Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 03
Nodes, Elements & Shapes Functions

by
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Hello, welcome to basics of finite element analysis, this is the third day of the first week. In our last lecture we had explained two important concepts; one is using this example of where we were trying to calculate the area under a complicated curve. We had explained what is meant by an interpolation function or a shape function that is one concept we had introduced, and the second thing we had shown was that the error at least in context of that particular example between the exact value and the numerical value of the area under a curve dependent on two parameters, one was the size of the element of a you know size of the element, so the larger the number of elements the lesser was the size and then the second way to reduce.

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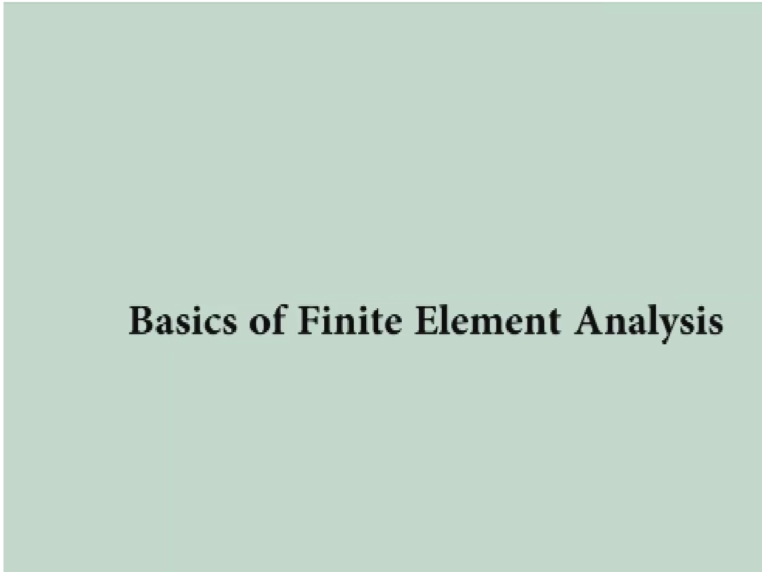

Error was to have a more accurate interpolation function or shape function. What we are going to do today is we will continue that discussion but in context of a different example, a different example.

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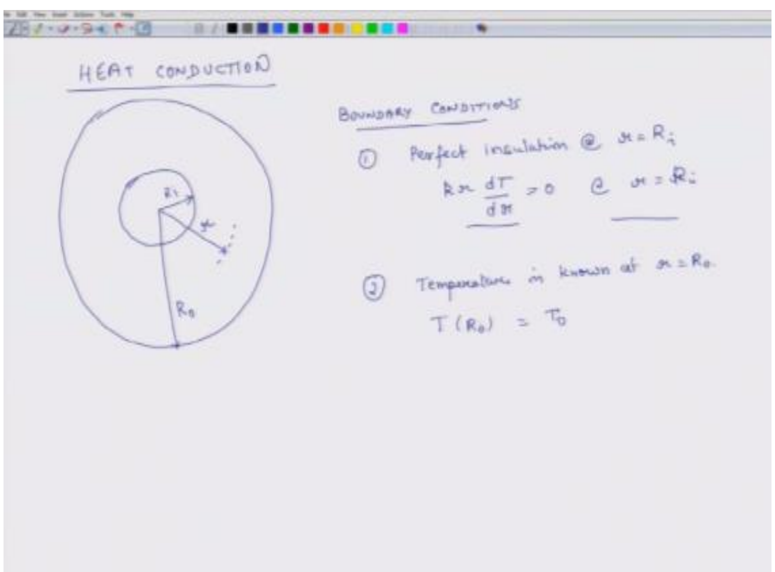

So, so what we will do is consider the example of heat conduction okay, so suppose I have a hollow pipe and this is the outside diameter, outside circle, the inside circle is having a radius of R_i and the outside circle has a radius of R_o and at some radius it is R. So you have a hollow pipe so what essentially you are seeing is the cross section of the pipe, and let us look at the boundary conditions.

The first boundary condition, so what is the boundary condition, whatever are the physical boundaries of the object, whatever conditions physical conditions are present there that is what is referred as boundary condition, so the first boundary condition we are specifying we know that is that on the inside surface we have perfect insulation at r $=R_i$ okay, so if you have perfect insulation there is no heat, no heat either going into the is not crossing this boundary either in outwards or inwards at r $=R_i$

Okay so mathematically I can express that condition as K times R dT/dr is equal to 0 at r $= R_i$ okay, so how does that come I mean for this you have to refer to some heat conduction equations, the focus in this class will not be explaining how these conditions are coming, but if we know the condition then how do we solve the differential equation? The second condition is that at r is equal to R_o temperature is specified okay.

So $R_{o,}$ so T so let us say temperature is designated by letter T so that is T at R_o is equal to T_0, so these are the two boundary conditions, there is no heat crossing the boundary on the inner surface, on the outer surface temperature is T_0 okay. Now we had explained earlier that if we have to solve n the other third thing is and that is not a boundary condition that there is heat being generated in, in this whole volume of the thing.

So when you look at this problem what you see is that the cross-section of this, of the pipe is axi symmetric right, cross-section is axi symmetric.

