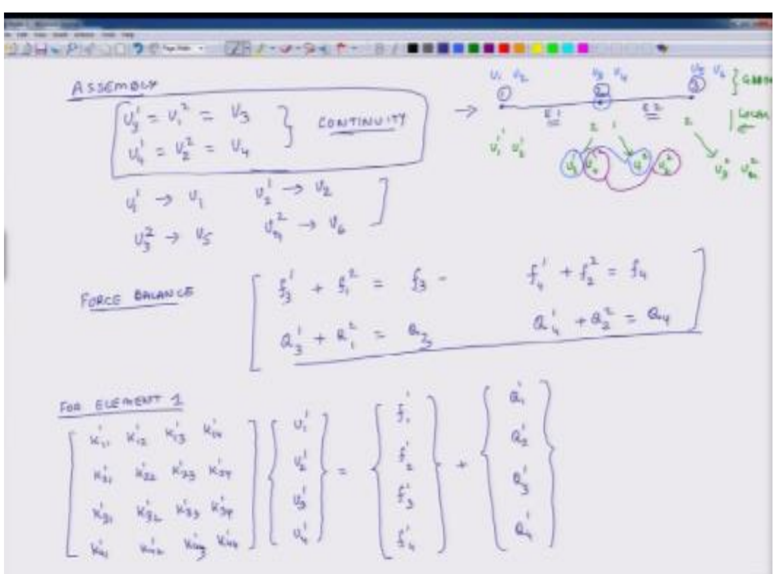
**Indian Institute of Technology Kanpur**
**National Programme on Technology Enhanced Learning (NPTEL)**
**Course Title**
**Basics of Finite Element Analysis**

**Lecture – 35**
**Assembly equations for Euler-Bernoulli beam**

**by**
**Prof. Nachiketa Tiwari**
**Dept. of Mechanical Engineering**
**IIT Kanpur**

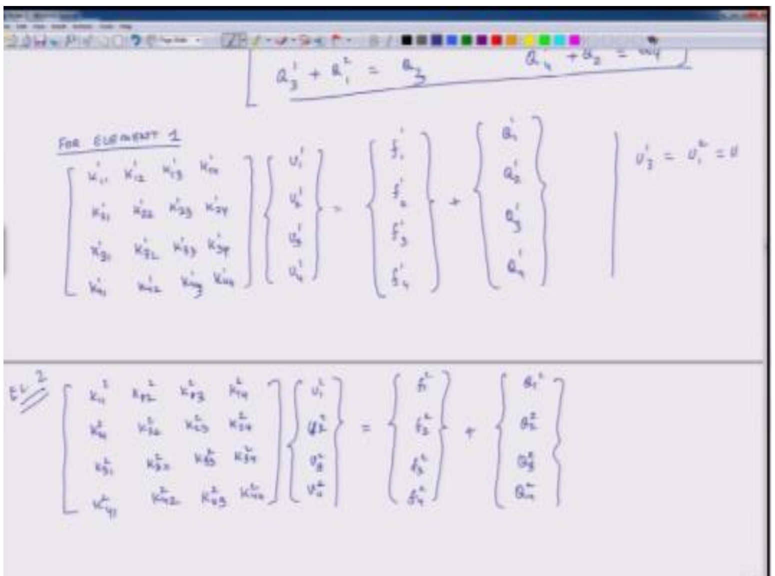
Hello, welcome to basics of finite element analysis, in the last class we had developed element level equations for the Euler-Bernoulli beam and also we had developed logic and expressions to enforce continuity of displacements across common nodes and also force violence relations which are relevant for common nodes, so what we will do today is continue that discussion further, and for a beam which is broken into 2 small elements we will write first 2 sets of element level equations and then assemble them using the logic which we had discussed in the last class.

