Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 30
One dimensional heat transfer

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Hello welcome to basics of FEA this is the last day of the current week which is fivth week and what we will discuss today is a little bit more detail about 1D heat conduction equation and in this context we will discuss 1D equation related to Cartesian frame of reference then also with respect to cylindrical and this spherical frame of reference and how do we actually implement boundary conditions through an actual example, so that is what we will do so first we will write the equations.

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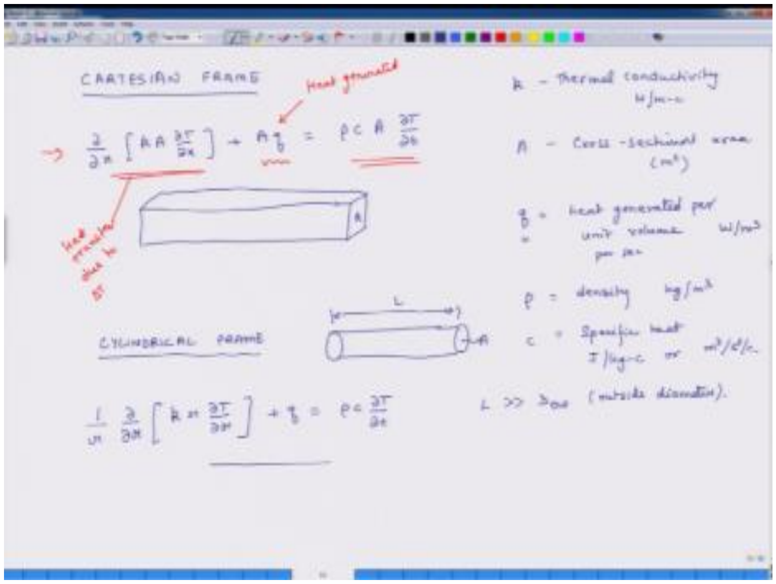

For heat conduction with respect to different farms of references, so if we have a Cartesian frame of reference then the 1D equation and this we have seen it earlier also is kA ∂T over ∂X +Aq = ρ CA ∂T over partial of temperature with respect time and K is thermal conductivity and it is units of are w/m-d centigrade then A is cross-sectional area so to give you a context suppose I have a some bar then the cross-sectional area of this bar represents A

So that will be in meters squared q is heat generated per unit volume so it is w/m^3 so to give you an example if you have a wire and it is carrying current then the total heat which will be generated will be i^2r right so this is actually heat generated per unit volume, per unit time per second okay. So the total heat generated will be i^2r and suppose the length of the wire is 10 meters then I can calculate the volume of that wire and I can divide this total power by the volume of the wire, and that will give me Q.

Okay, then ρ corresponds to density so that is in kilograms per m^3 and C corresponds to specific heat and that is J/K_g per degree centigrade or if I do a little bit more dimension analysis on this then I can also explicit as m^2 /s^2 /c, so in this equation this particular term relates to how much heat is getting transferred. Due to a ΔT temperature great in how much heat is getting transferred due to temperature and because of conductive effects okay?

This term represents how much heat is being generated so it captures the effect of heat getting generated and this is an equation also in time and this term it represents the effect of the raise of temperature with respect to time, so the temperature is not raising then there ΔT where Δpartial of temperature with respect to time will be 0, so this is very general equation for a 1D system now for a cylindrical frame of reference an example of a cylindrical system would be you have a cylinder.

Okay so that is my cross-sectional area A, and it is generating also heat along it is entire length and also there is heat conduction happening from 1 hired part to the lower part and so on and so forth, so all the physical phenomena heat transfer due to conduction heat generated and the raise in temperature everything is there and for that the governing equation is 1over r, so 1 over r partial of k r times ∂T over ∂r+ q which captures the effect of heat generated per unit volume times ρ time C, into ∂T over ∂T now this particular equation is good if the overall length of the system suppose this is L.

Then L has to be fairly large compared to let say D_{out} or outside diameter, okay this is the 2nd equation again 1D heat conduction equation but this is for a cylindrical coordinate system.

