Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 37
Shear deformable beams

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Hello, welcome to basics of finite element analysis, this is the seventh week of this particular course and what we will do in this week is we will start this week by discussing a special type of beam, so in last week we had developed relations finite-element relations for Euler Bernoulli beam, and this week and next few lectures we will be discussing a different type of beam known as shear deformable beam, so we will develop formulation for our sheer deformable beam, and then in the remaining part of this week we will start discussing and addressing Igan value problems. So today's lecture we are going to discuss shear deformable beam.

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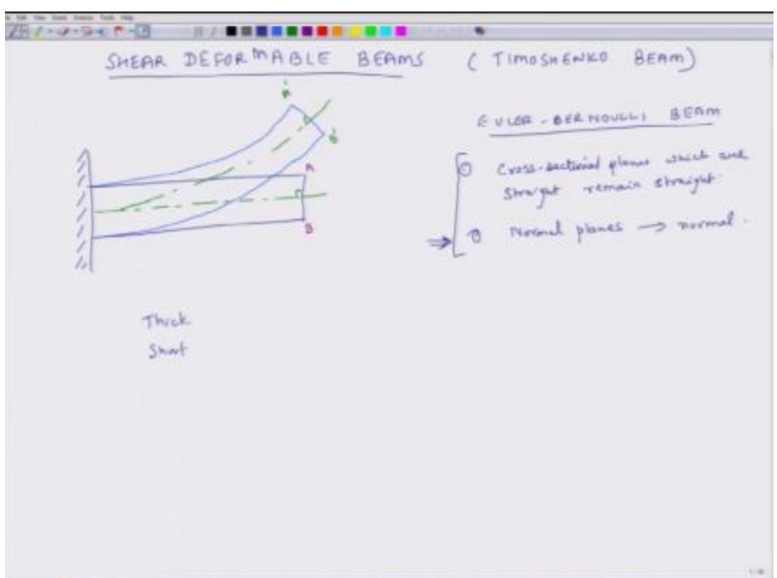

And these are also known as Timoshenko beams, and the theory which describes this particular beam is known as Timoshenko beam theory. So first let us recap what is an Euler beam, so suppose I have a straight beam and let us say this is a cantilever, so it is rigidly fixed at one end and when I apply a force then this beam bends and the light blue color depicts the bent shape of this beam.

Now, so the line in green is the neutral axis and also it is both in undeformed, n deformed situations, now when we have so this beam which I have depicted here is known as Euler Bernoulli beam, and the assumption in this Euler Bernoulli beam is that so the end of this beam and we will label it as A, B. So the end of the beam which is this line A, B is at 90 degrees to the mid plane of the beam disunited in the undeformed position and it is a straight line, it is a straight plane and it is at, it is perpendicular to the mid plane of the beam.

And once the beam has deformed it has moved to positions A' and B', and once again the angle between the end plane and, and the mid plane and the cross section of the beam it still remains 90 degrees so, so there are two assumptions in case of Euler Bernoulli beam, one is that cross-sectional planes which are straight remain straight, and the second is and the second condition is that planes, these planes if they are normal so if there are normal planes then they remain normal.

So if these two conditions are met then our beams go fall in this category called Euler Bernoulli beams okay. Now what happens is that especially if the beam is very fat or you can call it thick or if it is too short then this normality condition may not be exactly true. So I will give you a small demonstration, so we will consider this as a short beam and this is the cross section of this, so what I have done is I have made a red line and that let us say that represents the mid plane of the beam and the beam is straight.

So this edge of the beam is normal to the mid plane, and now this is a relatively short beam or you can also call it a thick beam does not matter and when I bend it, when I bend it and if because the beam is thick, the angle between the mid plane and the normal which was earlier it no longer remains normal to the mid plane okay. So, so that is what happens in case of thick beams.