(Refer Slide Time: 00:56)



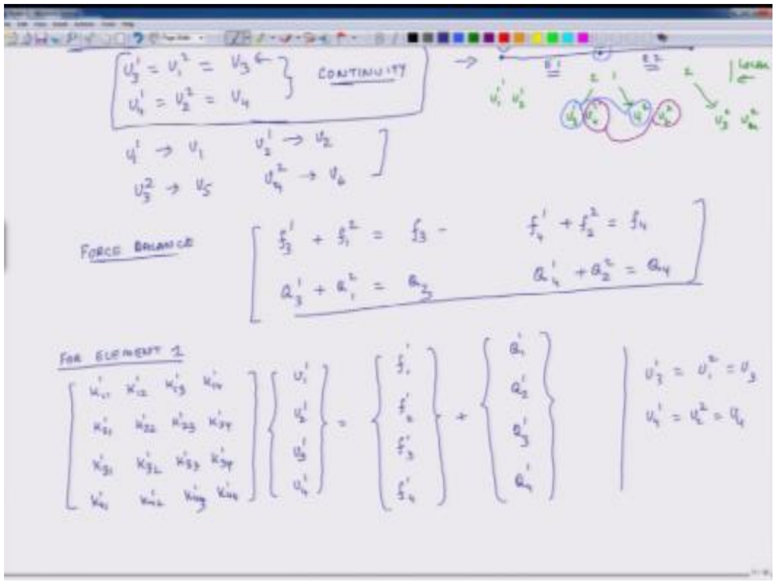
For element 1 so this is our beam and this is element 1 and this is the second element and the equations for element 1 for element1, the equations are $K_{11}$ and so you will have to be a little patient but I think it is important that I write these equations explicitly otherwise the understanding may not be clear, so all these relate to element 1 so I have a superscript 1 here and these are my generalized displacements which is the U vector, and then I have force vector corresponding to the first element + point forces and moments so these are the equations for the first element.

(Refer Slide Time: 02:58)



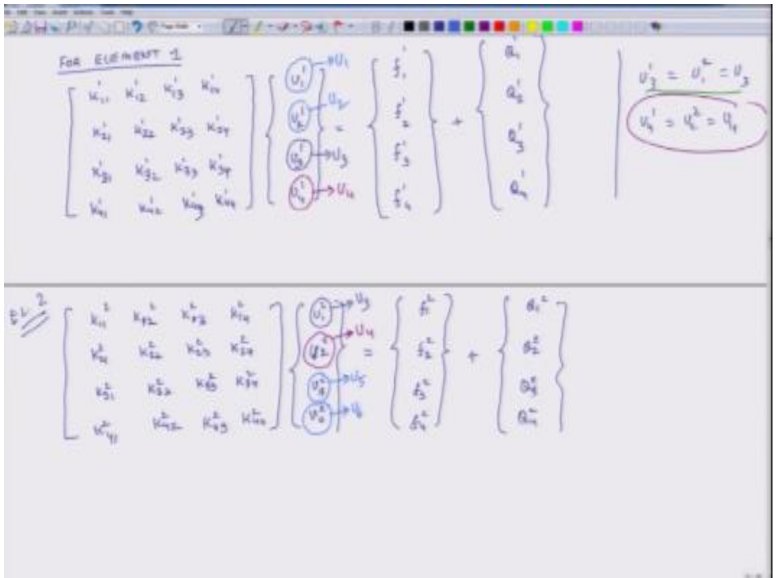

At the risk of sounding very repetitive but it is important, I will do the equations for second element and all these are related to the second element so there is a superscript 2 everywhere so this notation helps us avoid confusion and keep track of things, and these types of notations we frequently use when we actually write computer codes where we assign a number for each element, so we had so these are so this is for a element1and this is for element2 okay. Now our continuity condition was that $U^1{}_3$ is equal to $U_1{}^2$ and that equals U.