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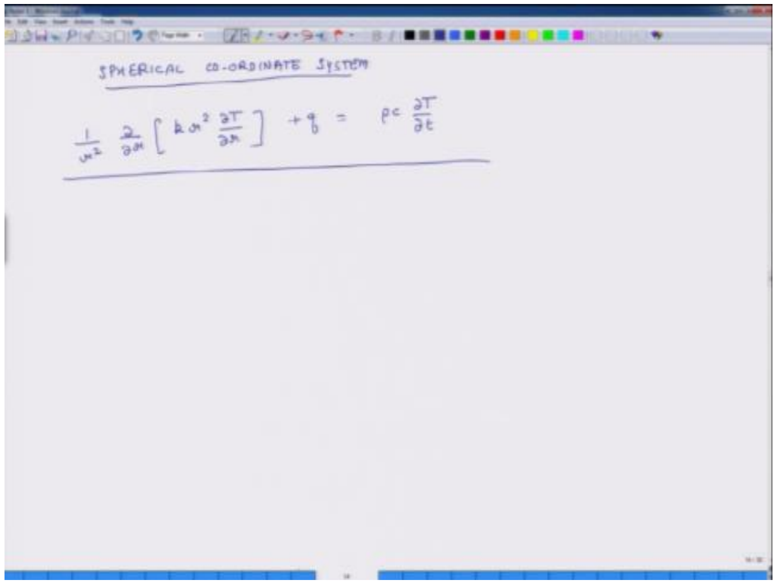

And then we have a spherical coordinate system and an example of this kind of heat transfer would be that they have small source of heat and it is surrounded by some homogeneous medium on all the sides. So I am generating heat and it is spreading out in all directions uniformly, so in that kind of situation the governing equation one-dimensional equation is 1over r^2 partial with respect to r , kr^2 partial of temperature with respect r + Q = ρ C partial of temperature with respective time.

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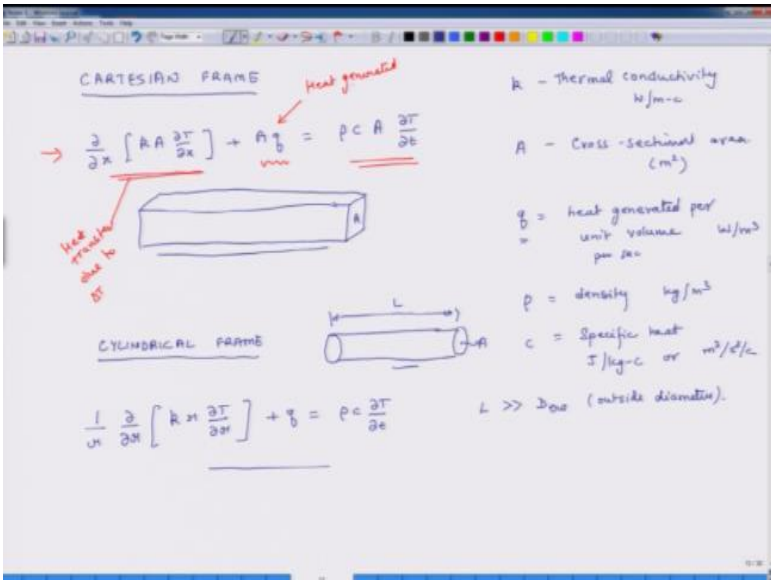

Now all these equations they do not account for convective heat transfer so for instance if you have this bar it could be surrounded by here right, and there may be not only heat transfer due to conduction but also because of conviction so he here will pick up some heat and it will that heat will get disputed into to here so here we do not have a convective term in this entire equation.

