Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 34
Finite element equations for Euler-Bernoulli beam

by
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Hello, welcome to basics of finite element analysis, today is the fourth day of our current week which is the sixth week and in our last class we had discussed as to how to develop interpolation functions for Euler-Bernoulli beam elements. Today we will actually use those interpolation functions to construct element level equations for an Euler-Bernoulli beam, and maybe also talk a little bit about the assembly procedure for these element level equations.

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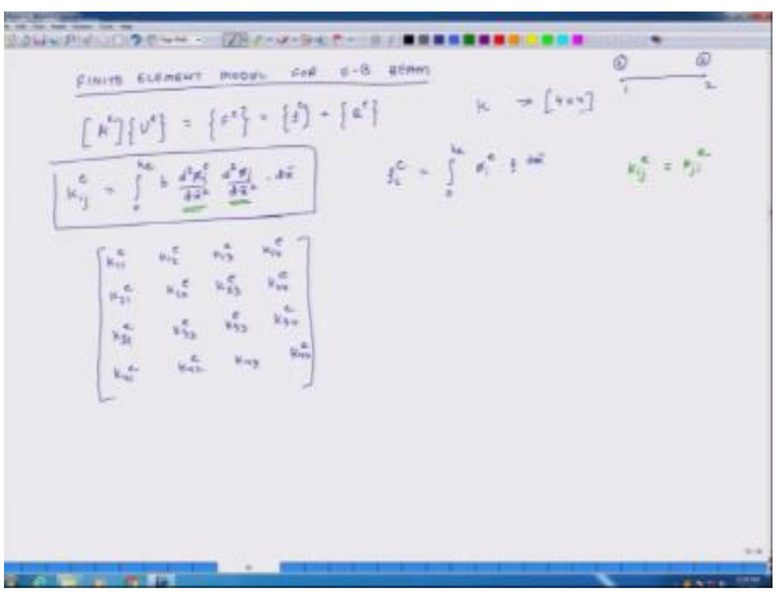

So we will develop finite element model for Euler Bernoulli beam, so our overall equation for a element is the stiffness matrix times generalized displacement vector equals a generalized force vector and our force vector is itself made up of two components contribution due to distributed normal force and contribution due to point forces and moments. Now k_{ij} so k, so if I have a two noded element then there are two degrees of freedom here and there are two degrees of freedom here.

So the size of this k matrix will be four by four, total number of four degrees of freedom so k_{ij} equals 0 to h_e b which equals e times I times second derivative of EI and actually again it should be a total derivative, so that is my definition of the ij^{th} element in this k^{th}, k matrix. f_i equals integral on the domain of $Ø_i^e$ times f which could be a function of times dx, and then of course I have Q1's, Q2, Q3, Q4 which I have already defind which constitute the K vector.

So in an overall sense I have K_{11}, K_{12}, K_{13}, $K_{14,}$ so this is my stiffness matrix for the e^{th} element and the way I will calculate each element is using this formula, and in this $Ø_i$ and $Ø_j$t will be the interpolation functions which we discussed in the last class. So we will use those Ø's as the interpolation functions, so that is how I will develop my K matrix, you please note that this key matrix. I will once again state it is bilinear in $Ø_i$ and $Ø_j$ and it is also symmetric in $Ø_i$ and $Ø_{j,}$ so if I replace $Ø_i$ with $Ø_j$ and $Ø_j$ with $Ø_i$ I will get the same value.

Which means that K_{ij} will be same as K_{ji}, so K matrix is again symmetric, and its primarily coming because my B functional was bilinear in symmetric.

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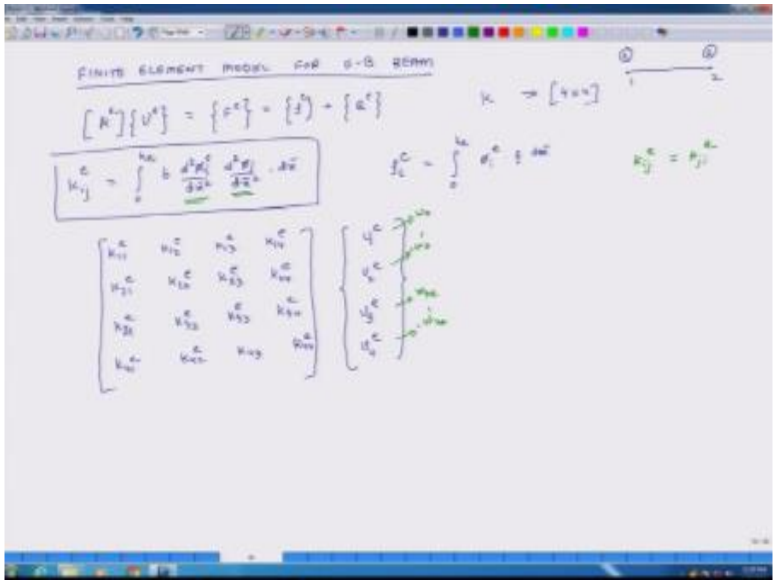

