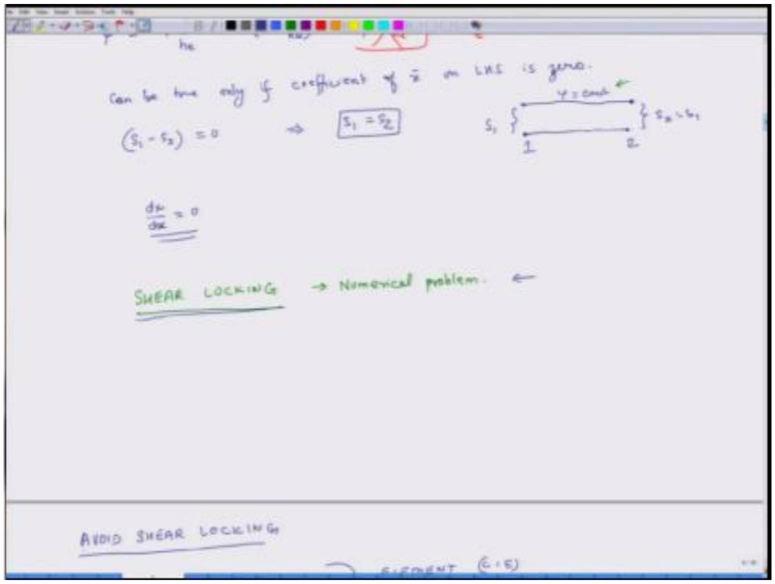

This vanishes.

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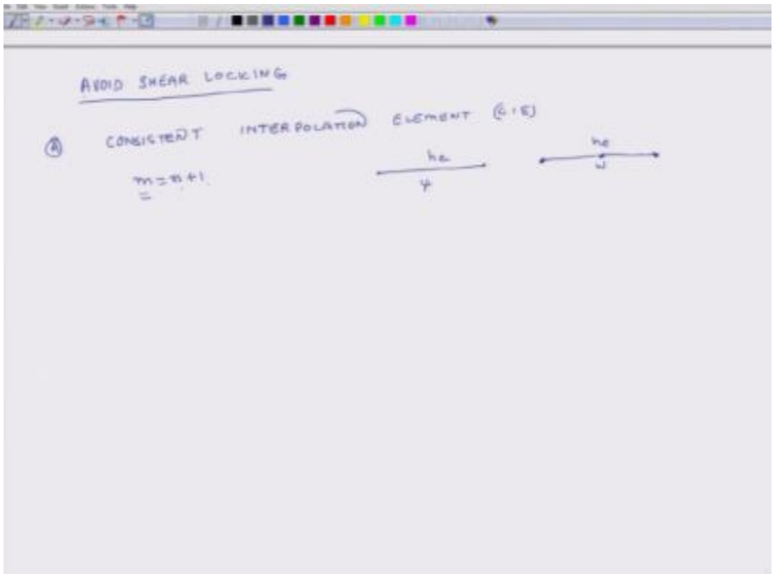

But once we do this we develop another problem and the problem is that suppose you have an element, two noded element and so for, for the same element which is a h_e long it will have two nodes for ψ if it is a linear element but for w the same element will have three nodes when I am doing the interpolation for w okay and I am doing the interpolation for w because m is n +1, now if n was two in case that is the order of ψ then m which is the order of w will be 2 + 1 which is 3 which means that I have to choose a quadratic function.

And if I choose a quadratic function I have an extra node in-between okay, now what is the problem because of this, the problem is that when we I do the assembly of all the elements when I do the assembly then it becomes tricky how we add up all the correct terms at the correct places right because then we have to be very careful and that becomes a tricky exercise but from an interpolation and from a theoretical standpoint.