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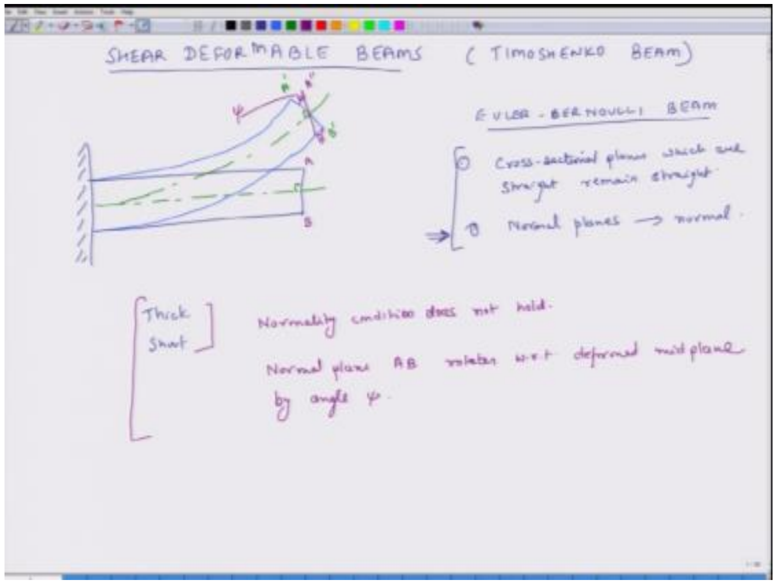

So in case of thick beams what happens is that this A, B, A', B' line is not necessarily at 90 degrees to, to the mid plane so what we have is a somewhat different orientation. So we will, so let us say this is A″ and B″ so in that case for thick and short beams this normality condition does not hold and as a consequence we have this extra rotation and I will call this rotation ψ, so we have this extra rotation, so normal plane AB rotates with respect to deformed mid plane by angle ψ.

And this happens because there is a shear deformation of the thing, so in the Euler Bernoulli beam because this angle remains at 90 degrees there is no shear deformation of the beam if there is a, because whenever we are talking about shear deformation the angles move and they no longer remain normal right, so there is a shear deformation

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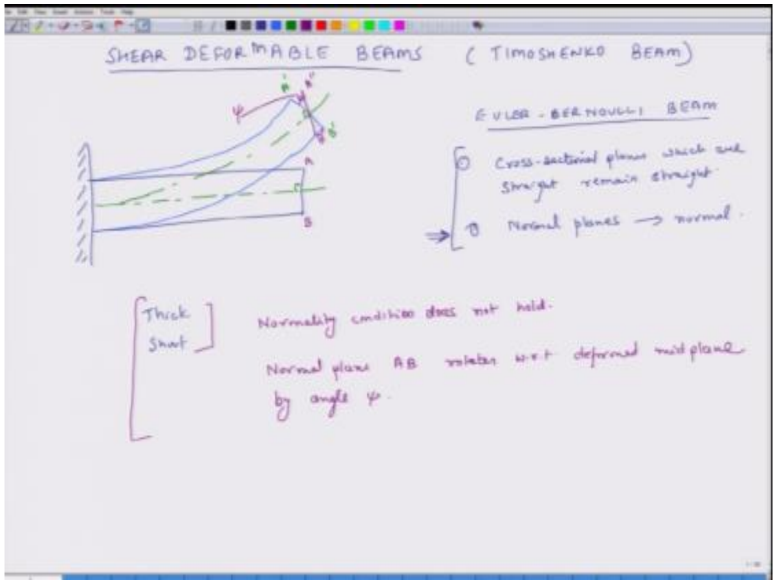

So that is why these types of beams are known as shear deformable beams, and typically you see this shear deformation effects in thick beams or short beams, mathematically they are the same.

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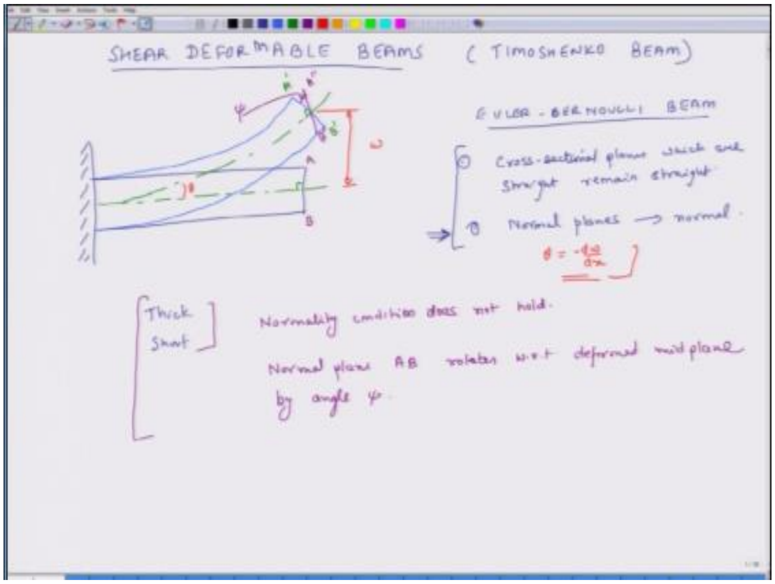

Another related thing is that in a regular beam or in an Euler Bernoulli beam, let us call this angle θ and let us say this displacement in the vertical direction is w, then for Euler Bernoulli beam θ equals dw over dx right, and so this is equal to dw over dx, and because I have I consider θ as positive which is clockwise so there is a negative sign here. But then again this condition is preserved in an Euler Bernoulli beam, but this θ may not be necessarily same as the rotation of a line AB with respect to line A'B'.

