Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 36
Boundary conditions for Euler-Bernoulli beam

by
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Hello, welcome to basics of finite element analysis, today is the last day of the sixth week of these lectures and what we will discuss today is how do we go around applying boundary conditions in context of finite element formulation of an Euler Bernoulli beam. So first we will look at different types of boundary conditions which are possible.

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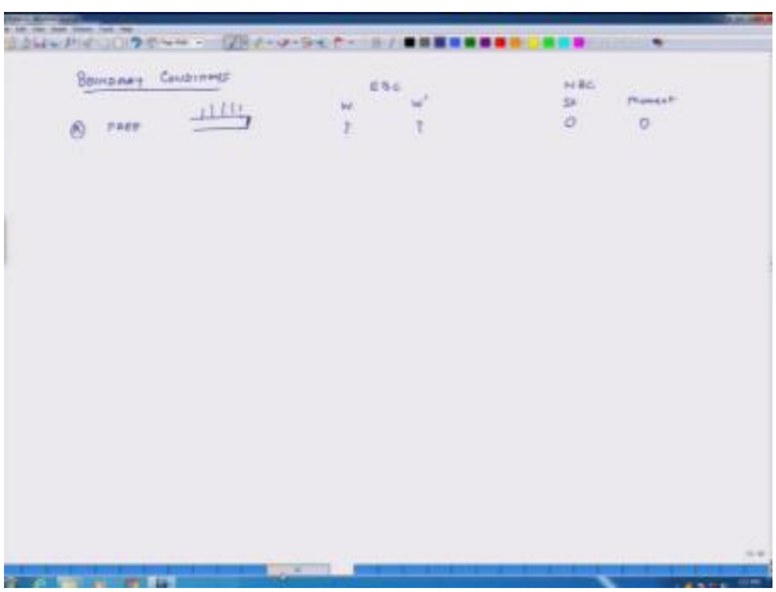

In context of beams okay, so the first condition is free end, what is the free end, suppose there is a beam end which is free and then we will look at essential boundary conditions and natural boundary conditions. There are two essential boundary conditions which are related to W and slope and then there are two natural boundary conditions which are related to shear force and moment right. So in case of a free end we do not know w, suppose I am having some load here we do not know what is the value of load w, we do not know slope also.

But what we do know is that at this end shear force is zero and also the moment is 0 okay so, so if this end corresponds to node2 then our equation.

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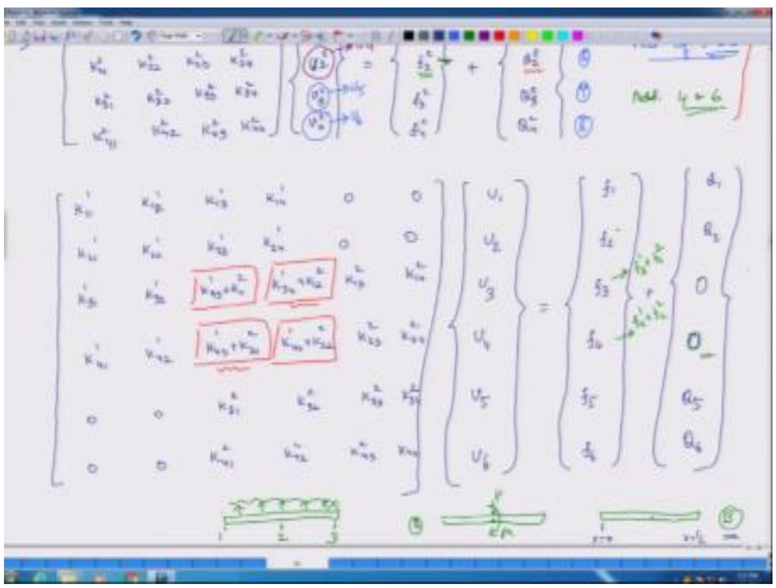

If that is the extreme end of the beam, then in our equation Q_5 and Q_6 will be prescribed as zero, Q_5 is the shear force and Q_6 is the moment okay.