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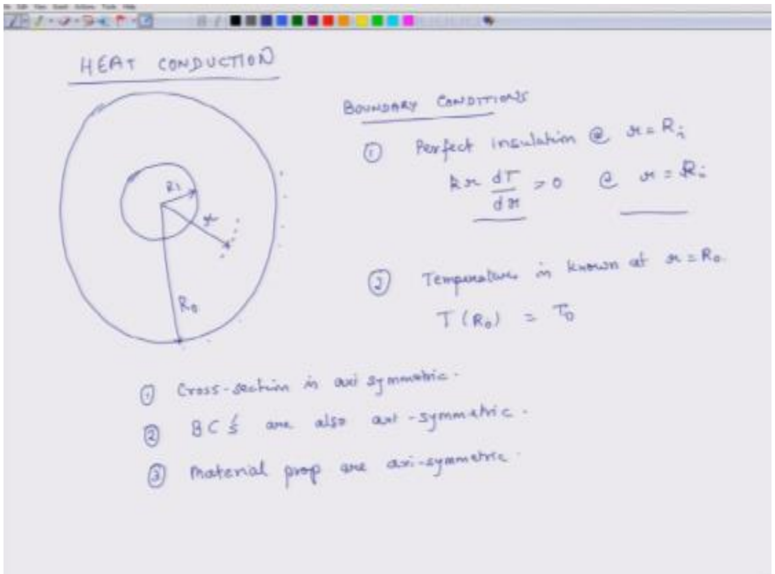

Which means there is no change in cross-section if I move in the θ direction, the second thing we notice that does not matter whether I am at this position, this position, this position, this position you know, the boundary conditions are also axi symmetric, so whether θ=0 degrees on the outside boundary temperature is T_0 and for any value of θ temperature is T_0 on the outside boundary condition boundary for any value of θ there is perfect insulation on the inside boundary.

So cross section is axi symmetric, boundary conditions are axi symmetric, and then I am telling you because we know this that material properties are axi symmetric. An example of this would be suppose you have a pipe and that pipe, on that pipe I have put another pipe, material properties are varying in the radial direction but in the θ direction they are not varying, the θ direction are not varying.

So this is one of those problems they are material properties this case conductivity K can vary in the radial direction but it is not varying in the θ direction, so material properties are also a thing

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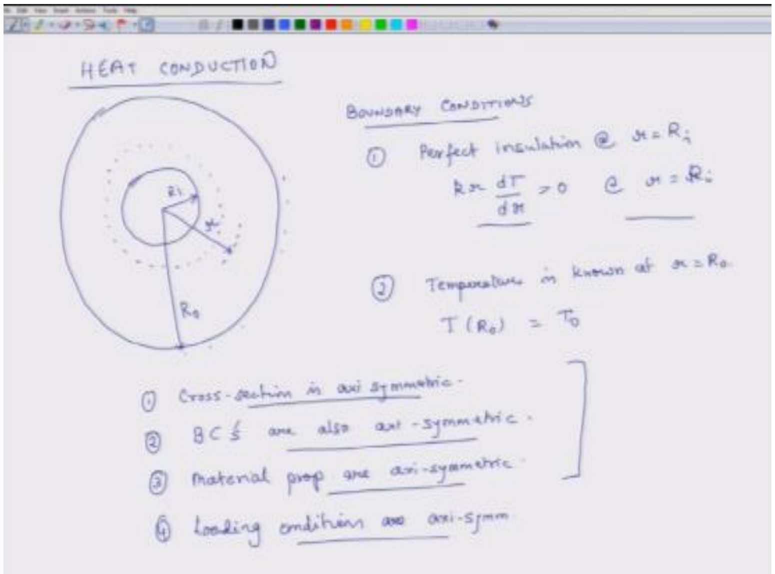

So if these three conditions are met where the geometry is axi symmetric, boundary condition is axi symmetric, and material properties are axi symmetric okay, and fourth is loading conditions are axi symmetric, so this is the fourth condition. What does I mean by loading condition, so it is in the volume of this pipe in some locations heat is being generated maybe heat is getting generated let us say R=Rh but it is getting generated at in an axi symmetric sort of way.

At a particular radius, one example could be you have a wire, a conductor wire, copper wire which is having several layers of insulation right and current is passing through it, so wire is a source of heat, that heat is being generated in an axi symmetric way okay it is getting generated in an axi symmetric way. The properties of the conductivity of the wire in the center it is that of copper, outside there is one insulation, outside of that insulation there may be another layer of insulation, so there may be three or four layers.

So the material properties are changing in radial direction but they are not changing in θ direction, also the cross section of the wire and all individual content is also axi symmetric so everything is axi symmetric in that.

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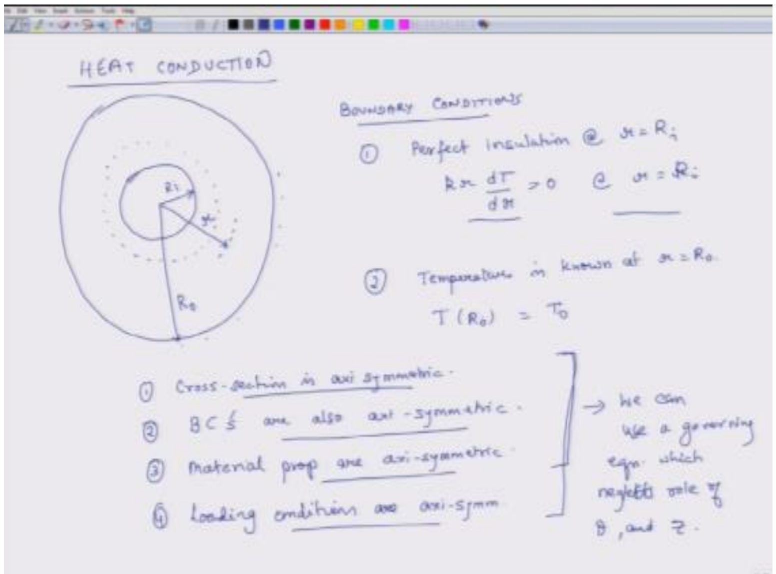