(Refer Slide Time: 05:13)



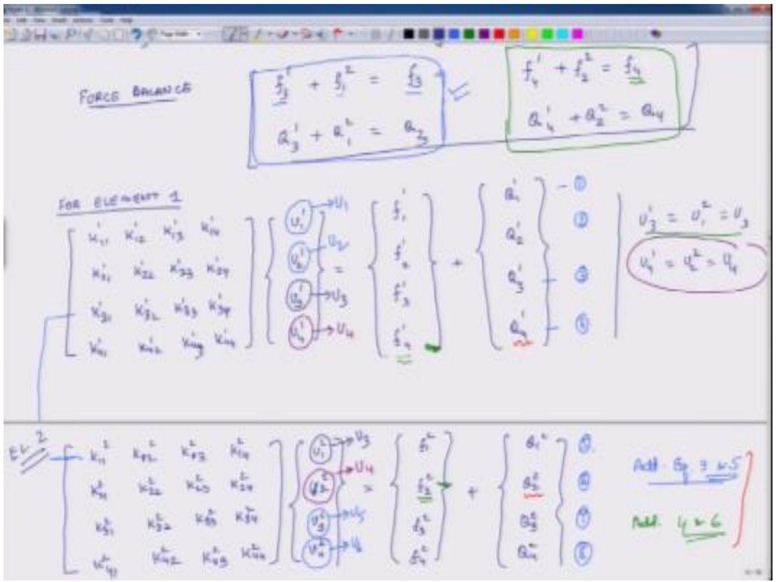
So this is the, this is the continuity condition I am going to use so that is = $U_3$ and the other continuity condition is $U_4^1 = U_2^2 = U_4$ okay.

(Refer Slide Time: 05:27)




So I replace this by $U_3$ and in the second equations for the second element I again replace $U_2^1$ by $U_3$ so that takes care of the first condition, then the second condition is $U_{4l}$ = U $_{22}$ = U4 so I replace this by $U_4$ and in the second set of equations I replace $U_2^2$ as U4, and then I know that this is my global degree of freedom which is $U_1$ so I change it to $U_1$ and the global degree of freedom as which is which right now is $U_1^2$ is $U_2$ and the same thing goes for $U_2^3$ this is $U_5$ and this is $U_{6,}$ so this is the first set of changes I do before I actually merge these two equations okay. So now you have 8 equations 4 for first element and 4 for second equation second element and I have 6 degrees of freedom so I have a total of 2 extra equations which I have to eliminate, so for that we use.

(Refer Slide Time: 07:02)

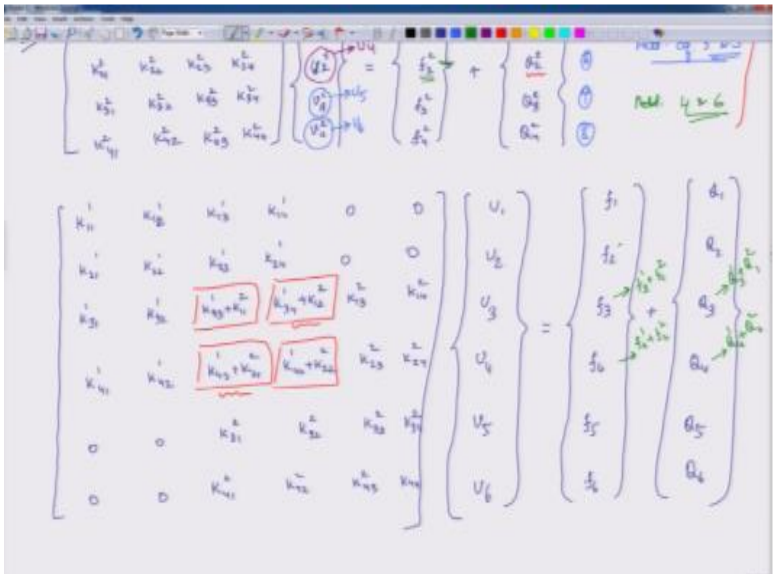


The force balance relation okay, so we see that f 13 + f2 1 equals f3so the equation where this f13 appears so is, I will call this equation 1 or I will actually number these equations so this is 1,2 and I am going to number them 3,4,5,6,7 and 8 so right now I have 8 equations and if I see, if I look at these two conditions then the right side of, if I add the right side of equation 3 with right side of equation 5 I will write right side of equation 3 with right side of equation 5 then that will be the total force so I have to add these equations.

