Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Basics of Finite Element Analysis

Lecture – 31
1D-Heat conduction with convective effects : examples

by
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Hello, welcome to basics of finite element analysis, this is the 6th week of the current ongoing mook course on FA and this week we will start with continuing our discussion for 1D heat conduction problem with convective effects, and in today's lecture we will actually close that topic by solving for a problem, and then in the remaining 5 lectures we will explore beam theory and how it is solved using the finite element method. So for this lecture we will first write down the equations which we had developed in the last lecture.

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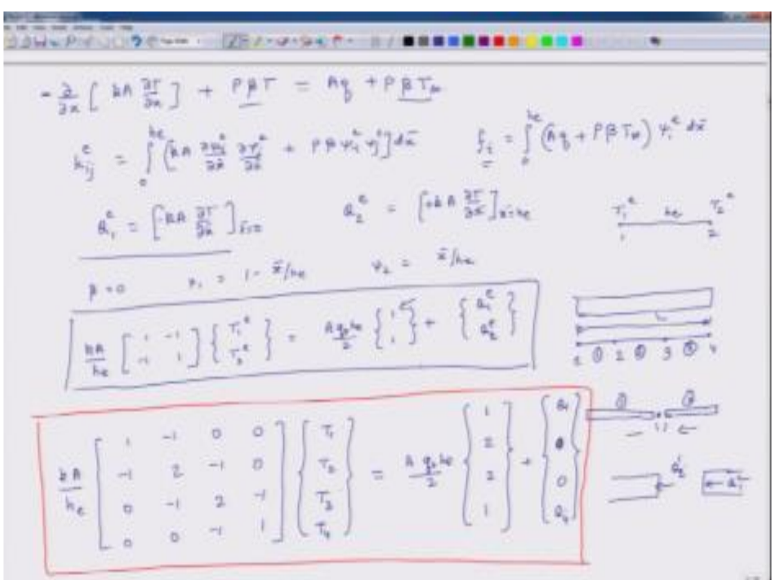

And the overall governing equation for 1D heat conduction with convective effects was this thing P, so these 2 terms where β appears they are because of perfect of convection okay, and for this particular equation the stiffness matrix elements we had defined were, so that is my stiffness element, then in the force vector the terms are A_Q + P β T∞ and because it is the E^{th} element I have to put a subscript P on all the parameters and then the Q is Q_1 was kA ∂T over ∂X, and there is minus sign here at x-bar equals 0 and Q_2 for the E^{th} element is exactly the same relation but with a positive sign and evaluated at x=he, x bar=he, and these are all X's are in local coordinate system.

So I am not using X but I am using X bar and now what we will do is, we will actually develop these equations and we will have a convective boundary condition but before we handle the convective boundary condition we will do this problem without having the convective term okay. So in that case let us assume that β = 0 and then we have seen that the shape function so if it is a 2 noded element this is node 1, node, 2 length of the element is h_e then my approximation function is 1, ψ1=1 - X bar / he and ψ2 = X bar/he.

So using these functions I can also calculate their derivatives and then my stiffness matrix comes out as, so my overall equation I will write that for the element is kA/ he times 1 – 1 -1 11and if I have a linear element so then the temperature at linear element at first node is T_1^e and temperature at the 2^{nd} node is $T_2^{e,}$ and these are unknowns so T_1^e, T_2^e and this equals the F vector and the values in that vector are Aq_0 he over 2,11+q_1^e which is a point load or a point source of heat. Now suppose I have a bar and for purposes of discussion in this class I am breaking this bar.

So I am idealizing this as a line so total bar length is L okay, and I am breaking it up into 3 elements, element number1, element number2, element number3 and the global nodes are 1,2,3,4 okay, so I can write 4 sets of equations at the element level which will look like this and then using the condition of continuity 3 sets of equations here because they are 3 elements I am sorry 3 sets of equations at the element level and then using the continuity condition, and the continuity condition is that T_2^e in the first element = $T_1^e$of the 2^{nd} element, similarly T_2^e of the second element =T_1^e of the 3^{rd} element so that is one condition.

So I eliminate 2 variables from there and I also know that whatever is the source term whatever is that, whatever is the heat generated so suppose let us say so this is my first element and this is the second element and in reality these two elements are sticking at this point right, so the heat generated because of Q is expressed here okay, at node 1 and this is at node2 so the total heat which is being generated at this location will be contribution from this side and contribution from this side so I add up the contributions at the intermediate nodes from 2 elements.

So I add up the equations the same way we added up equations in case of solid mechanics problem the same logic, so if I do all that my overall equation at the element level it becomes at the not the element level at the assembly level it becomes KA over h_e 1 – 100 – 12 – 100- 12 – 1 00 – 11, so this is my global stiffness matrix global k matrix that multiplied by T_1,T_2 these are my global degrees of freedom which are not known and this equals a term, a vector attributable to distributed heat load.