So in an Euler Bernoulli beam line AB rotates by the same angle but in our shear deformation, shear deformable beam that rotation is not same as θ, which is equal to dw over dx. So this is, so this is the,

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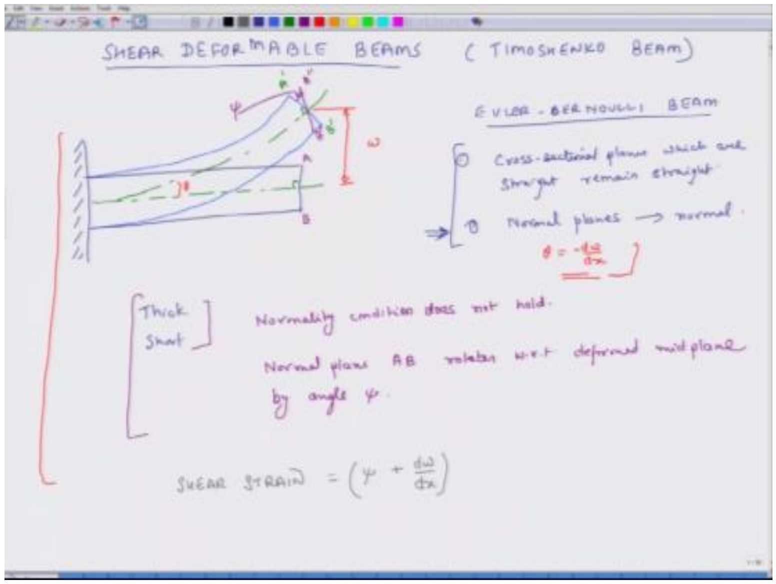

Detail about a typical shear deformable beam, and in this case the shear strain is equal to ψ + dw over dx that is the shear strain, so with this background what we will do is, we will now develop a finite element formulation for this type of a beam, and the reason I am doing.

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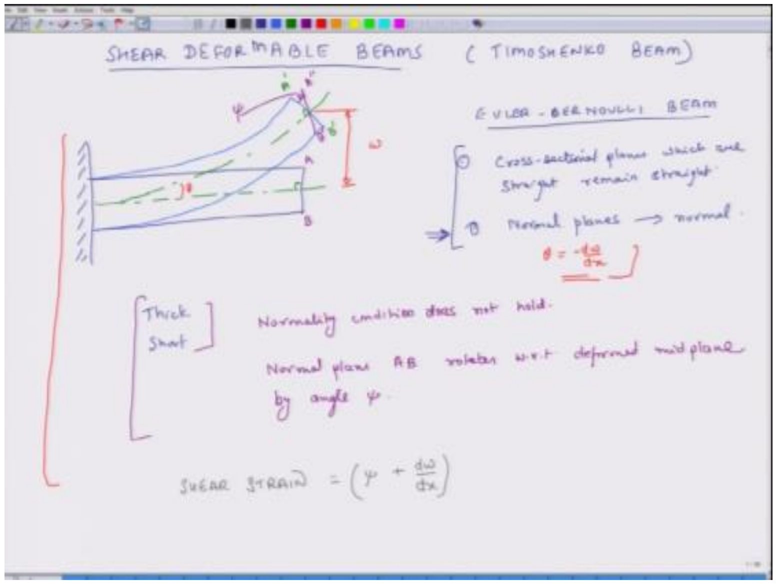

This is that not only we get to learn a little bit more about beams, but also because of the fact that in this particular case when we create our stiffness matrix we have to resort to some different methods which are known as reduced integration methods when we are computing the stiffness matrix, so I wanted you to become aware of these methods also as you get deeper into finite element analysis.