So this matrix is multiplied by a generalized displacement vector which are u_1, u2, u_3 and u_4 okay, physically u_1 represents w at 0, u_2 represents slope at zero actually negative of that, u_3 represents w at h_e and u_4 represents negative of slope at h_e. But in our computational algorithm we do not differentiate, we just put u_1, u_2, u_3 because it is easy to program.

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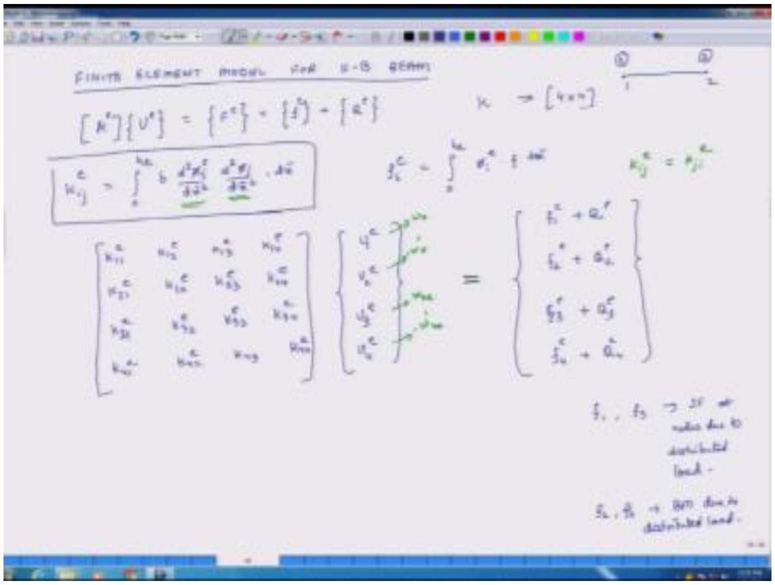

So this equals a force vector f_1+Q_1, f_2+Q_2, f_3+Q_3, f_4+Q_4 and there is a subscript, superscript e at all of these, because this is for the e^{th} element. Here f's, f1and f3 represent shear forces at specific nodes due to distributed load Q, and f_2 and f_4 they represent moments, bending moments again due to distributed load, again at the nodes. So you have a distributed load and its contribution at node 1 and node 2 that is what f_2 and f_4 represent, and then Q_1 and Q_3 they represent shear forces at the extreme points of the element Q_1 and Q_3 and Q_2 and Q_4 they represent point moments at the nodes 1 and 2.

So f's they represent consequential shear forces and bending moments due to distributed load and Q's they represent shear forces and bending moments due to point forces. If these point forces are absent then when we do the assembly process these things will vanish okay.

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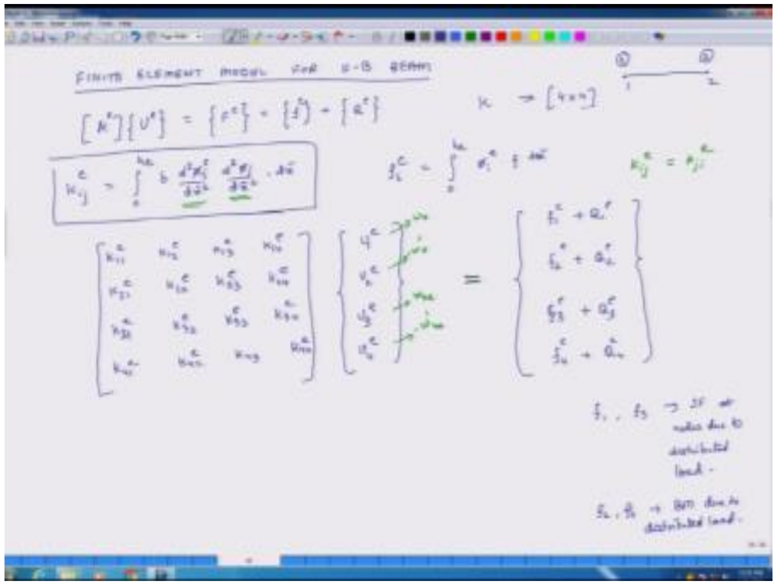