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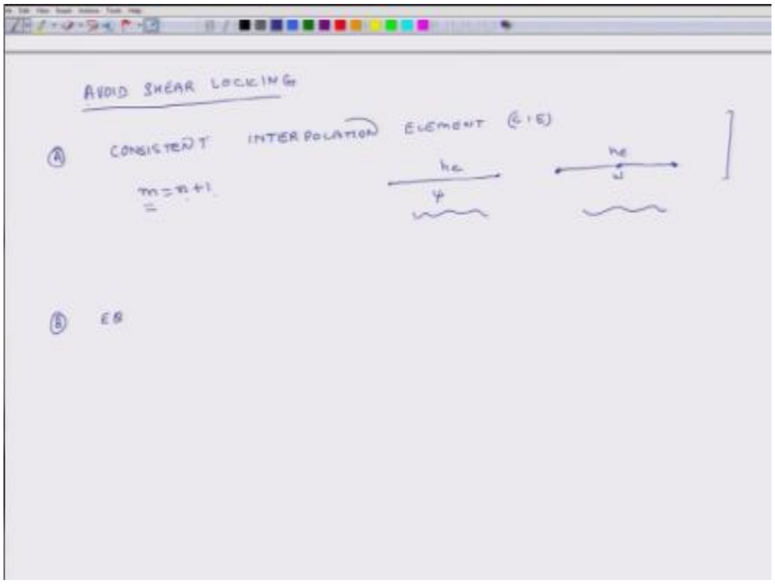

This approach is just fine it is just fine, but to avoid this assembly problem people have come up with another method and that is called equal interpolation, so in the first case it is consistent interpolation what does it mean, consistent interpolation means that whatever interpolation order is it is consistent with the mathematics required right and the mathematic require that m should be.

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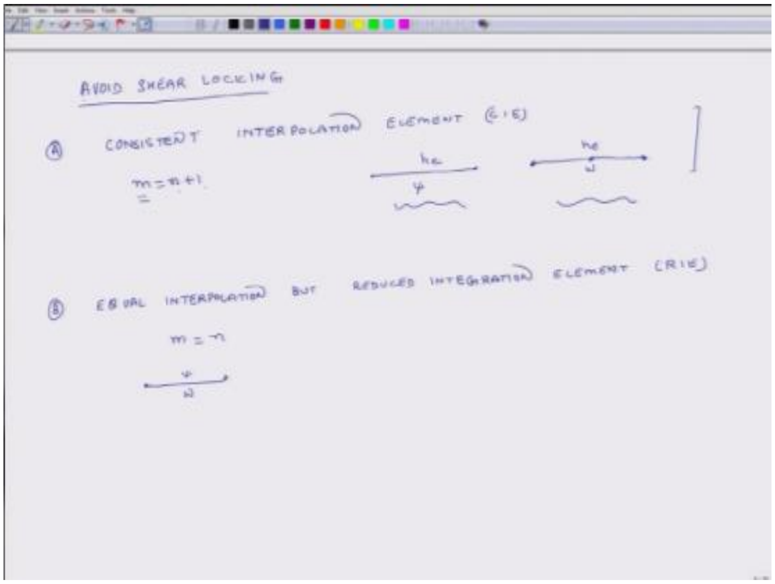

Equal to n plus 1 so that is the consistent interpolation method, in the second method which avoids this assembly problem this method is known as equal interpolation but reduced integration, but reduced integration element RIE so here we, equal interpolation means that we still take m equals n, m equals n but if we just take m equals n and did nothing then we would have the shear locking problem which we have discussed.

So but we still want to avoid the shear locking problem and because m is equal to n the number of nodes r_2 whether it is ψ or whether it is w or if it is, if n is equal to three then it is if n is equal to three then it is a quadratic element for w as well as ψ, so in terms of assembly that issue is not there but then we still have shear locking so to avoid shear locking.

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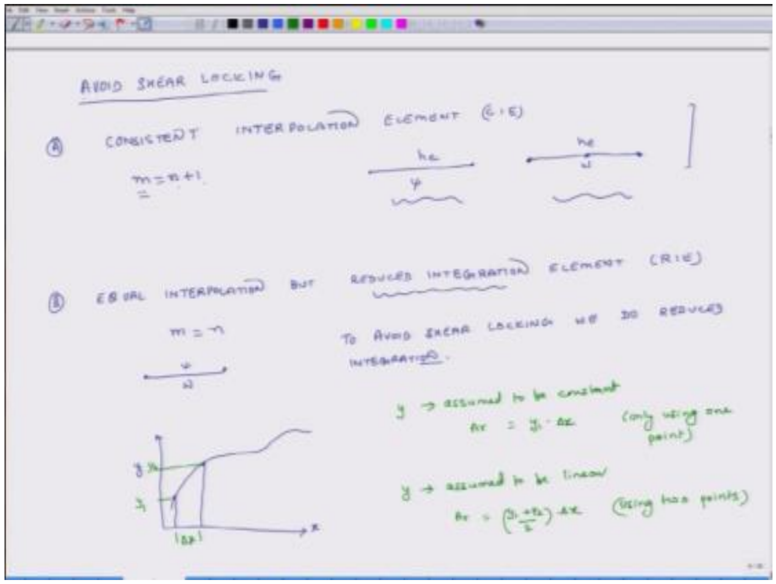