So this is 1, 2, 2, and 1 + point heat sources which is Q_1 and here this will be 0 because the contribution from left side and contribution from right side it will be balanced right, what does that mean in context of heat transfer equation, suppose heat is coming in like this so that is something I will call Q_2^1 right and in the adjacent element the same amount of heat is going out right. So that will be Q_1^2 and when I add these 2 heat terms they will become 0 and net heat generation is 0 so at the interfaces these terms will be 0 and here it will be Q_4, so that is my equation.

At the assembly level okay, now I see that in this equation I have 4 unknowns T_1, T_2,T_3 and T_4 and then I have 2 more unknowns Q_1 and Q_4 so there are total of 6 equations 6 unknowns and 4 equations, so I need two extra conditions and these two extra conditions will be gotten through application of the boundary conditions.

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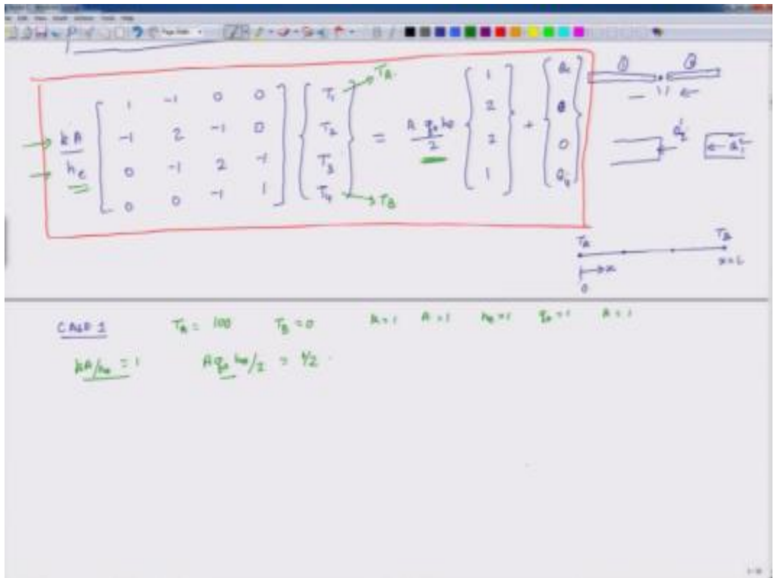

So we will solve this equation for 2 different sets of boundary conditions in case one, in case one we will say that T, so this is my overall bar which I have broken into 3 elements right and here this is my global degree of freedom so and this is coordinate is X so at X = 0 I said the temperature is T_A and at X=L temperature is T_B so I am saying that I have a bar and the temperature at left end is prescribed as T_A and the temperature at right end is prescribed is TB.

So this is one set of boundary conditions so if I do that we will prescribe this $T_{A,}$ this will be prescribed as T_A and this will be prescribed as T_B right, so now I have so these are now knowns T_A and T_B because we know these values so now I have 4 unknowns T_2,$T_{3,}$ Q_1 and Q_4 and I have 4 equations, so now I can solve for these 4 equations, for these 4 unknowns using these 4 equations so we will actually do that okay, so let us say that T_A equals but I assume 100 degrees T_B = 0 and then for sake of simplicity.

I have assumed a lot of things as unity so I say k =1 , K =1 h_e = 1 so that our computation becomes faster that is all and Q_0 =1 and A is also = 1 okay, so this ka / h e =1 so this term and this term is also = 1 it is the this term becomes 1 over 2, this term becomes 1 over 2 AQ_0 he/2 over 2 is 1 over 2. So what we are doing is that we are going to prescribe KH, KA / he as1and the other term this term as 1/2 and solve for T_2 and $T_{3,}$ so we will write down equations from these 4 equations for T_2 and T_3 so the first equation which is the second equation .

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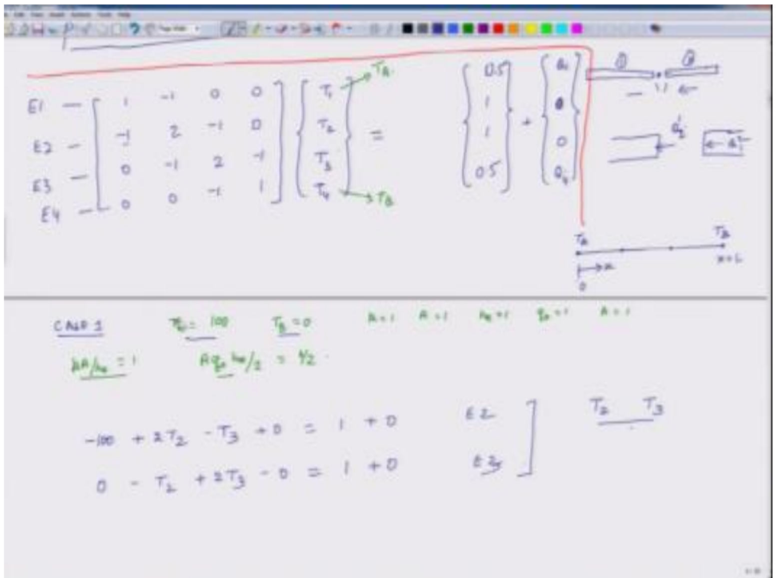