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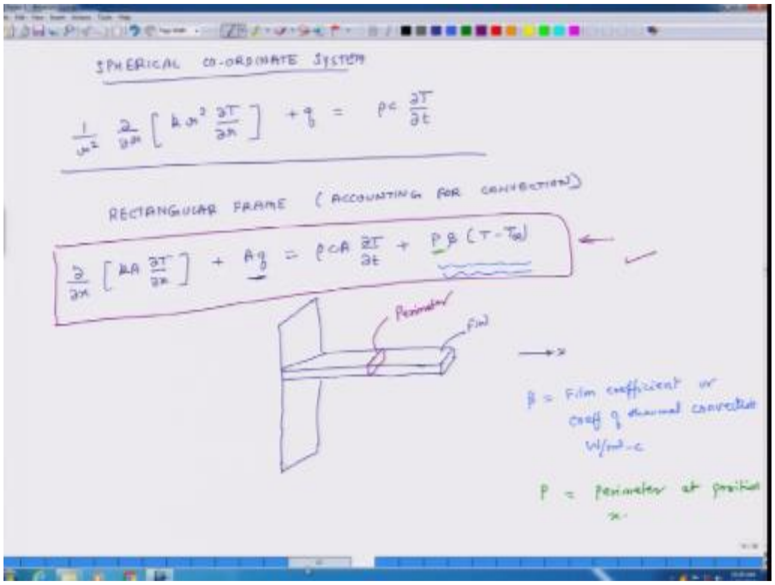

So to account for convective heat transfer and we will develop write the equation only for rectangular coordinate system rectangular frame and accounting for convection also my governing equation is so the governing equation is pretty much same but we add one extra term and the extra term on account of convection is P times B and $T - T_\infty$ and I will explain that so to give you an example suppose, suppose I have this FIN a thermal FIN and I want to see how temperature is changing in this thermal FIN .

And in this thermal FIN I have a heat source also which is uniformly distributed and the strength of this heat source is Q, Q W^2 for per cubic meter and then because of conduction heat is getting transferred along the length so this is my x direction but also because there is it is surrounded by ar so heat is also getting dissipated due to convictive transfer so then that extra effect due to conviction is captured by this term and here β is film coefficient or coefficient of thermal convection and it is units are in W/m^2 per degrees centigrade.

And then I have this term call P so P is perimeter at position x so what is that mean so suppose I have this point P then at this point that is my perimeter okay, so this differential equation is true at a point right. Differential equations always to at a particular point and we have to find solutions for the point-to-point basis, so at that particular point P this is the differential equation which governs it and at that point I have to find P by actually imagining the FIN and I have to put that number in the differential equation okay.

So P is known P can vary with the as with respective x because here the FIN is of a uniform cross-section but it can change, so P can be a function of x but we, we know P, we know P now all these equations this equation.

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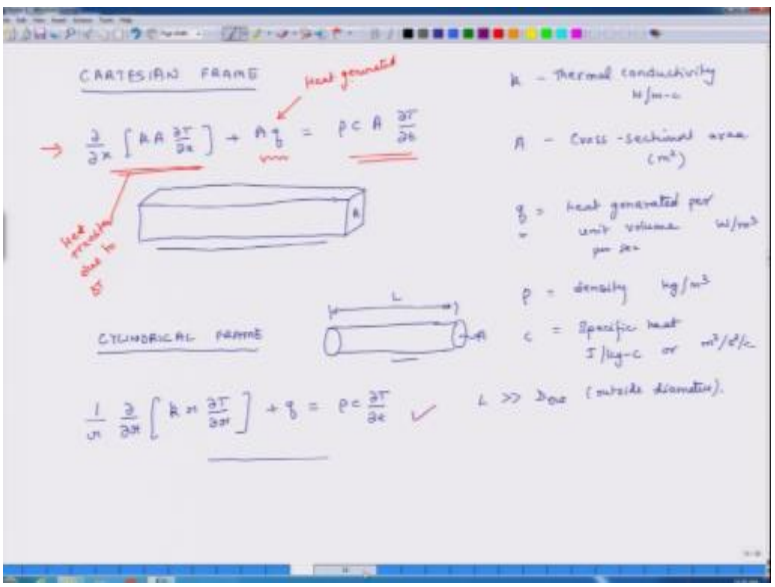

And the other 2 equations which we had developed through this equation.