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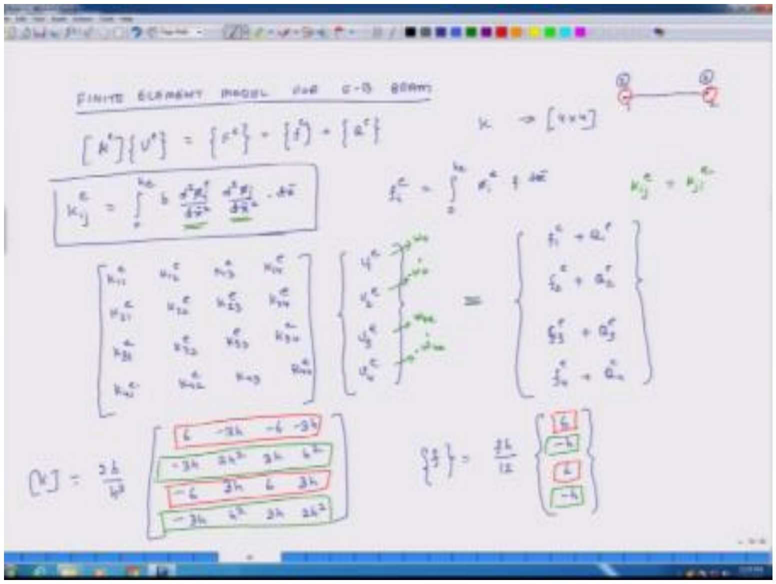

If I do the math and use all these interpolation functions which we discussed earlier in the last class then my stiffness matrix and this f matrix or f vector it looks something like this, so k equals 2b over hq for the moment I will just drop that supposed superscript e for purpose of convenience only, and these values of terms inside the matrix are something like this, and my f vector is, so I would like you to note two things, this row the second row and the fourth row they have the same terms, same thing is here also.

Second row and fourth row they have the same terms and then the first row and the third row they have the same terms both on left and right side, and the reason for this is you may have already guessed that the second row corresponds to rotational degrees of freedom and they are associated with moments, so it does not matter whether you are looking at node 1 and node 2, whatever is the rotational degree and associated bending stiffness of the element will be same right, the first term.

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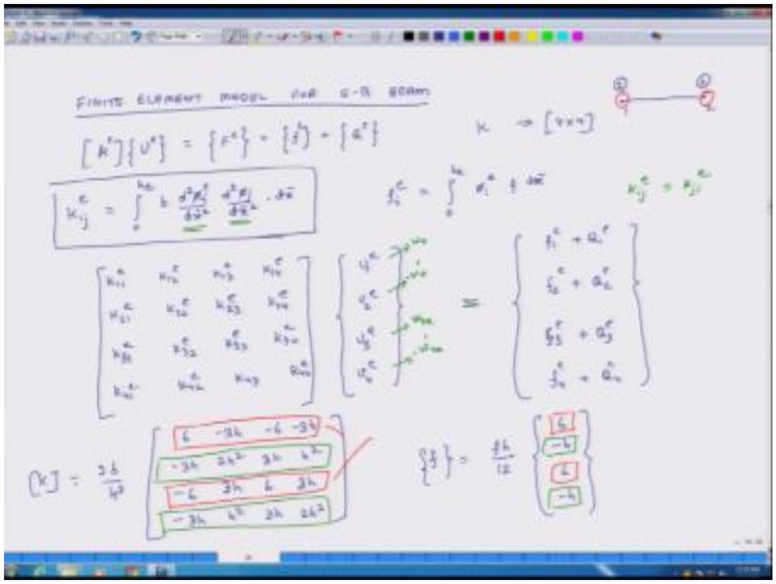

First row and the third row they are associated with the displacement. And they are related the displacement so that is why they look similar, the same thing on the right side of the equation also, that you have same terms there. So even though these things in the u vector they are generalized displacements but physically they are different entities, and the dimensions of elements in the second row and fourth row are different than the dimensions of elements in that first and third row.