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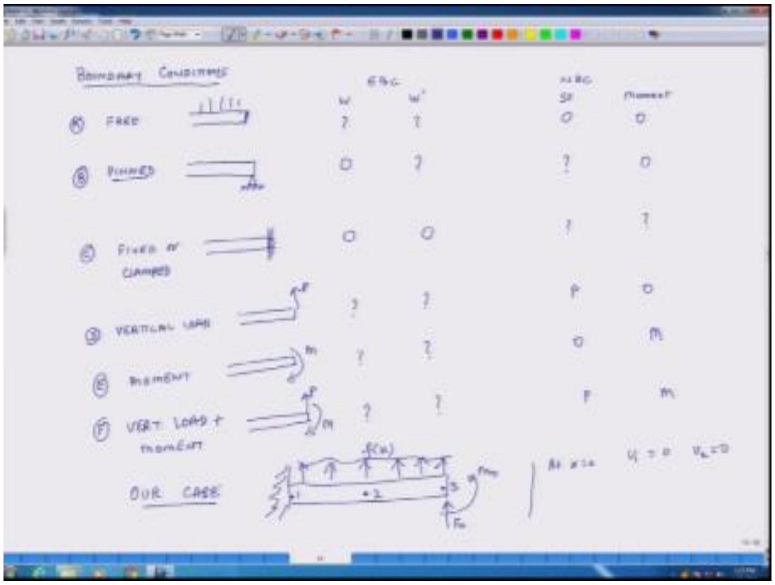

So we have to look at each of the boundary and see what things we are going to prescribe, the second condition possible condition could be pinned end, so pinned end could be something like this, in this case the value of W is 0, and the slope is unknown and because we can at any point the shear force now we do not know because this pin is exacting some shear force we do not know.

So this is unknown, but the beam is free to rotate at the pinned end so moment is 0. The third boundary condition is fixed end. So what is the fixed end, it is something like this and here I have rigidly fixed it, so in this case w is 0 and so is slope but shear force is unknown and moment is unknown. The fourth condition is vertical load, what is the case here I have end here and I apply a force P, in this case w is unknown and so is the slope. But the shear force is p and moment is 0, moment at the end which I am applying external is zero.

The fifth condition is moment, so was I am putting a moment of m here, then again w and slopes are unknown, but this is 0 and this is m, and then I can have another condition which is vertical load plus moment. So I am applying a force P and I am applying a moment M, then again w and its slope are unknown but these entities we know okay. In our case let us look at our case, we will consider that the beam is clammed at one end and it has some uniform distributed load or it may or may not be uniform but it is f(x), this is node one, this is node two, this node three and I have a moment of m_0 and I also have a force of f_0.

So for this situation this case corresponds to that at x is equal to 0 right, w and w′ are 0 which means that u_1 is 0 and u_2 is equal to 0.

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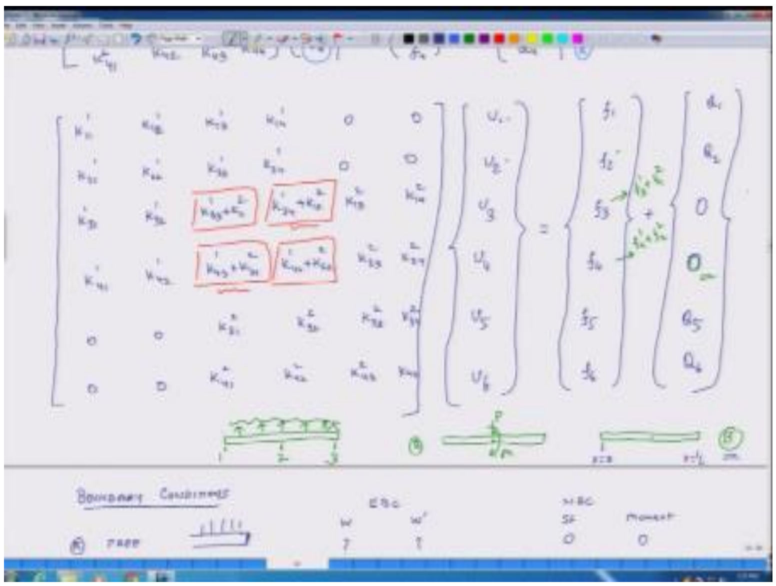

What is u_1 and u_2, these are degree of freedom right.