What we do is we do reduced integration and I will explain that in a moment okay, I will explain that in a moment to avoid so we still take equal number of nodes order is same m is equal to n but to avoid shear locking we do something known as reduced integration on some specific terms, so what does reduced integration mean to understand this consider this, so suppose I have a curve okay and I want to find the area under the under this curve between two points okay.

So let us say this is my x and this is some function y so I want to find the area which is here okay. Now if I consider only one point when I am integrating it then I will get one answer what does that mean, so suppose this is y_1 and this is y_2 okay if I assume, if I assume that y is constant then the area is if y is assumed to be constant then area is equal to y_1 times suppose this Δ x, Δ x so this is one estimate of the area it may not be accurate but this is one estimate of the area and in this case I am only using one point.

Right I am only using one point over the element to get the area one point in this case is y_1 okay, if I say that Y is assumed to be linear then area is equal to y_1 + y_2/2 times Δx okay. Here I am using two points so I am using two points for integration right, in the first case I am using only one point for integration in the second case I am using two points for integration okay. I can extend this logic.

If I assume y to be quadratic then I will consider three points in the domain when I am trying to find the area and we can work out the formula that is not the problem right we can.

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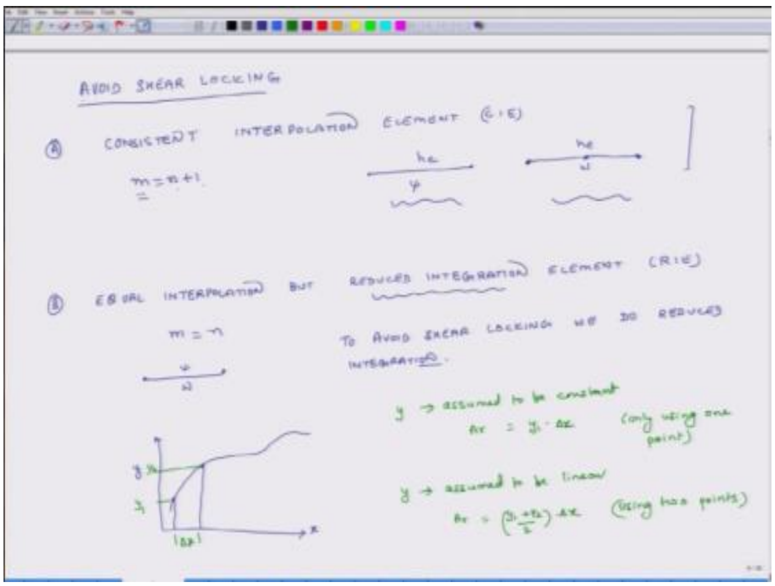

So we need three points so these, so the number of points which we consider when we are doing this integration it tells us whether the function is constant or it is a linear element or it is a quadratic element or it is a cubic element. The number of points which are considered for integration defines whether the function is linear or constant or quadratic or so on and so forth okay.

So these see these number of points in and that is when we are doing so in our.

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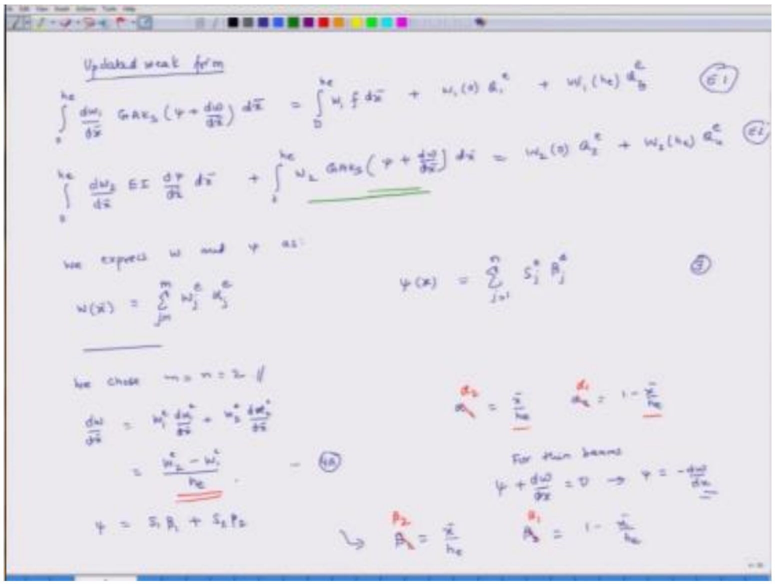