So you should remember that, so when we do assembly we have to make sure that we do not accidentally assemble second and third row or first and second row, because they are dimensionally different entities.

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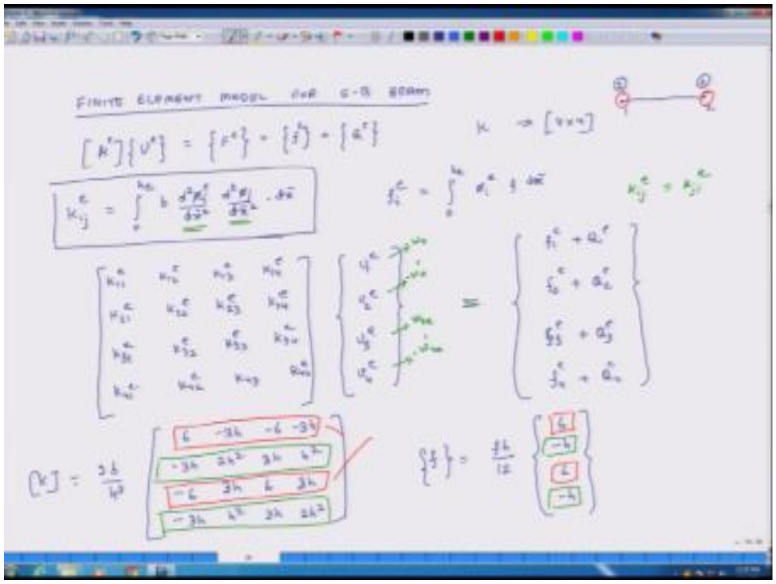

So next what we will do is we will start discussing the assembly procedure.

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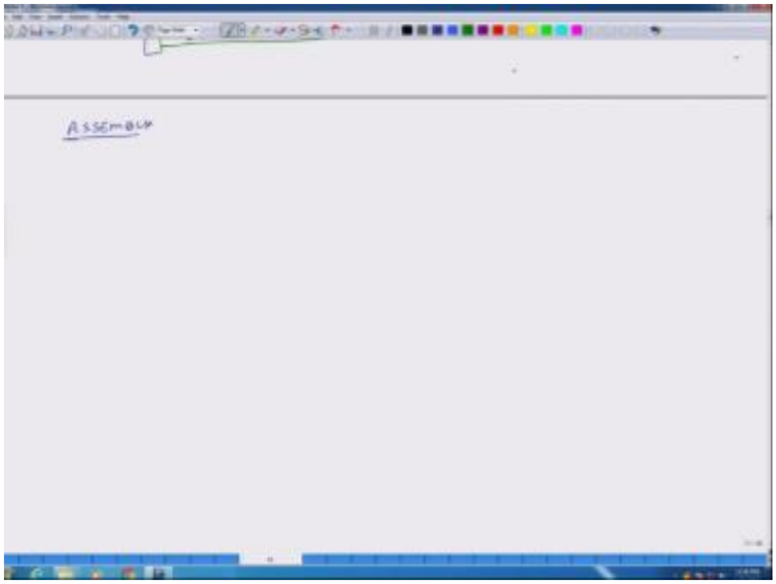

We will start discussing the assembly procedure, so suppose in this case what we will assume is that I have.

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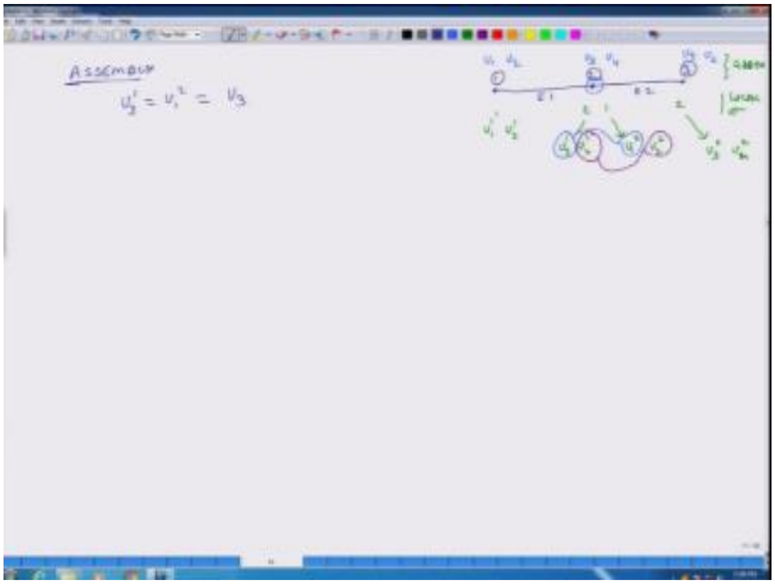