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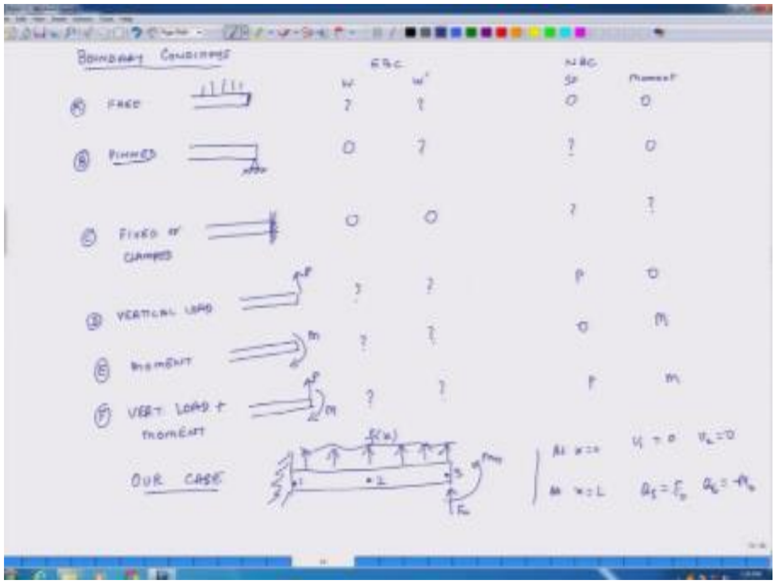

And in the second case, so these are two boundary conditions then that x is equal to l, Q_5 is equal to p and Q_6 is equal to minus M, oh it is actually f_0.

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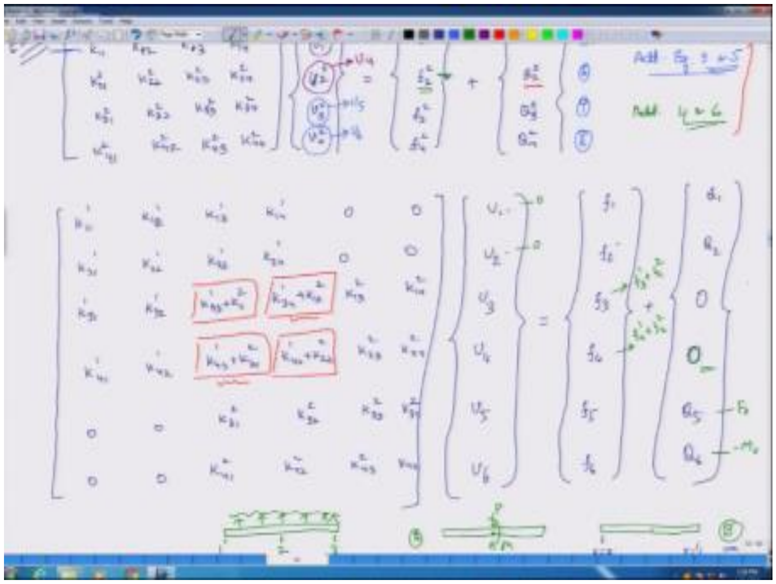

So these are the four conditions which I can put here, here I can put minus M_0 instead of Q_5, here I can put f_0 here I can put 0, and here I can put 0, and once I put that then I have six equations and six unknowns which are u_3, u_4, u_5, u_6 and Q_1 and Q_2. So now I have six equations section knows I can solve and get all these values okay. There is another boundary condition which I did not talk about.