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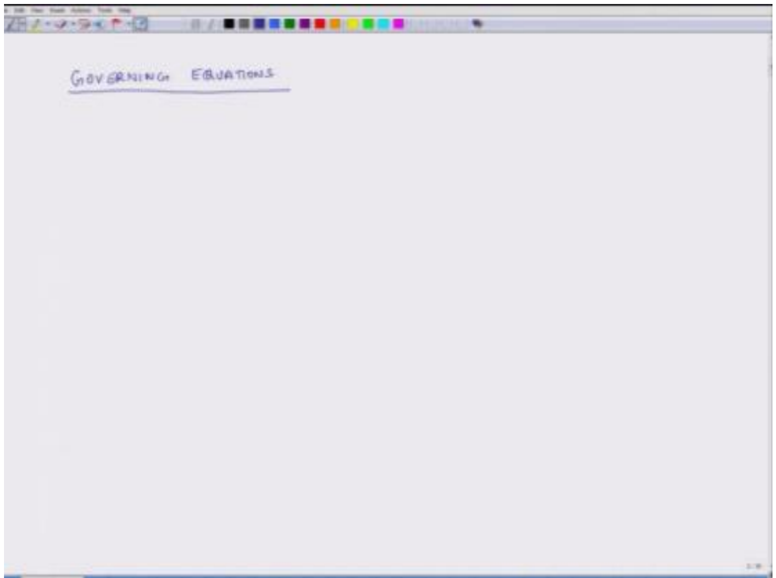

So what we will do is we will first write the governing equations for such a beam. Now in case of shear deformable beam, in case of an Euler Bernoulli beam there was only one unknown which was w right and the rotation could be expressed earlier in terms of w. But in case of shear deformable beam not only we have the vertical displacement but the rotation of a line element is not directly dependent on just w alone so there is on independent parameter also known as ψ, so we have two unknowns w in ψ, and consequently we need two equations of equilibrium. So these equations of equilibrium are.

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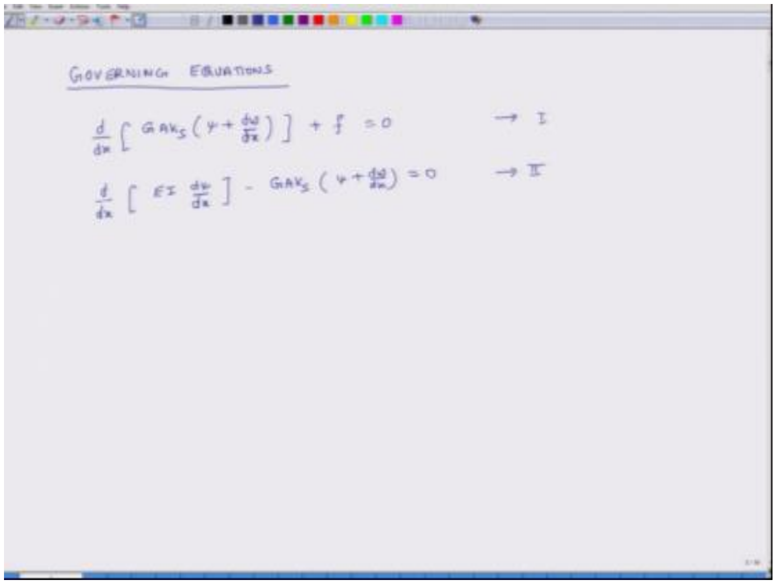

d over dx, GAK_s times ψ plus dw over dx plus f equals zero, and the second differential equation is EI dψ over dx minus GAK_s ψ plus dw over dx equals zero okay. So this equation the first one relates to the equilibrium of forces and the second equation relates to equilibrium of moments okay. So first one ensures the w direction degree of freedom has equilibrium, and in the second case the rotational degree of freedom is addressed.

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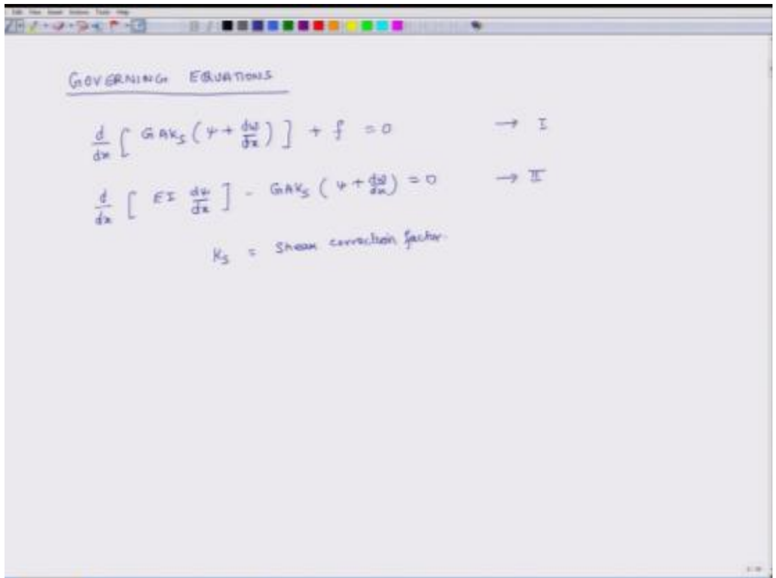