This is the beam, I am breaking it up into two elements, this is first element, this is second element and the global degrees of freedom global node numbers are 1, 2 and 3, the global degrees of freedom are u_1 and u_2, u_3 and u_4, u_5 and u_6 locally the local degrees of freedom okay, so before I write these degrees of freedom local, local nodes are one and two for first element and one and two for second element.

So this is all global and this is all local, so these are the node numbers and the associated degrees of freedom are u_1^1 and u_2^1 associated with this I have u_3^1 and u_4^1 right, associated with this I have u_1 second element and u_2 of second element and associated with this degree of free this node we have u_3 second element and u_4 second element okay. So at this node, this degree of freedom is equal to this degree of freedom at the common node, and this degree of freedom is equal to this degree of freedom.

So the conditions at the common node are u_3^1 equals u_1^2 and that equals my global degree of freedom which is u_3, similarly.

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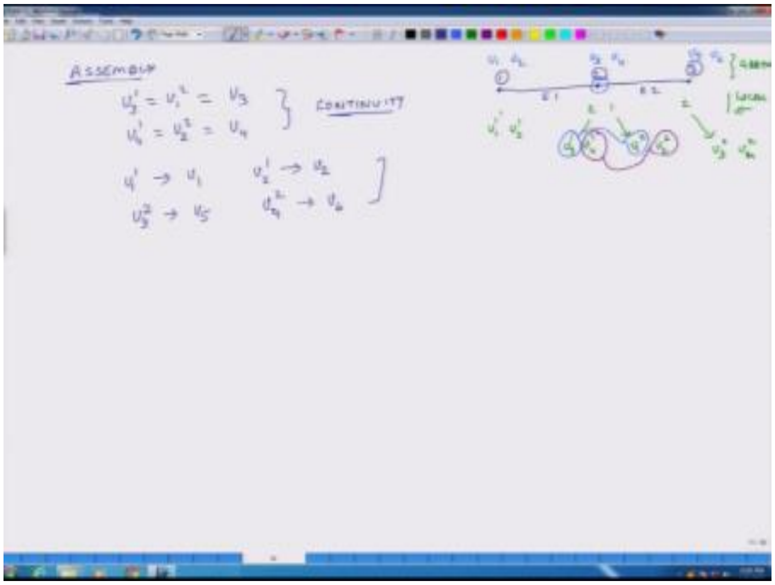

u_4^1 equals u_2^2 and that equals u_4, and because I am equating these two conditions these are the conditions for continuity, and of course other conditions when I am doing the assembly I will eventually I will replace u_1^1 by u_1 which is the global degree of freedom, u_2^1 by u_2, and then at node 3 it will be u_3^2 by u_5 and u_4^2 by u_6. So I will rename these in the, at the assembly level these things, so I have.

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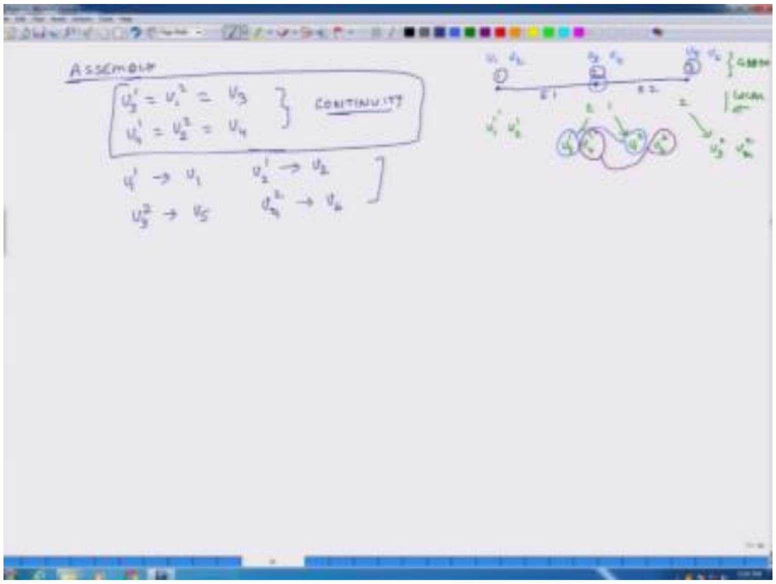

These are my continuity conditions okay, the other condition at the common node is that the total force at this common node from element 1 plus the total force from the side of element two, if I add them up that will be the actual force at node 2 right.