Situation in case of wire and same thing is here, so if all these four conditions are met, cross-section, boundary conditions, material properties and loading conditions, all of them they have to, you have to meet. Then we can use a governing equation which neglects role of θ and z okay, so z is the direction of axis and θ is the direction of angle, the hoop angle so we can use a governing equation which is which is also axi symmetric in nature but if even one of these conditions was not axi symmetric, suppose material was not axi symmetric.

Then the governing equation we use cannot be of one dimensional nature okay, so now I am going to write down the governing equation for this problem because that is the first step that we have to figure out what is the governing equation, so we will not derive that equation in this course we will just take it from some other book, so for axi symmetric.

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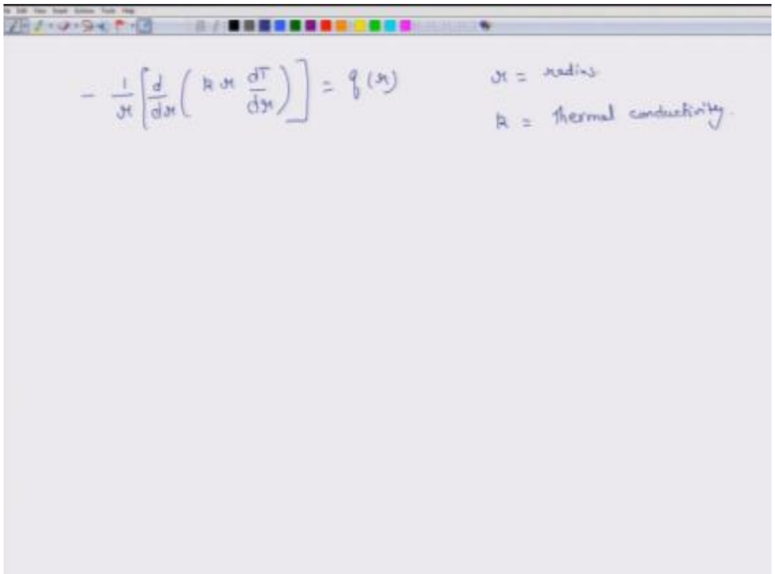

Heat conduction problem steady state heat conduction problem the governing equation is 1/ r [d / dr (k times r dT) over dr] is equal to q(r) and I will explain that, so here r is the radius so if I want to calculate the value of temperature at radius R that is what it means okay, k is thermal conductivity and this thermal conductivity does not change with θ direction but it can change in the radial direction that is why.

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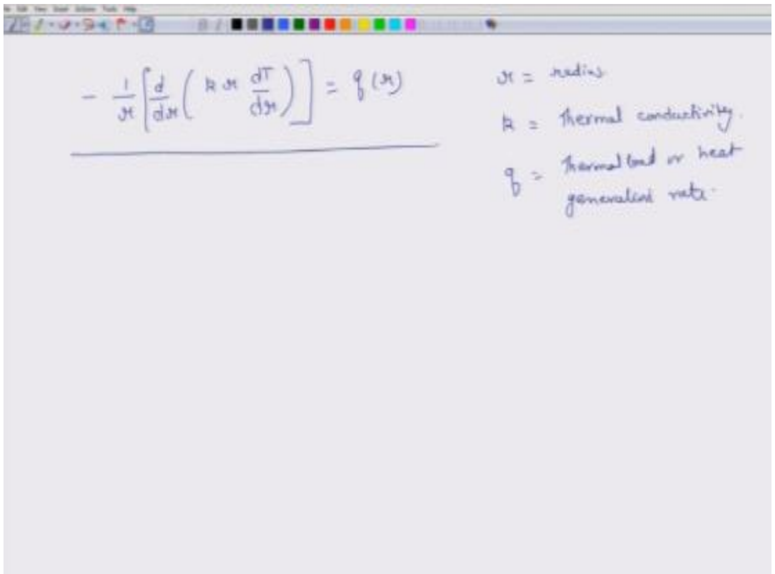

It is in the brackets otherwise it would have been a constant and it would have been out of the brackets. And q is thermal load or heat generation rate I will say, so this is the governing equation and we have also.