Now here we have we have this term called K_s and K_s is a constant and it is known as shear correction factor. So this is a correction factor appear number and the way we figure out this number is either through sophisticated analytical methods or through experiments based on the cross section of the beam and the length of the beam. We actually evaluate this

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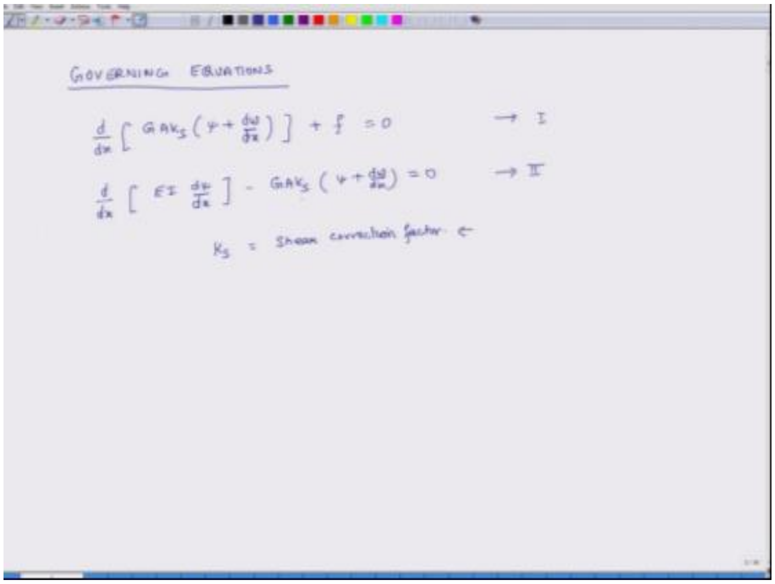

Correction factor and we incorporate into the equation. So it is some number some factor and we incorporate it based on some analytical methods or based on some experimental approaches. So that is our shear addition factor.

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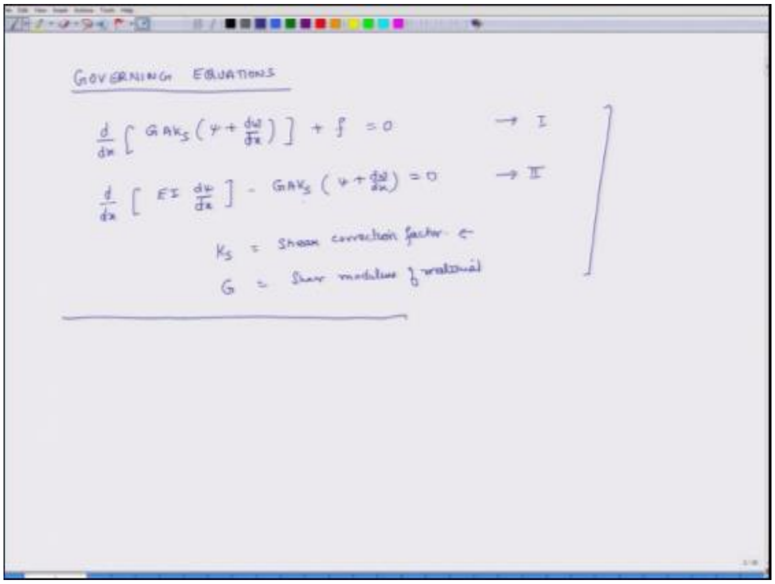

G is the shear modules of, of material and A is the cross sectional area of the beam. So these are the governing equations and next what we will do is we will develop a weak formulation for these governing equations. So one, immediately one different thing which you notice about this particular problem is that in all the problems which we have discussed still so far we had only one equilibrium equation.

And we were handling, we were, we had figured out how to handle these, this one single equilibrium equation and how to develop its weak formulation. Now here we have two equilibrium equations and we have two fundamental unknowns ψ and w, so through this example not only we will learn a little bit more about beams and the theory of beams.