Differential equation what are we doing we are integrating over the domain of the elements some functions right that is all we are doing we are integrating some functions over the domain of the element.

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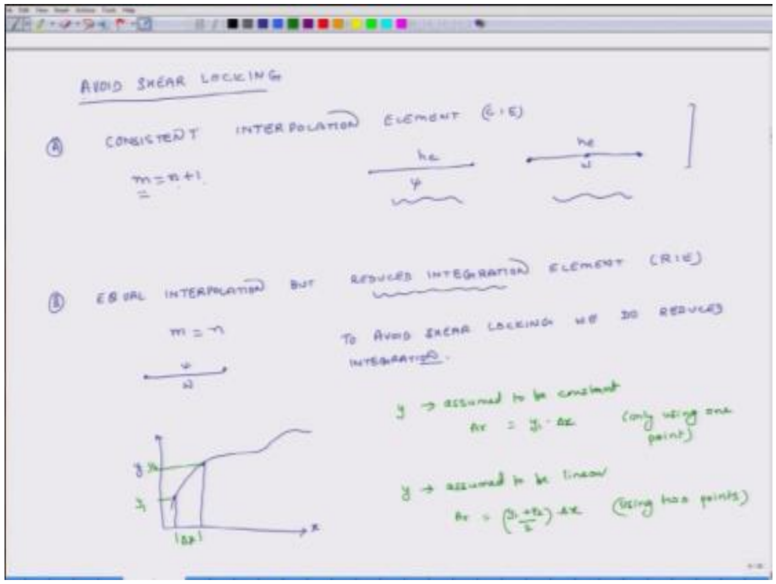

So what we are doing here is we are using equal interpolation, so that gets rid of this problem related to unequal number of nodes for w and ψ but for.

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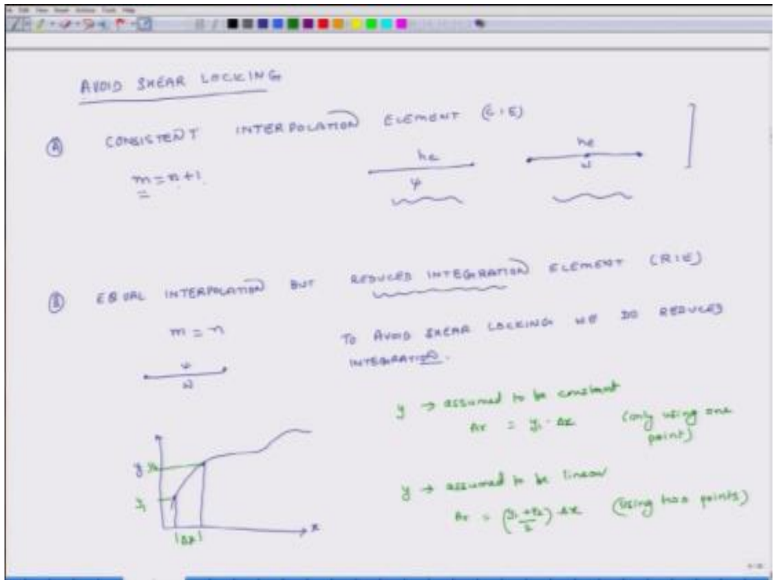

Specific terms which have to be integrated which relate to which caused shear locking so the integral the numerical integration of a function is achieved by, it depends on the number of points considered in the domain of integration when we are doing that thing okay, so that is what we are doing.