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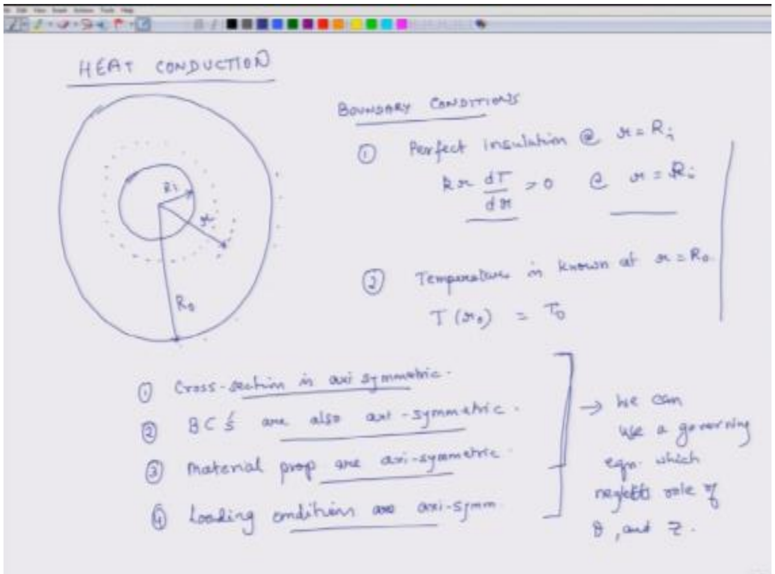

We have already explained that these are the boundary conditions. So this is the geometry, these are the boundary conditions also heat is getting generated somewhere in the body in a axi symmetric way.

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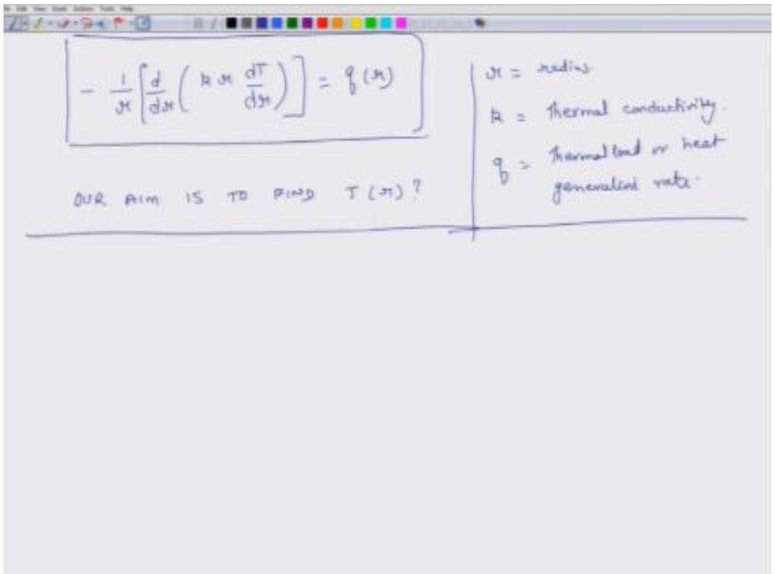

So the governing equation for this problem is this, and our aim is to find T as a function of r okay, so what we are interested in finding out is how does T change as I move from R_i to R_0 from inside radius to outside radius, that is what my aim is okay, so.

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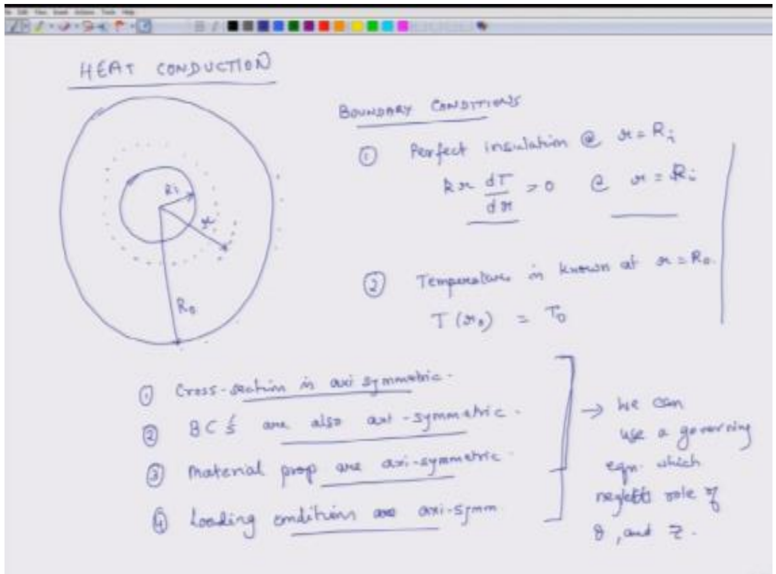

If you try to solve this problem using some standard way in an exact form, in an exact form you will not be able to solve that problem, so what we will do is.

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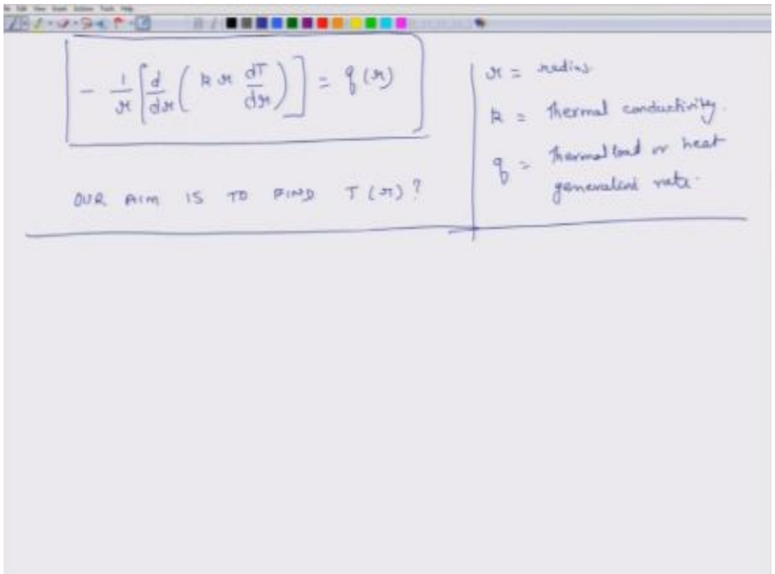