So this is, so first thing what I will do is I will erase this KA over he because it is nothing but 1 so that goes away and this term it becomes 1/2 and if I remove this ½ and put it inside so this becomes 0.5 and this becomes 11 and 0.5. So I will call this equation 1, call this equation 2, equation 3, and this is equation 4, so what I am going to do is with these prescribed boundary conditions I am going to write equations 2 and 3, so my equation 2 is – 1 times TA and TA is 100 so it is -100+2 times T_2 -1times T_3 and 0 times T_4 and that = f_2 which is 1 +Q_2 which is 0 okay, so this is my equation 2. The 2nd equation is equation 3 so I am going to write it is

So that is 0 times T_A so that is 0-1 times T_2 and then 2 times T_3 and then - 1 times T_4&T_4 is 0 degrees right which is T_B so this is 0 and this equals again f3 1 +$Q_3$0 so this is my 2nd equation. Now 3rd equation, so equation2 and equation 3, so I solve for these two equations and I can calculateT_2 and T_3 and then I can plug these values back in my equations for E_1 equations E_1 and

E_2 and I can find the values of Q_1 and Q_4, so this is my first case where I have prescribed at both the ends the primary variables which is 0 and 100 degrees okay.

Now what we will do is we will do another case where we will have a convective boundary condition at one end and another conduct source heat source at the other end and we will again solve these equations, so I am going to erase all this

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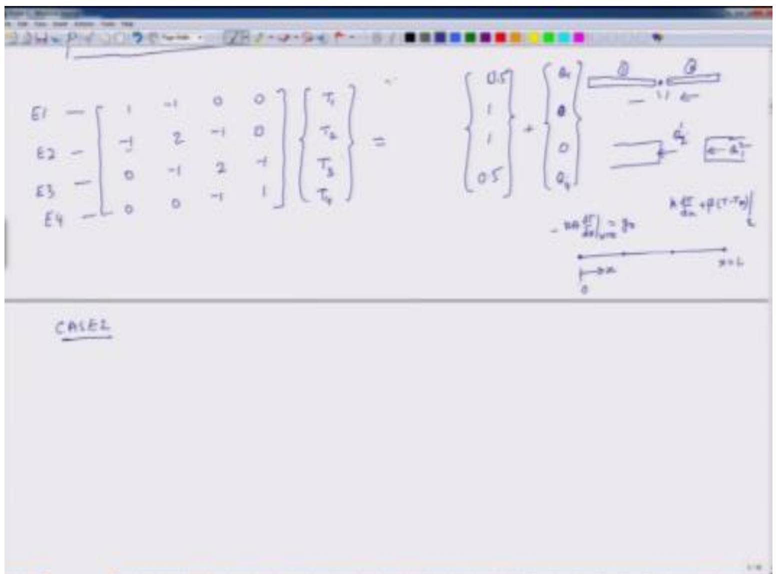

And the second boundary condition is case 2 so in case 2 my boundary condition so this is all this also goes away so once again in case 2 the boundary condition here is not known temperatures but at X=0 I know how much heat I am injecting in into the system so that is -KA∂T over ∂X at X=0 is some constant G_0 I know that, that also at this point at X=0 I am pumping some heat into the system and the value of that heat is G, G_0 okay that is what this physically means - KA times ∂T over ∂X, ∂T over ∂X is temperature gradient k is conductivity is area of cross section.

So that is the amount of heat transferred because of temperature difference right and at this point at X=L the boundary condition is that K times ∂T over ∂X, so one thing I would like to do is that instead of partial derivatives I will express them as total derivatives because the only variable left is now X, temperature is not there so partial derivative is same as total derivative so on the right side at X=L the boundary condition is K ∂T over ∂X +β T- $T\infty$ at X= L and this I say is 0.