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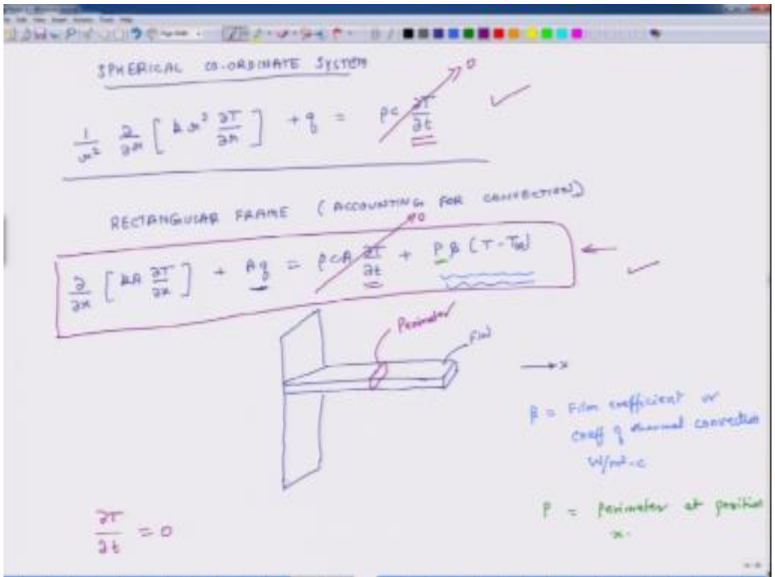

And this equation they all have this partial of temperature with respective time so this is for a system where we are looking for transient solutions, where solution is changing with respective time if the solution does not change it time then it will be a steady state system and steady-state system partial of temperature with respective time will be 0, so in this case this term goes to 0 for a spherical coordinate system this term goes to 0.

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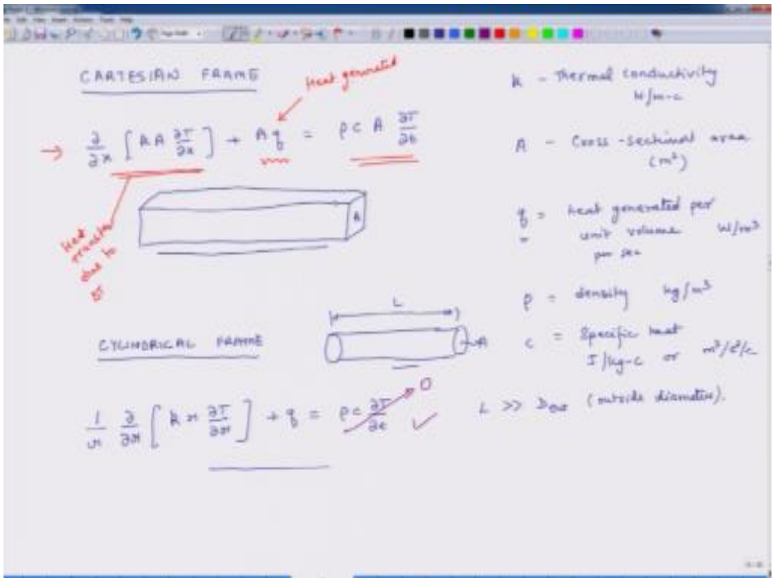

For a cylindrical frame of reference this term goes to 0 so if I just drop the temperature derivative of the time derivative of temperature then I can get steady state equations.