So we add up equation 3 and 5 okay, and I while adding up please remember that I know everything in this F column because my function f is known and I have to just multiply it by shape function integrated to over the element so this F vector is known so everything here is known right, so we get we add up equations 3 and 5 and enforce the first force balance relation and similarly we add up equations 2 and 6 to take care of the second set of force balance relations. What does it say f 1 of 4 which is this term+ f 2, f 2 which is this term right when I add these up that will be the total force which I will call f4.

So this should be 4 so this term has to be added with this term and similarly this term has to be added with no not that one sorry this term, so then I get so then in that way I get a total of 6 equations, so that is what I will do in the next step.

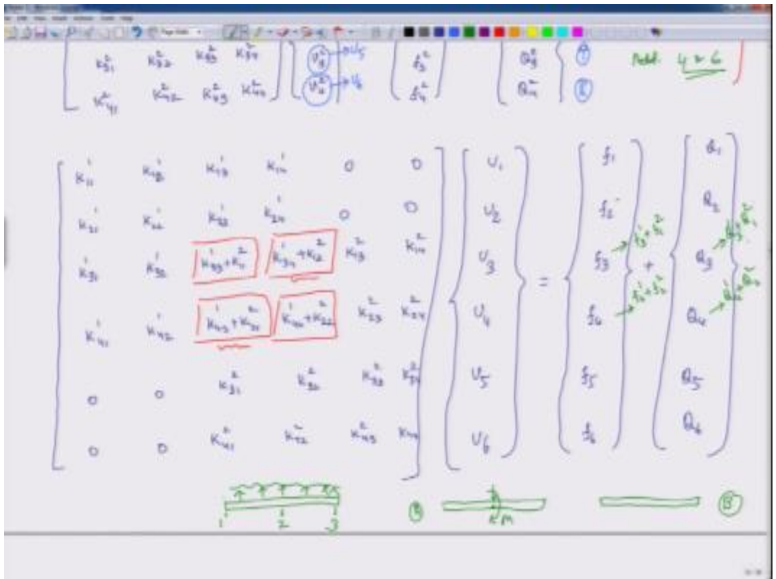
(Refer Slide Time: 10:02)



So I get a bigger metrics k 11, k 121, k 131, k 141, 00 so I will have a 6/6 now because I have eliminated 2 equations and actually I will make a little more space here, and my global degrees of freedom are U1, U2, U3, U4 ,U5 ,U6 and this equals f1 f2 f3 f4 f5 f6+ Q1, Q2, Q3, Q4, Q5 Q6 okay, this term is actually = f $3_1$ + $f_1$2 and this guy is f 41 + f 22, so what you see here is that there are certain stiffness terms associated with the common node where the stiffness gets added. Also the other thing you will notice that this assembled stiffness matrix is also symmetric because when you look at this term.

And these 2 terms k43 and K 134 they are same and k 2 1 and k12 are also same so when I at them as the overall excuse me, stiffness matrix is also symmetric so for a bilinear symmetric system whether it is a second order fourth order system the stiffness matrix will also come over will always come up as symmetric okay. On the right side I have the force vector due to distributed force and also a force vector due to concentrated loads generalized loads, so this one Q3 is = Q 3 1 + Q 12 and Q4 is = Q41 + Q22, okay.