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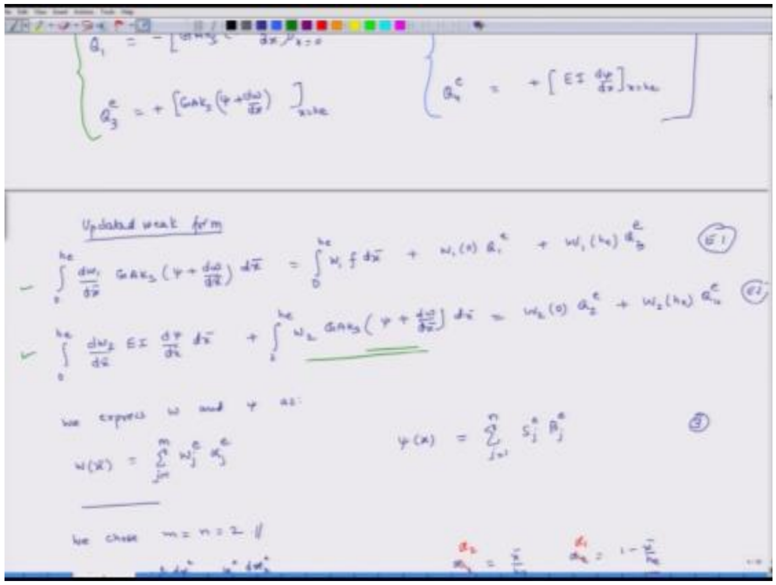

So what we are doing here is that there may be in our differential equation or in the weak form some terms so this is, these are the two weak forms and some of these terms may be causing shear locking and the shear locking is happening because we have assumed that M equals N equals 2, if this equality was not there if M was more than n we could have avoided this.

Now in this case we still want m to be same as n and that is fine but then.

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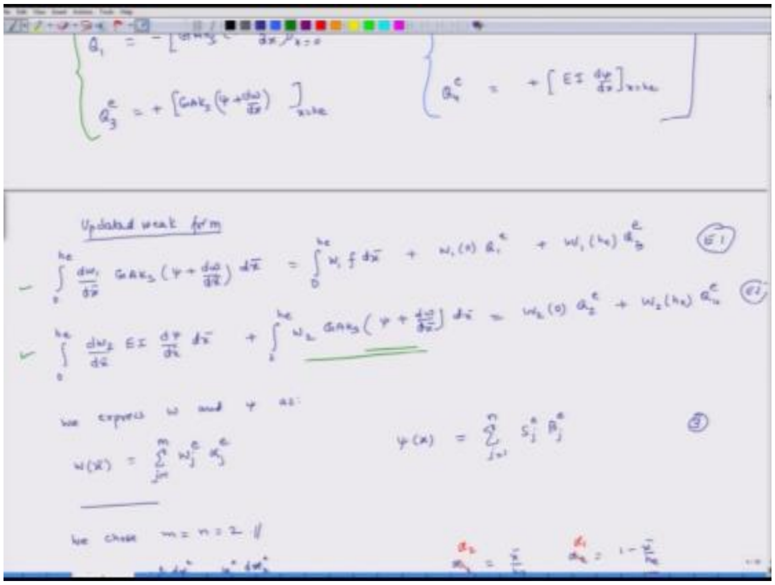

If there are terms, for instance this particular term which causes shear locking then why when I integrate the contribution of this term I consider one less number of point when I am doing the integration. Now you will how do we do that we will learn it but when I am integrating this term.

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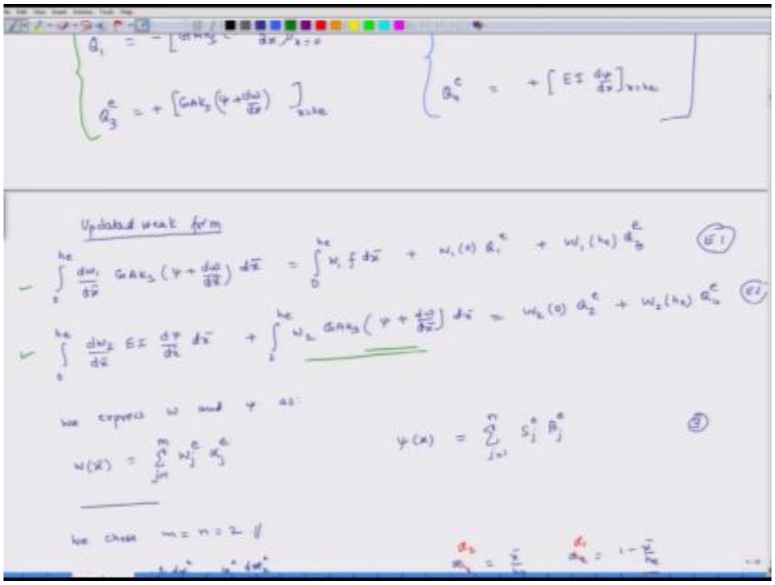

Then I evaluate the function at an extra number of points, when I integrate this first term then I evaluate the function at one less number of points so what does that help us, it helps us maintain the number of nodes is same for ψ as well as w so we do not have the assembly problem and we still are able to avoid.