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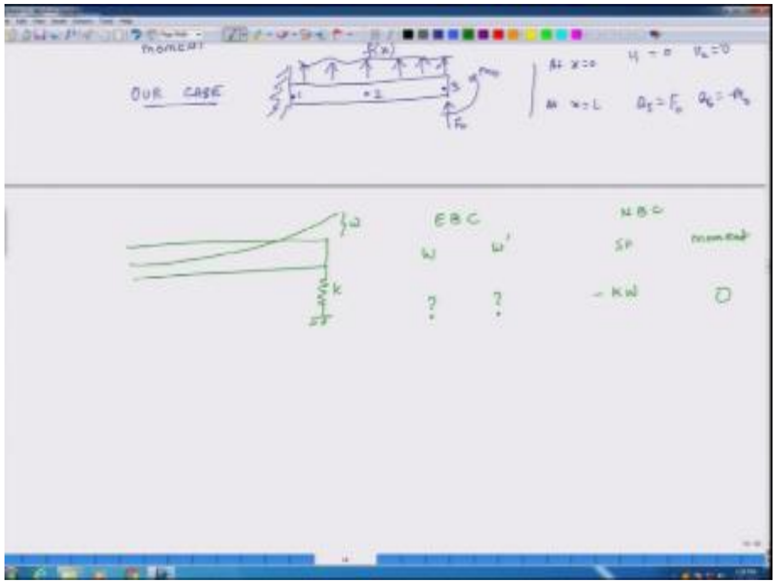

And that is suppose you have a spring at the end, then what will be the condition at the boundary condition? So in this case our natural essential boundary conditions and then we have natural boundary conditions right, so this is our shear force and this is our moment. So w is known or unknown, w is unknown right, we do not know what is w, we do not know also the slope, we know that moment at the end is 0, there is no moment being prescribed.

But there is also a shear force and this shear force is equal to if, if the deflection is like this, this is w then the shear force will be trying to pull it, so it will be equal to minus k times w, K is the stiffness of the spring and w is the deflection at this end, k times w it opposes.

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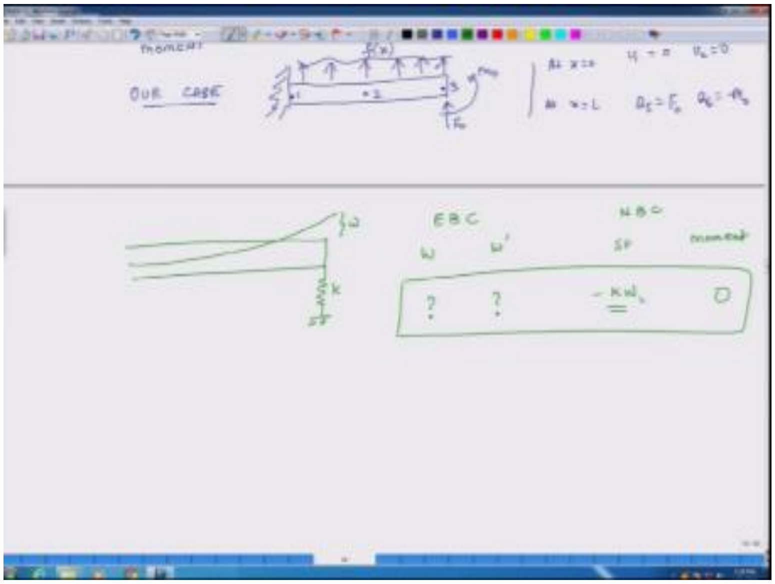

Suppose different deflection is w here, then this spring will try to pull it back and that is why it is opposing the direction of w_l okay. So we can apply these boundary conditions in our original equation, so that is what we will do.

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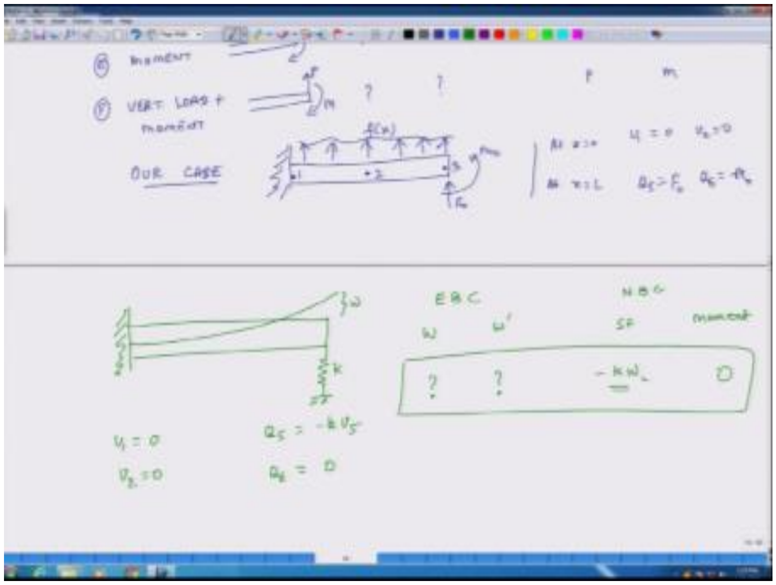

So support my beam was clammed at this end then what is what are the boundary conditions we will apply, u_1 is equal to 0, u_2 is equal to 0, and then Q_5 equals minus K times u_5 right, and Q_6 equals 0, these are the four conditions.