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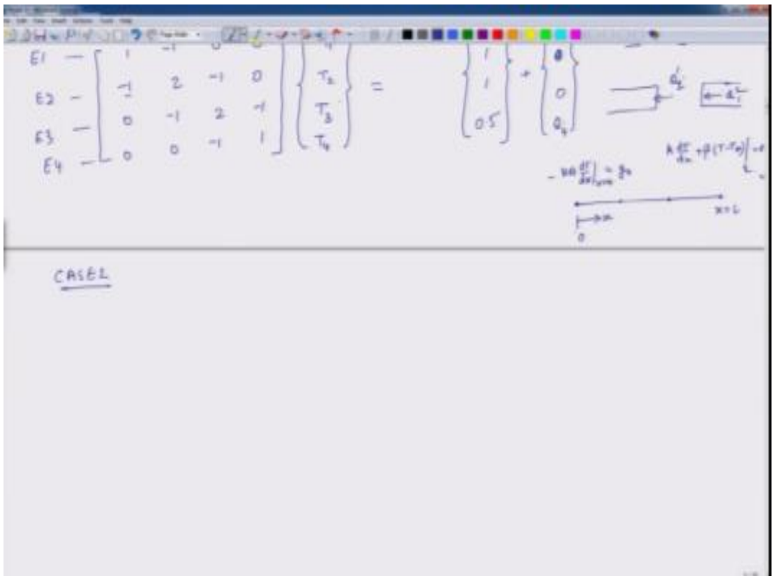

So what does that mean?

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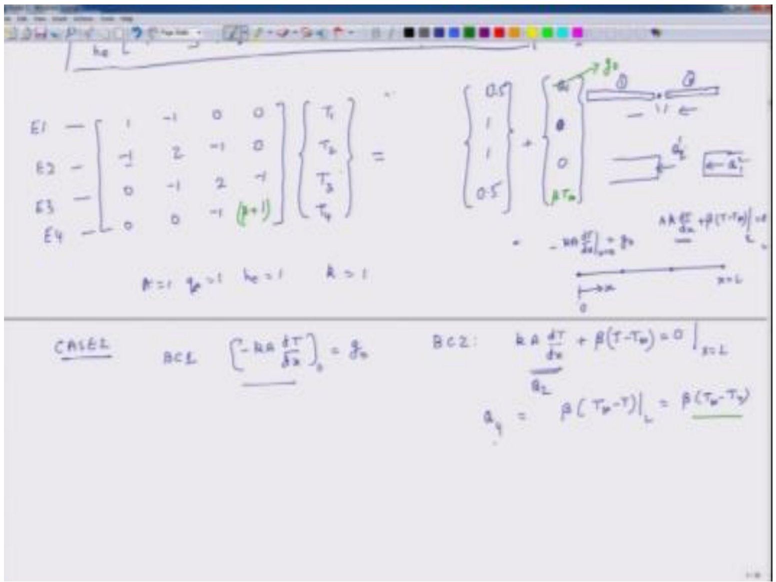

That there is some convective transfer happening, this K∂T over ∂X is heat which is going out right and that equals the convective heat transfer, that equals convective heat transfer so then how we will see how to account for this convective heat transfer in our boundary conditions. So once again my 4 equations and we still assume that A = 1 Q_0 = 1 he =1 and what else and we will, K, K=1 so these equations are still valid okay, so BC1 is – KA ∂T over ∂X at 0 equals G_0 and we know that the definition of Q_1 is this thing okay.

So this term becomes $G_{0.}$ The second boundary condition BC2 is KA and I again I think I messed it up this should be complete total derivative ∂T over ∂T +β T- T∞ = 0. Now what I do is this equals Q_2 right so I express it as Q_2 equals – β or I can express it as my β times T∞ -T and this is at X=L and at X=L this is β times T∞ - T_4 because at X=L temperature is T_4 okay. So I replaced this Q_4 by this term β T∞ - T_4 agreed, this should be Q_4 and I missed this A term here so this A also should be there so what I have seen is that I have replaced Q_1 by G_0 and I have replaced Q_4 by β times T ∞ - T_4 okay.

Now once again in these 4 equations, these are 4 equations and I have now 4 unknowns on the left side I have T_1, T_2,T_3,T_4 these are 4 unknowns, on the right side also there is no unknown which is extra because I know β I know T∞,T∞ is the temperature of here far away from the FIN or this thing andT_4 is an unknown so, but that is already accounted for, so 4 equations, 4 unknowns. So the next step which I do is that I transfer this T_4 on the left side I transfer T_4 to the left side and when I transfer it, so, so first step was replacing $_Q4$ by this term and now I transfer this.

So on my right side is I still have β times T∞ and on my left side I get an extra term β here okay, so the effect of convective heat transfer at the end of the bar is accounted by changing or modification by because of modification of this K matrix so β comes here and now I can solve these 4 equations for 4 unknowns in a fairly straightforward way. So this is what I wanted to discuss in context of heat conduction and equation which also has a convective heat transfer term and this captures or covers this particular topic, in the next class we will start discussing the beam theory and how we can use finite element analysis to solve beam related problems, thank you very much.

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