We can solve this problem however using finite element method, so now what I will do is I will just conduct first few three, four steps not in detail but at a slightly more detailed level compared to the example which we saw earlier where we were having, we were trying to calculate the area under a curve, so the first

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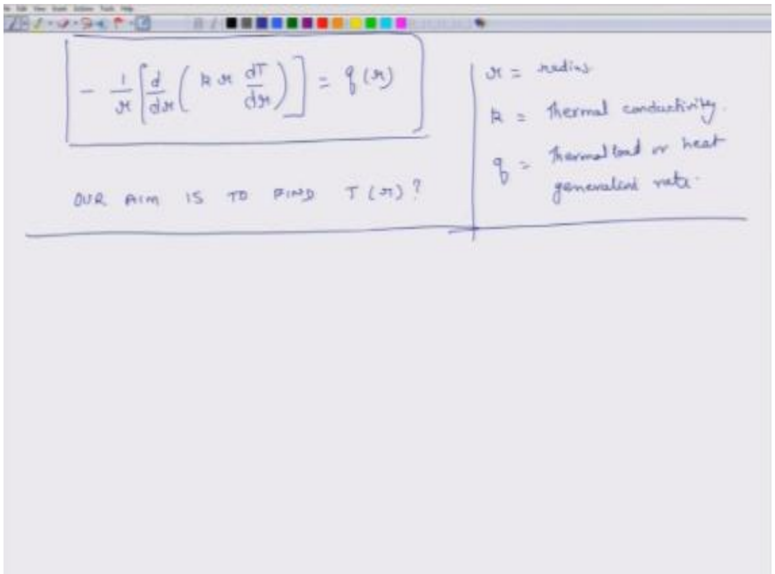

Thing we do is, so what I said that we are trying to.

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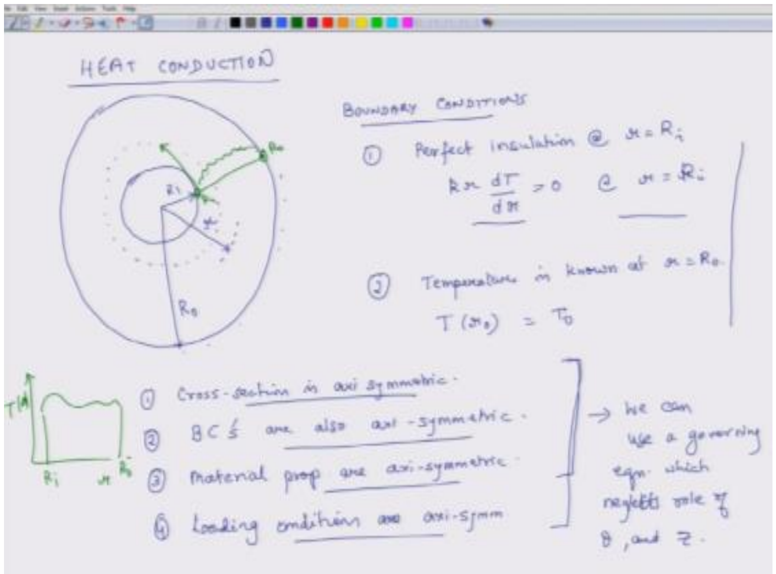

We are trying to calculate how is temperature, see here $r=R_i$ and here $r=R_O$ here $r=R_i$ how is temperature, so temperature varying, it could vary this readily in the radial direction, so this is so I am trying to make a plot, x axis of the plot represents radius, y axis represents temperature, so I am trying to get a plot of this nature, this is radius, this is temperature as a function of r and the domain is R_i to $R_{o,}$ inner radius to outer radius okay.

My domain is I am trying to compute the value of temperature for all values of r between R_i and R_o .

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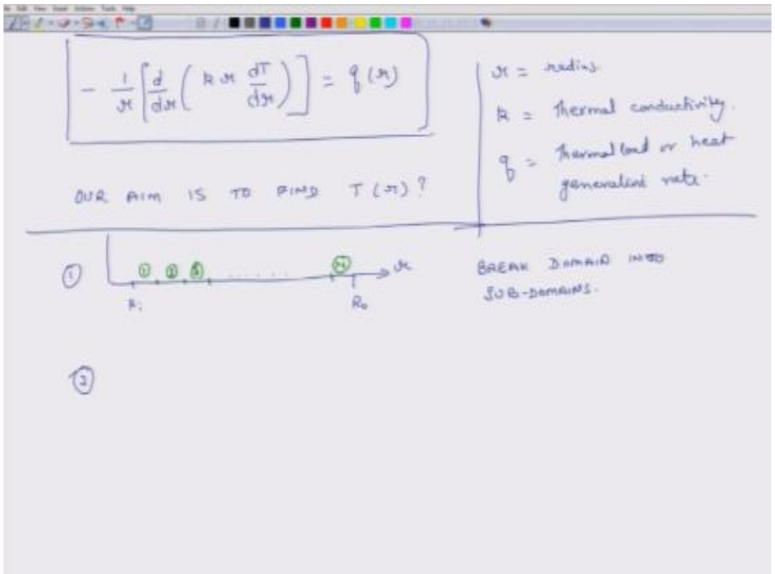

So my domain is this, this is R_i this is R_O okay, this is my axis for radius and this is my domain okay, so the first step we do is we break this R_i to R_o this is my domain, so I break domain into sub-domains, sub-domains I, so what, what I am doing, I will break it up into let us say n elements, so this is first boundary of first element, boundary of second element, boundary of third element and so on and so forth and this is the boundary of n^{th} element.