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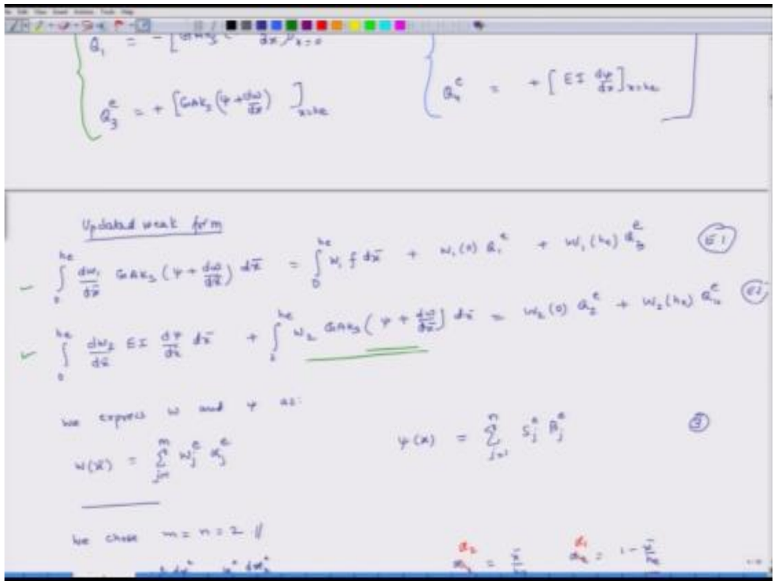

Because we have for specific terms we are we are reducing the order of integration okay and we will learn a little bit more about this when maybe next week when we discuss numerical integration procedures called Gaussian by using Gaussian Quadrature method. there also we evaluate the function at a certain number of points and if it is a constant function then we evaluate at one point, if we want to do a linear quadrature then we do it at two points and so on and so forth.

So based on how many number of points we evaluate the function and use those values to compute the integral that is the way.

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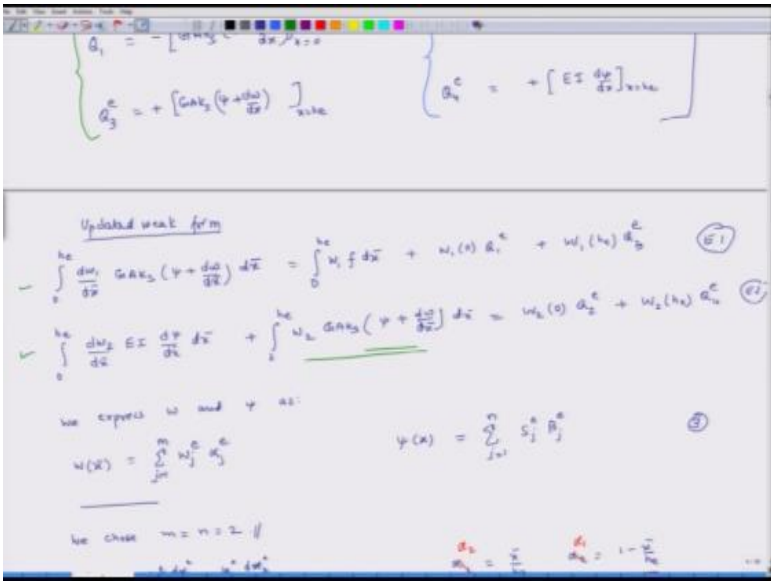

We solve this numerical problem of shear locking okay, so that is how we accomplish this reduced integration, so we reduce the level of integration on specific terms but when we are doing interpolation we still have same number of functions okay, so this method is called reduced integration method or the element on which this function is carried out this is called reduced integration element. So I think what we will do is we will close this discussion at this point of time and in the next class we will actually do an exercise on reduced integration so that it becomes more clear to all of us, so thank you and we will meet tomorrow, bye.

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