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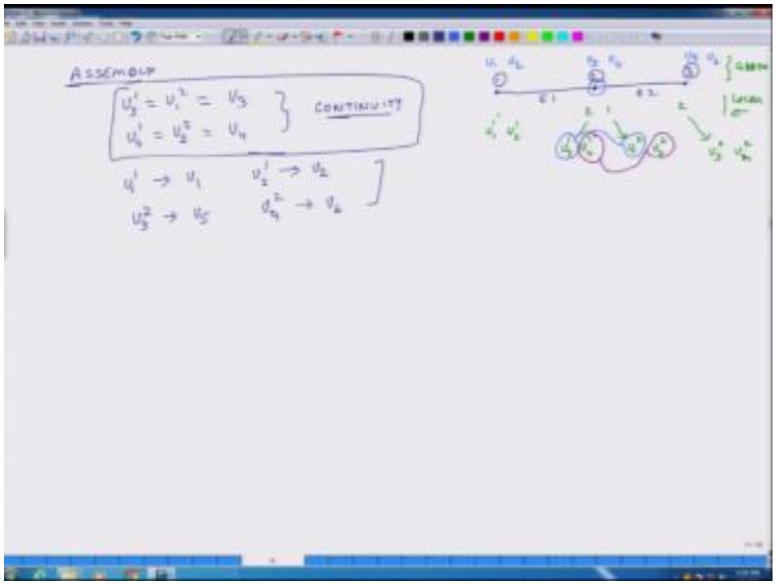

If I just look at element 1 I will have only partial contribution of the load, so if I am looking at.

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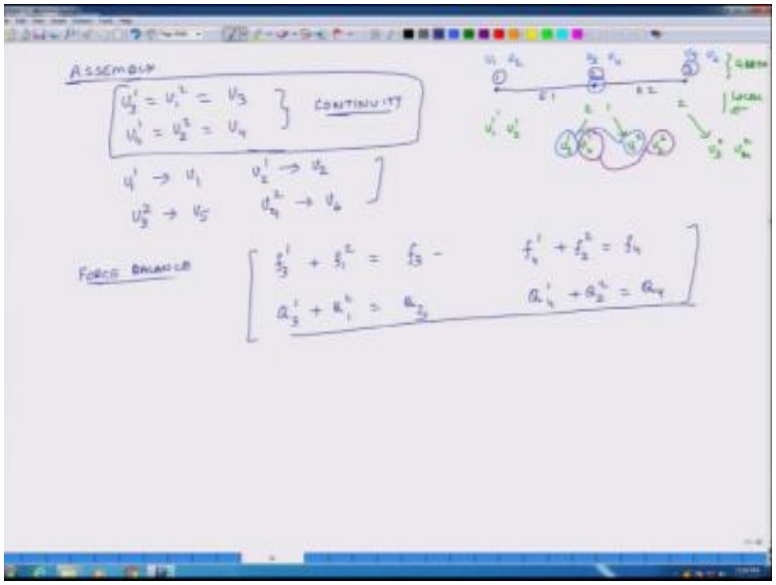

The at the assembly level total load from side one and total load from side 2, if I add them up then that will be the total force at so force balance. So what are the equations for force balance $f_3^1 + f_1^2$ this equals f_3, similarly for the moment so this is about sheer force f_3 and then if I have to add up moments then $f_4^1 + f_2^2$ equals f_4 that is that generalized force which is moment, and then when I look at Q's, $Q_3^1 + Q_1^2$ equals Q_3 and $Q_4^1 + Q_2^2$ equals Q_4. So these are the equations for making sure that we account for forces correctly.

So what we will do is that in the next class we will write down the equations for both these elements and then using these relations actually assemble those elements, so that you can understand how this actual assembly process is going to happen. So thanks very, thanks a lot and we will meet once again tomorrow. Thank you.

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