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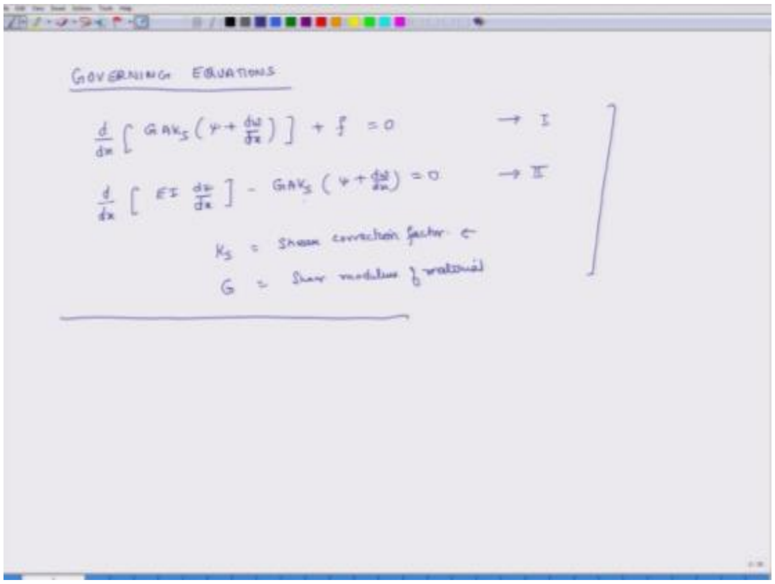

But we will also learn from in context of finite element analysis as to how multiple degrees of freedoms, systems are handled.

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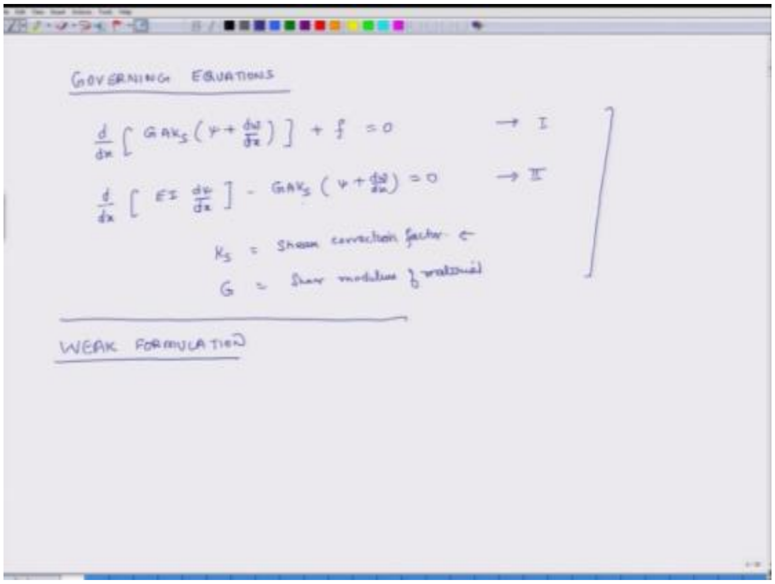

So next what we are going to do is we will develop a weak formulation okay, oh by the way.

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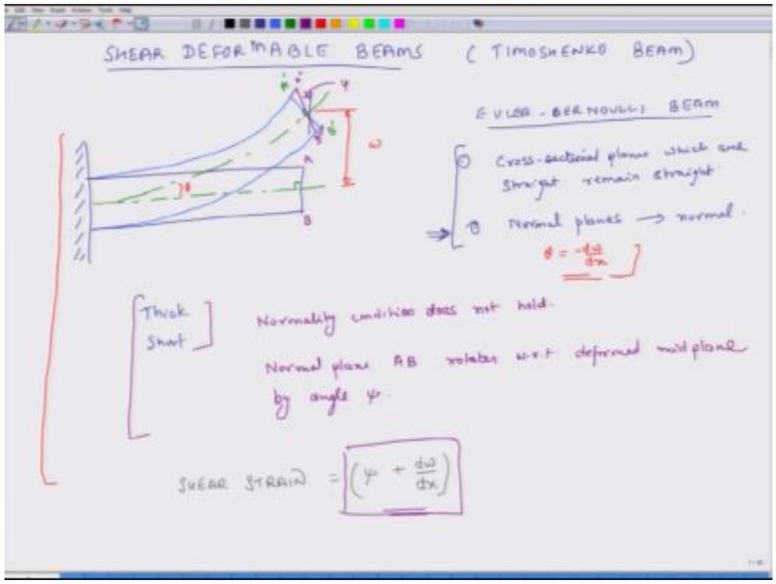

I think there is a small error in this that this angle which is ψ is with respect to the original. So this was my original situation, so ψ I must apologize for all this mess, so this is the deformed beam and this angle. So this is the deformed beam in case of Euler Bernoulli, and if it is a Timoshenko beam or a shear deformable beam then I had said that this A' it is actually A″ and B″ and this angle is ψ, this is important note yeah. Because only then this will be my shear strain, only then this is going to be my shear strain and for a Euler Bernoulli beam if I add these two up it will be zero okay.