So this is my first element, second element, third element, and so on and so forth and I get my n^{th} element here, is this clear, this is the first step. Second step, so like in case of area when we are trying to compute between the first boundary of the end point of the first element and the end point of and the second end point of the second first element we assumed how that function is changing and we could make lots of choices for those functions, in first case we said that that function was constant, in second case we said that function was linear, in third case we said function was quadratic.

We make a, we have to make similar choices here okay, so what we do is

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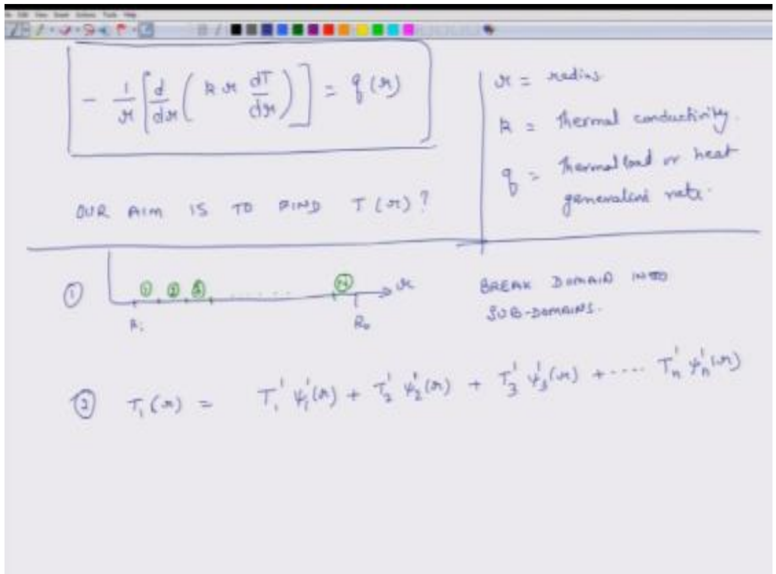

We develop those relations, so we say that temperature in the first element T_1 is first element at radius r is equal to okay T_1 $\psi_{1,}$ and I will explain all this so just be patient and there is a + T_2 ψ_2 r excuse me+$T^1_{3,}$ what have I written, so before I explain what all this means thus so I have a bunch of T's and a bunch of ψ's okay. The superscript of T designates the element number. The superscript of T and also the superscript of ψ it designates the element number; the superscript designates the element number because this is the terminology we will follow consistently through the course.

So this is important to we have to get it right okay, so we are trying in this equation to figure out how is temperature changing from the first point of from the first between the first node of element number 1 to the second node of element number 1 right, and that is why we have written

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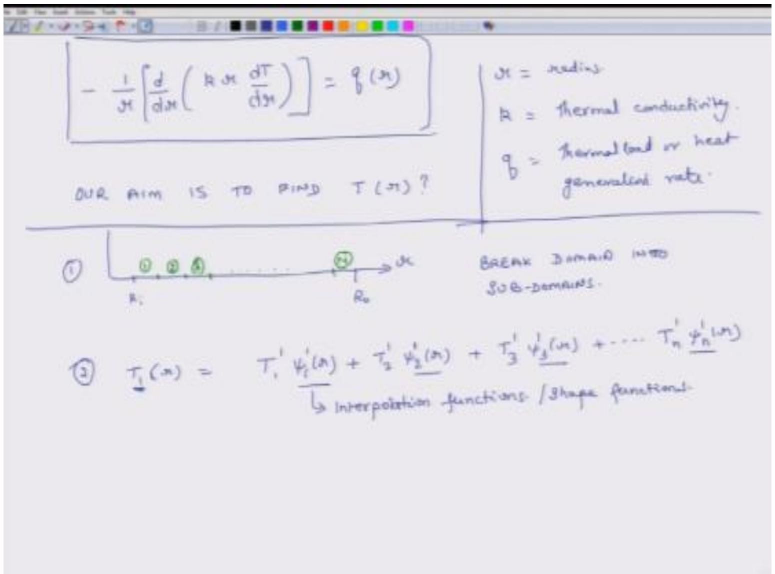

T_1 so this expression is it tells us how is temperature changing over the first element that is all it does, second thing these ψ they also have a superscript and subscript. The superscript again for these ψ's designates that element number and thus and they also have a subscript, these ψ's are interpolation functions and they are not any significantly different than the functions which we had considered when we were computing the area under the curve you can assume that it is a constant function you can assume.

That it is a linear function, you can assume it is a quadratic function so this i could be constant and actually we will when we will do computing since greater after four five lectures we will actually use only these simple functions, we will not use some very complicated functions, so these are interpolation functions or shape functions understood, and I can have n different functions, it is there is no rule which says that only one function will help me explain how the shape is changing okay.

So I can have a constant function and I can also add that constant function to a linear function and I can also add that to a quadratic function and so on and so forth right, so I can have n number of functions.