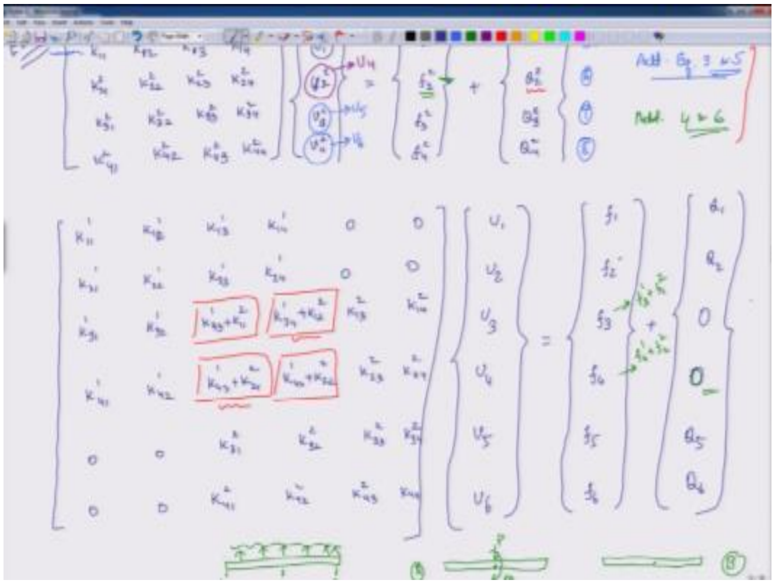
(Refer Slide Time: 15:00)



So at this stage we are ready to apply the boundary conditions but before we apply the boundary conditions we makes couple of other observations, so if suppose this is my beam and this is node1, this is global node2, this is global node 3 right, and I have some distributed force here then the F vector takes care of the force loading you know the forcing situation due to this distributed load, now if in our beam I do not have at these intermediate nodes any point external any point force or a point moment right.

See there could be a beam there could be 2 cases, case one or case A and this is case B, in case A at node2 suppose I am having some point load and also suppose I am having some moment M then I will replace this.

(Refer Slide Time: 16:23)



This entire thing by the first one is a  point load let us say this is P then I will replace it by - p because it is pointing downwards and thisQ4 I will replace it by -  M because again it is no this is clockwise so I will replace it by M and + M yeah so, so this is case A, in case B at this common node there is no external force, what that means is that this when I add these up this was $Q_3$ they will themselves cancel again each other and that total will be 0 so in that case this term will become 0 and so will $Q_4$ okay.

So unless I have at the intermediate nodes common notes some external point forces these point generalized forces which could be their shape of force or a moment the Q terms at common nodes will add up and they will become 0 so that is why these things go away, these things go away. So again I mean we do not just make it 0 in all the cases if we have point loads then we have to enforce those conditions okay, so assume that our case is same as that of B then I have I know all f1 f2 f3 f4 f5 f6 you know because I know this the nature of external load.

The things which I do not know are $U_1$ through $U_6$ so 6 unknowns and then I also do not know $Q_1$, $Q_2$, $Q_5$ and $Q_6$ these are the, so total I have 10 unknowns and 6 equations so again I need 4 extra conditions and these 4 extra conditions will come through our  boundary conditions, these 4 extra conditions will be related to the boundary conditions, so this beam has 2 boundaries X = 0 and X= L and at each end I will have 2 boundary conditions, either they will be related to the primary variables which are W and its slope or they will be related to associated secondary degrees of you know secondary variables which are shear force and bending moments but at each I have to specify these 2 conditions then I will be able to predict how this system is going to behave, so that is what we will do in our next class and thanks for today and we will again meet tomorrow, thank you.


**Acknowledgement**
**Ministry of Human Resource & Development**

**Prof. Satyaki Roy**
**Co-ordinator, NPTEL IIT Kanpur**

**NPTEL Team**
**Sanjay Pal**
**Ashish Singh**
**Badal Pradhan**
**Tapobrata Das**
**Ram Chandra**
**Dilip Tripathi**
**Manoj Shrivastava**
**Padam Shukla**
**Sanjay Mishra**
**Shubham Rawat**
**Shikha Gupta**
**K. K. Mishra**
**Aradhana Singh**
**Sweta**
**Ashutosh Gairola**
**Dilip Katiyar**
**Sharwan**
**Hari Ram**
**Bhadra Rao**
**Puneet Kumar Bajpai**
**Lalty Dutta**
**Ajay Kanaujia**
**Shivendra Kumar Tiwari**

**an IIT Kanpur Production**