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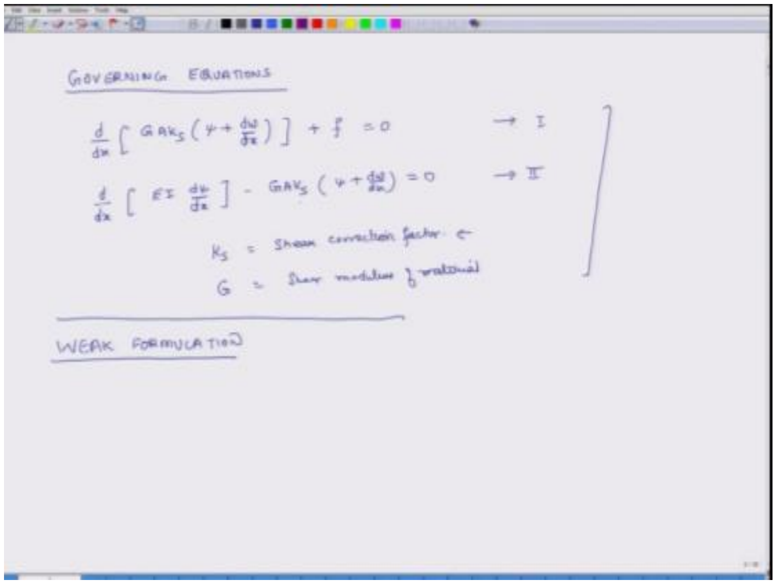

So with that correction now we go back to the weak formulation and what we note here is that there are two variables ψ and w, and the way we construct a weak formulation is that we first construct develop a relations for the residue multiplied by a weight function, integrate the weight function over the whole domain, in this case domain of the element and then equate that integral to zero, and then do integration by parts to develop the weak form, so that is what we will do here.

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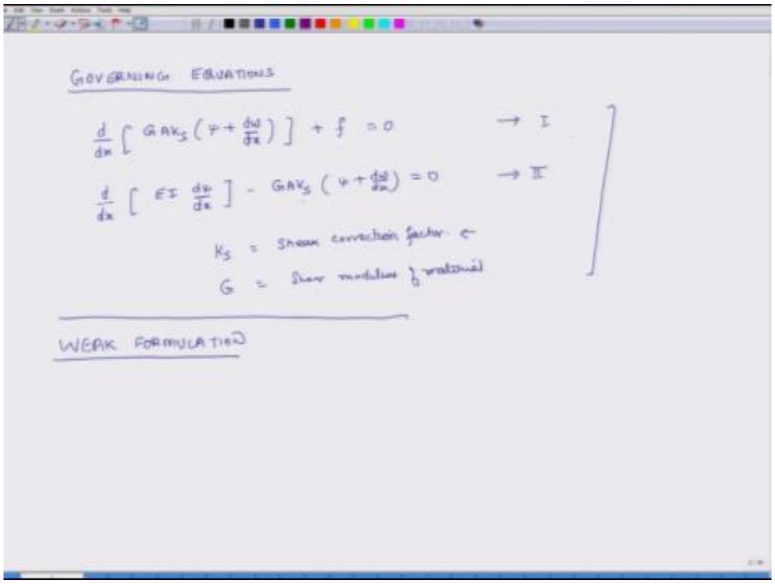

And because we have two equations we will have two statements for weak formulation, so the first equation.