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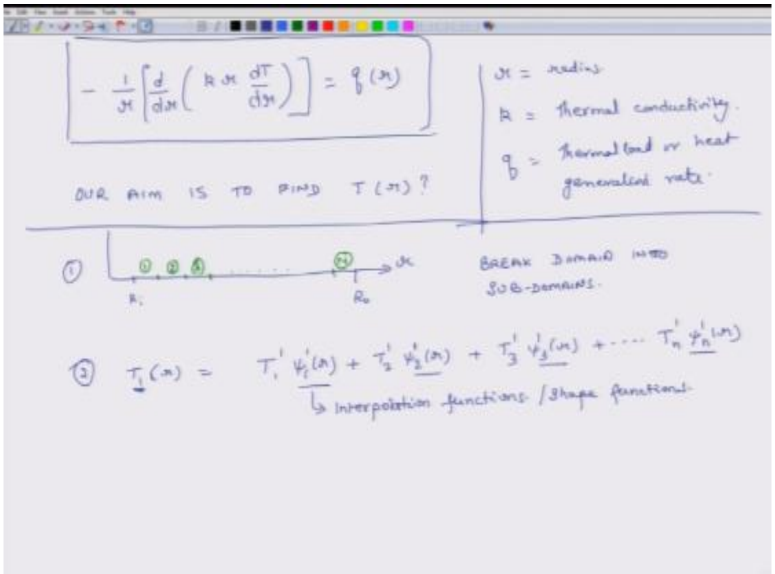

I can have n number of functions which help us understand how the temperature is changing over the length of the element not the overall cylinder, we are right now only at the first element understood, so these are interpolation functions which help us understand how temperature is changing over the first element. They can be ψ functions what is the amplitude of a ψ function, one right?

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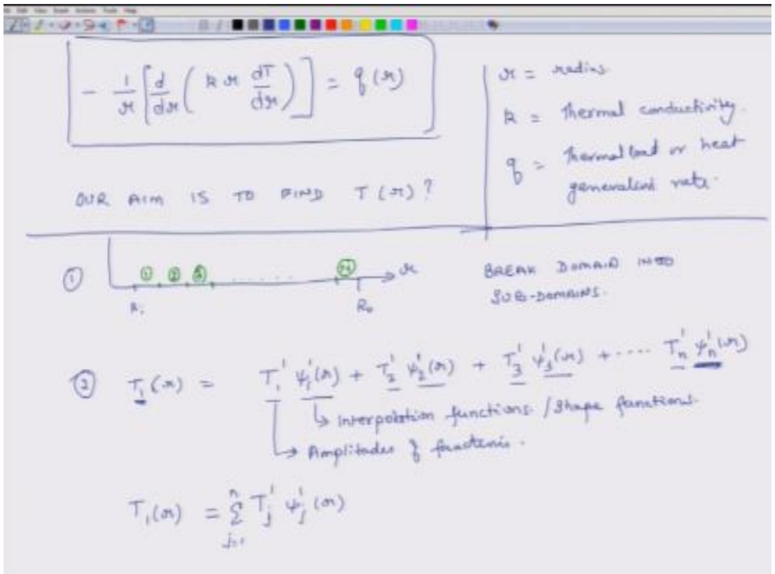

These T's are the amplitudes of these functions, these T's are their amplitudes and there, we will interpret these T's in some other way also, so I can compress this thing as T_1r is nothing but T_1 instead of 1 I say j ψ_j r Σ 1 1 and j is equal to 1 2 small n, small is thus the number of functions which we are considering to capture the variation of that function on an element okay, big N is the number of elements, this is big N.

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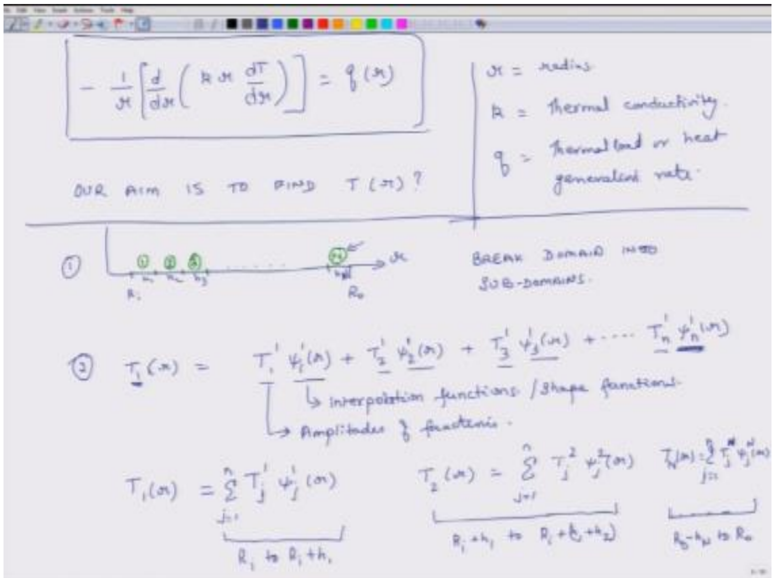

Capital N is the number of elements, small n is the number of functions which we are using on each specific element understood okay. Similarly T_2r and y ever do I have bracket in parentheses because these size they are not constants, they are functions and they are dependent on r right, similarity T_2r is j =1 to n is T_j ψ_j so you have to be patient while writing these because there are a lot of subscripts and superscripts and the superscript will be 2.