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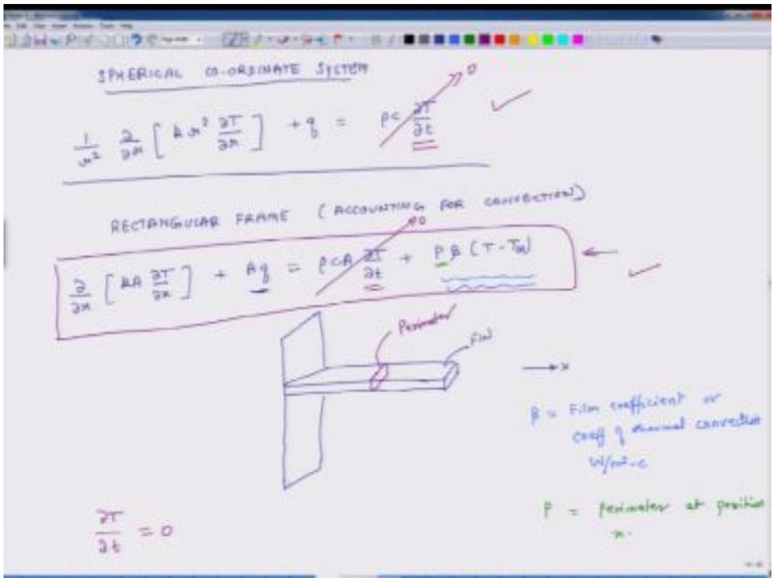

For all these 3 coordinate frames.

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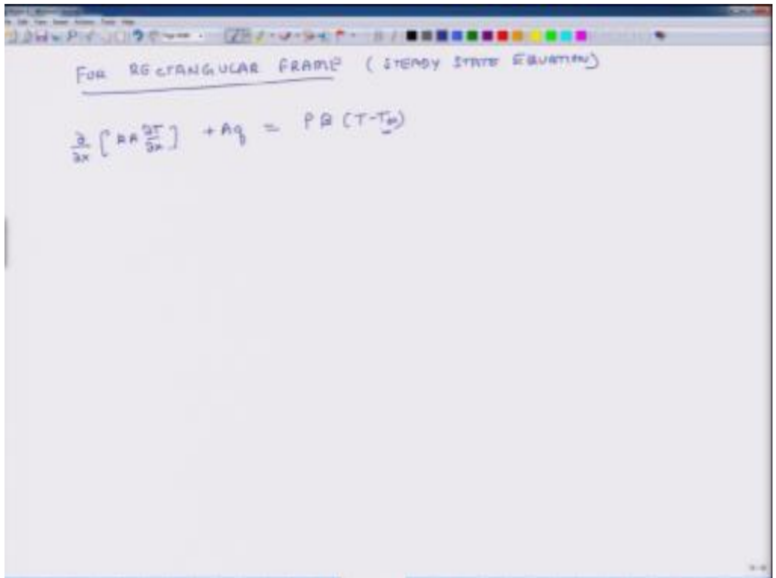

So for , for rectangular frame okay my governing equation steady state equation the governing equation is partial with respective x KA ∂T over ∂x + Aq = Pβ times T – T_∞ by the way I did not explain what is T_∞ .

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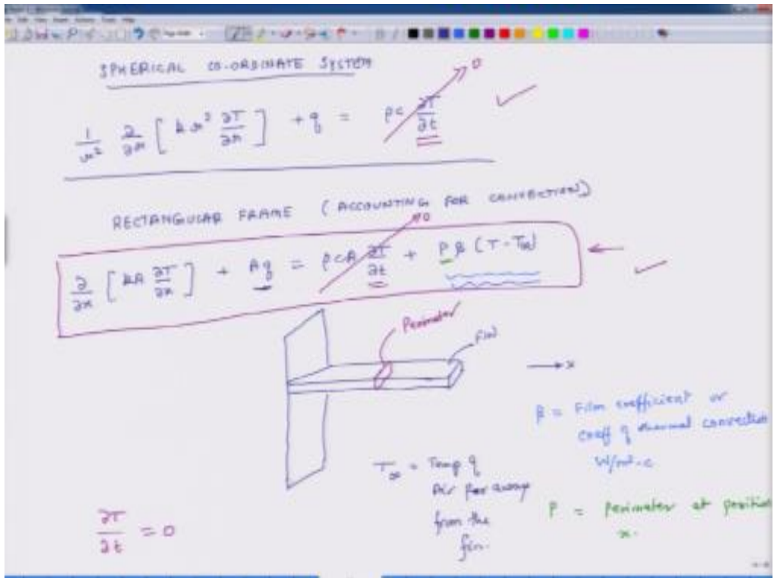

So in this case T_∞ is the temperature of surrounding here so it is temperature of air and it is infinity because we are measuring this temperature quite far away from the FIN temperature of air far away from the FIN So that is my T_∞ if T_∞ is small then we will have more convective heat transfer if it is more than convective effects will be less and β is I explain it, it is film coefficient.