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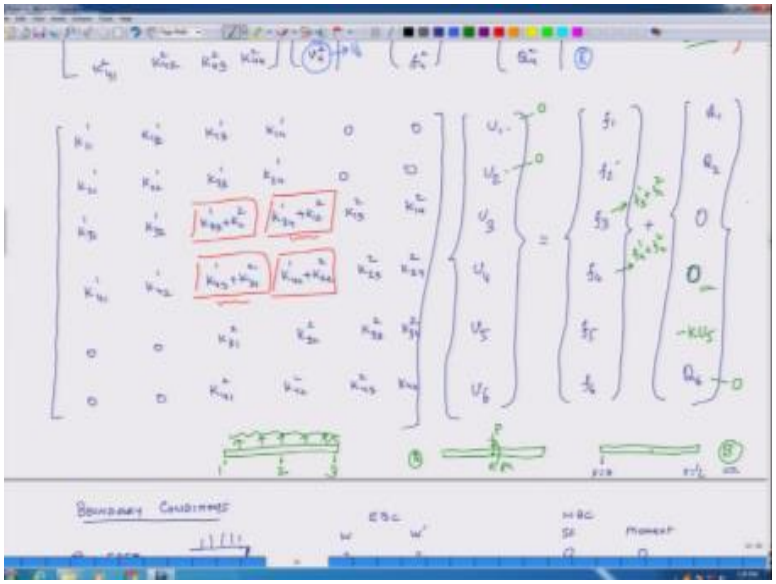

So we go back and again put these modified boundary conditions, so u_1 we said is 0 because it is clammed and u_2 is zero, because it is clammed in and moment was 0, and this we specify as minus K times u_5 okay. Now when I solve this equation I have to move.

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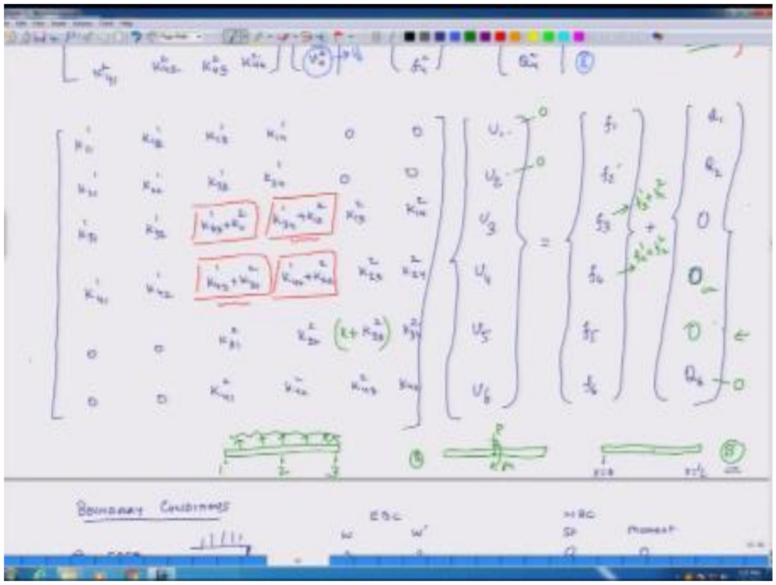

This u_5 to the left side okay, so where does it go, where does it, go first thing is that it will appear in the fifth equation this is the fifth equation right. So when I bring this K in the fifth equation on this side it will add up here, and once I have transferred it then this side becomes 0, does not mean that there is no shear force acting here. But it is that I have transferred that shear force here, and the later if we have to calculate shear force I have to find Q_5 u_5 multiplied by k and that is my shear force okay.