And for the n^{th} that is big N^{th} element T_Nr it will be Σ T_j and N ψ_j and I am summing up over 1 to n is this clear, right this function is valid from x_0 to $x_0 + h_1$ let us say the h_1 is the length of the first element, so let us say this length is h_1 this h_2 this h_3 and so on so forth this is h_N begin, so the first function is valid after that the element does not exist so we do not have to worry about that function.

So the first function will be computed only from x is equal to x_0 to x so I am sorry it should not be x_0 it is R_i to R_i +h_1, so it is valid from R_i which is the internal radius to R_i +h_1 which is the length of the first element. This function is valid from R_i +h_1 to $R_i + h_1 + h_2$ is this clear, and so on and so forth. Similarly we can find this nth^T will be valid from

R_o either you add up $h_1 + h_2 +$ you know h_{n-1} or from the other side it will be R_o-h_n to R_o does not matter okay, so for the

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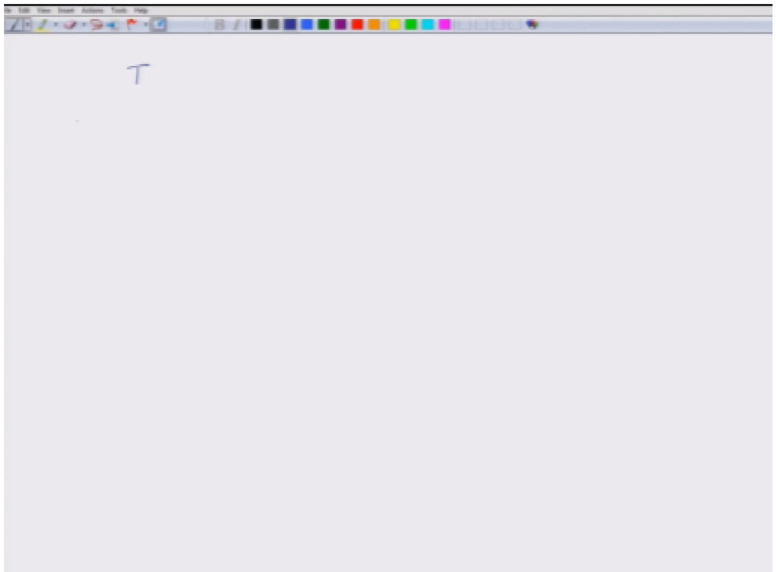

E^{th} element so

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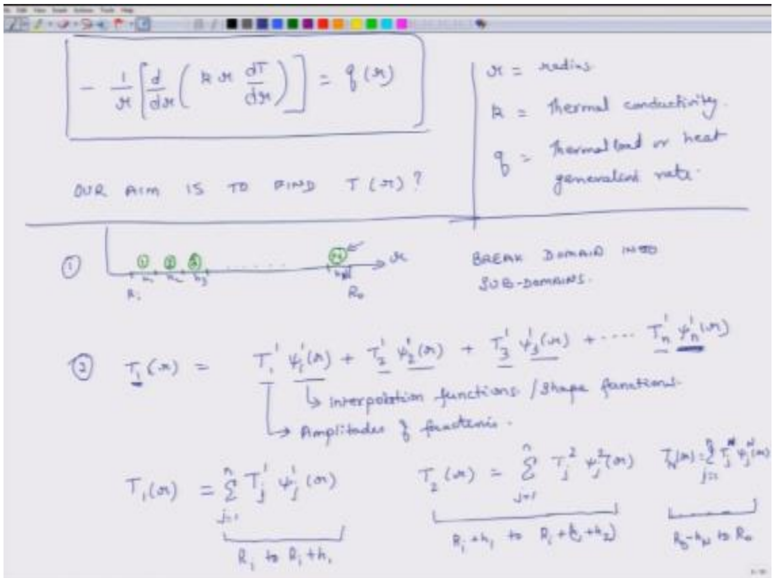

This is T_1 this T_2r this T_3r suppose I have to compute the value of T for e[th] element.

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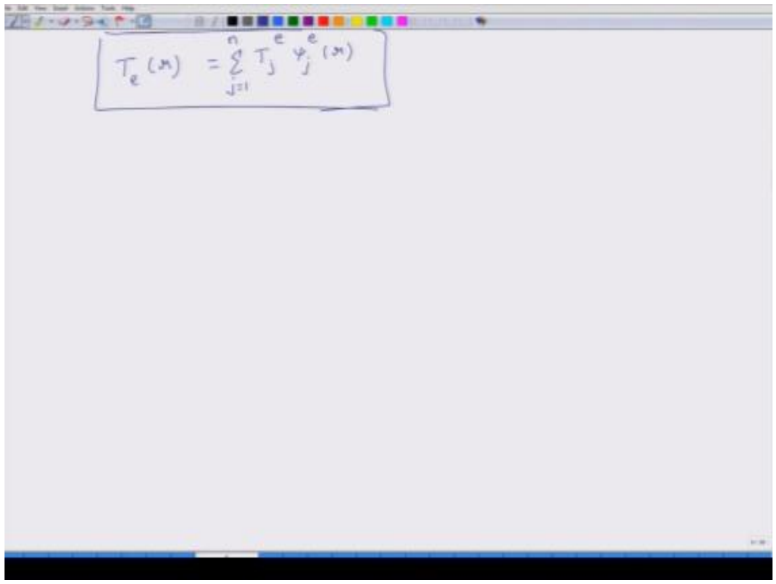

Then $T_e r = T_j \psi_j r$ e,e n okay, so I think we will close at this stage today and we will continue this discussion tomorrow.

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