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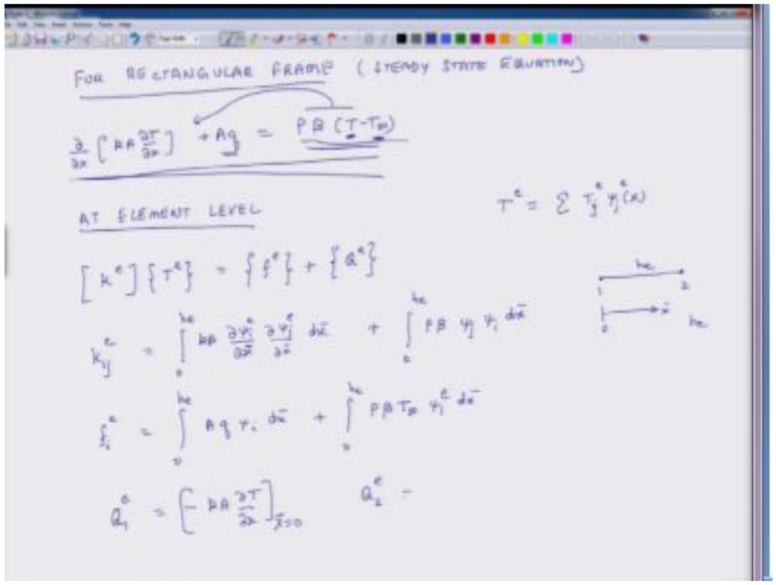

So that is my governing equation for the rectangular frame of reference now what we will do is we will actually solve a problem 1D heat conduction problem and where we will also explain how we incorporate this convective heat transfer condition into the overall differential equations so for, for rectangular frame of reference we can express at element level right. At element level my equation will be k^e times T^e = f^e + some source term Q^e and the value of K_{ij} will be integral over the domain of the element which is 0 to h^e.

KA so these are all local coordinate systems, so I have not x but I have $\bar{x}$ okay. So that is my K_{ij} + now you see that this T is an unknown so we always typically move all these unknown primary variables on the left side so if I move this term PB times T here and I multiply this by weight function and integrated to over the domain that will also be a part of K_{ij} so there is a term related to K_{ij} and that is 0 to h^e P times β and because we say that T temperature is equal to ∑ of T_j, ψ_j x right that is how this is for the e^{th} element.

So I forgot so in this case this will be equal to the convective part will be ψ_j times ψ_i d_X this is all Rayleigh Ritz formulation so weight function is same as ψ_s so by default we will be following Rayleigh Ritz formulation in this whole system then my f_i this equals 0 to h_e so f is the source term which relates to how much heat is being generated so that is a times Q times, ψ_i dx and then there is another source term which is related to T_∞ right. So this + 0 to h_e P β T∞ so formulation is exactly the same if you go back to a earlier lectures only thing which we have added is this PB, P times β times T - T∞.

And when you find the residue multiplied it by a weight function integrated over the domain and do the weak formulation you will get all these expressions, so that process is exactly the same and then Q_1^e so suppose we have 2 noded element this is 1 this is node 2 and the length of the element is h_e and Q_1^e = - KA ∂T over ∂x at x=0 so this is my local coordinate system $\overline{x}$, is 0 here and it is he here and Q_2^e = KA , ∂T over ∂X at $\overline{x}$ =h_e so this is these are the in element level equations.

Now what we will do is we will actually in our next lecture we will assemble all these elements and then solve for an actual problem we are we will specify the boundary conditions and see how we account for actual 1D heat conduction equation in which of you also have convictive effects, so this closes our discussion for today and we will continue this particular discussion in the next week which is the 6^{th} week of this particular book course thank you very much and have a great day.

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