So this is, so my stiffness matrix gets adjusted and this k appears here, and now I can solve for again u_3, u_4, u_5, u_6 and solve for it.

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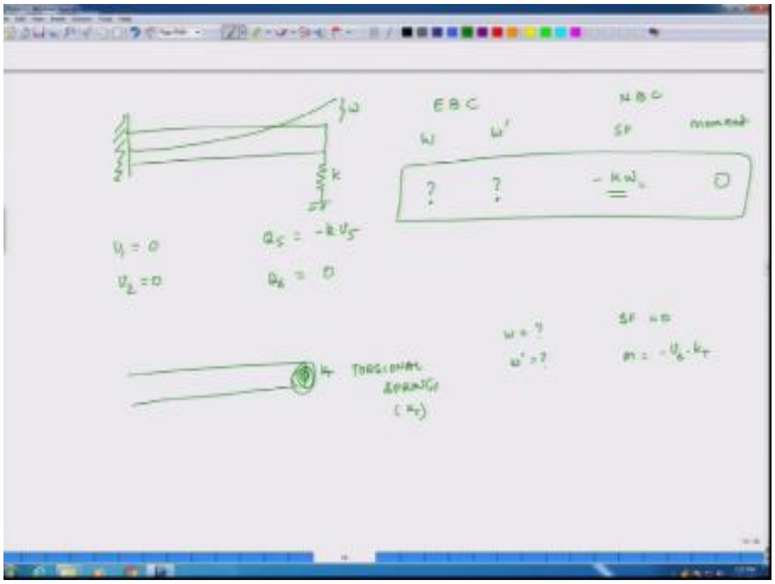

Likewise, I have another boundary condition, and it is that when it is bending I have a torsional stiffness, k_T, in this case the moment in this you know this is a torsional spring, in this case again w is unknown w′ is unknown shear force, I am not putting explicitly any shear force so that is zero. But moment is equal to what? Rotation which is u_6 times k_T, and it opposes the rotation so it is minus. So suppose the torsional stiffness is k_T then this is here okay.

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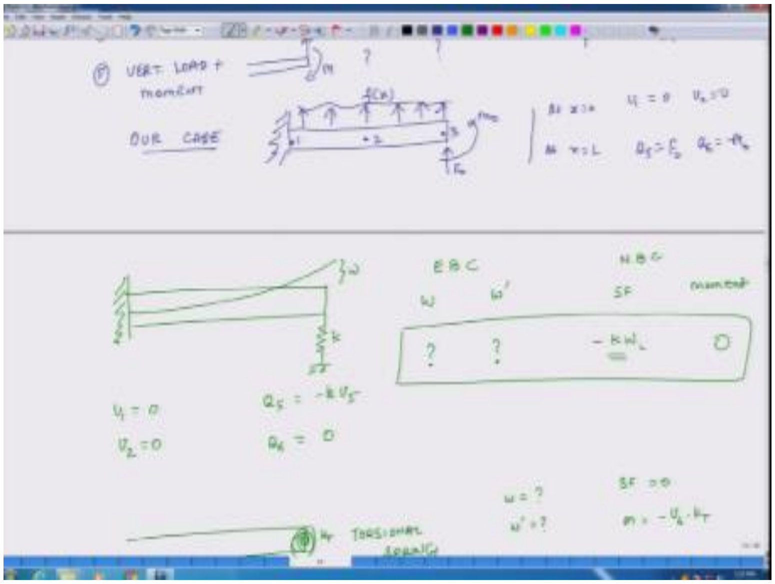

So then again if you want to implement this boundary condition it will come in place of u_6 and then you have to remove that k_T and add it to k_4^4 to solve for it okay. So there is this is a lot of times we have springs also attached on beams, so if there is a spring attached on a beam then this is how you have come for the stiffness of the spring. So this completes our treatment for the Euler-Bernoulli's beams and I hope you have understood what I wanted to explain. In the next week we will have a small discussion on another category of beams known as demotion go beams, and then we will start discussing about Eigen values and time-dependent problems. So thanks a lot and have a great weekend. Bye.

<u>Acknowledgement</u>
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