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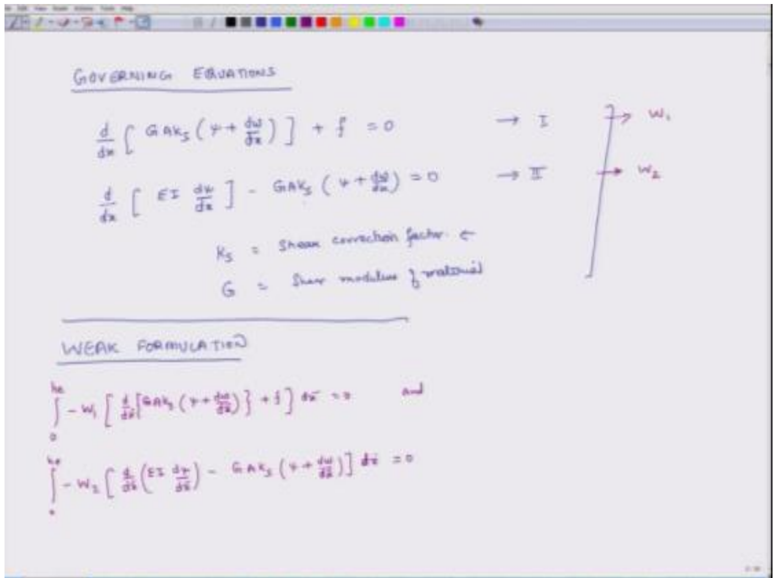

We are going to multiply by a weight function called w_1, and the second equation will be multiplied by another weight function call w_2 okay. So let us say I am going to integrate it over the domain of an element, then this is equal to minus w_1 d by dx $GAK_s\psi$ + dw over dx plus f dx equals zero, and the second statement for zero residue is minus of w_2 which is the weight function times d/dxe EI dψ /dx X minus GAK_s times ψ+ dw over dx is equal to times d$\bar{x}$ is equal to zero, and because I am integrating it over the element using local coordinate system I am going to replace x by $\bar{x}$ okay.

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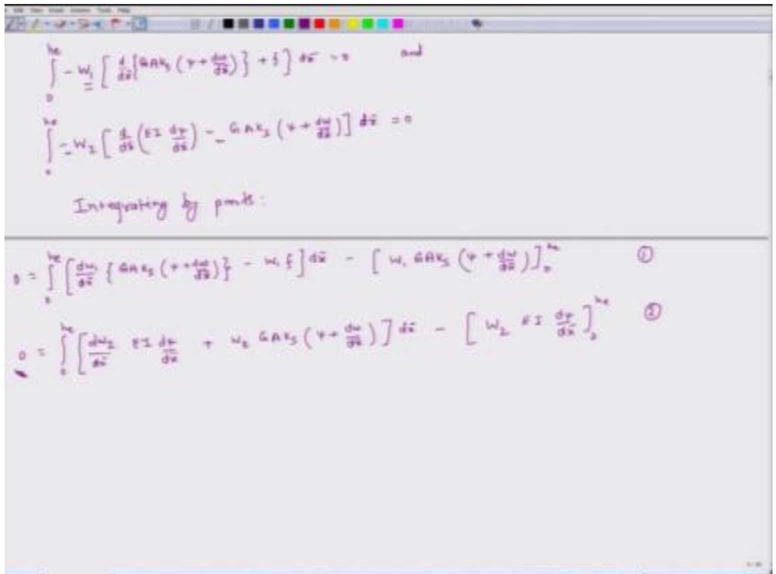

So please note that w_1 and w_2 are weight functions so they are not constants they are weight functions and they are mutually independent, they are mutually independent so it is not that w_1 is a linear multiple of w_2 or something like that, so it is an independent weight function. So now what we do is we integrate these equations by parts, so what we get is again two equations 0 to h_e, and here in the first equation we shift the differentiability operator d over dx to w_1, so we get dw_1 over the $\bar{x}$ times GAK_s ψ plus dw over $d\bar{x}$ minus w_1f, the entire thing is being integrated over the domain, and this is equal to zero, and then I have some boundary terms when I do integration by parts and the boundary times are w_1 GAK_s ψ plus dw over $d\bar{x}$ and this boundary term will be evaluated at the limits of the element 0 and h_e.

So this is equation1, and the second equation is again integral from 0 to h_e dw_2 over $d\bar{x}$ EI dψ over dx plus, so this minus and this minus becomes plus so I get w_2 GAK_s ψ plus dw over the $d\bar{x}$ and this entire thing I am going to integrate over that domain and then again I have a boundary term w2 EI dψ over $d\bar{x}$ 0 to $h_{e,}$ this is equation 2. So the 0 is already there, so this is there zero is there okay. So I think what we will do is we will close our lecture today and tomorrow we will continue this discussion in the next lecture okay. Thank you.

<u>Acknowledgement</u>
Ministry of Human Resource & Development

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Co-ordinator, NPTEL IIT Kanpur

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