Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 29
Radially symmetric problems

by
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Hello, welcome to basics of finite element analysis. Today is the fifth day of a this week and what we are going to discuss today is, Radially symmetric problem. So till so far whatever problems which we had been doing where using a Cartesian frame of reference but today we will actually do again a boundary value problem of second order. But here we will use a cylindrical coordinate system.

Examples of such problem could be that suppose you have, a very long pipe and some hot fluid is passing through and what you are interested in knowing, is that how is temperature changing in the radial direction. If the material properties of this pipe are radially symmetric and also if the cross section of the pipe, is uniform and radially symmetric even though and it also does not change in the z- direction.

Then we can reduce this three-dimensional problem to a one dimensional problem. So in general for several types of such types of problems, the governing equation could be written something like this.

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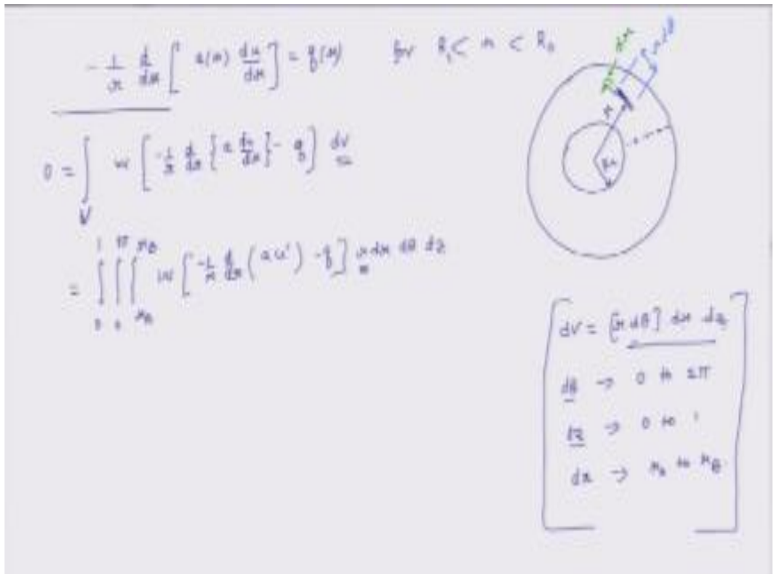

And this governing equation is valid for values of R, which are larger than internal radius of it could be zero or non-zero entity and an external radius. So first the first step in this whole process if we have to develop its K matrix is to develop a, integral formulation for the whole thing. So 0 equals W times [-1/r d/dr] and at this stage we have to realize that we are integrating it over the whole volume of the system, okay.

So suppose this is my cross section and I am looking at a small element of this, right? So that is my R_i, the location of this element is r, the thickness of this element is dr. And the length of this element is rdθ. So then the volume of an element will be. So this I will be integrating with respective volume and this dv is equal to r times dθ times dr which is the area of this cross-section small element and then in the z-direction it is dz.

Also when I so, so this is a volume integral and also the limits of this volume integral will be, when I am integrating on θ it will be from 0 to 2Π. When I am integrating on z, in this case I assume that the length of this pipe or cylindrical cross-section is one, so just for simplicity purpose that is the only reason but if it is l then I can make it 0 to l. So d when I integrate in for the on z the limits will be 0 to 1. Because dv is having dθ, dr and dz.

So I will have three success integrals and when I am integrating it on R, I am discretizing. This whole radius R_i minus into small elements, okay. So it will be from R_A to R_B. So my weighted integral is basically three integrals w[-1/d. d/dr (au')-q] and then the first thing I am integrating it is with respective r dr. So I am replacing this dv by this entire thing, then I am integrating it with respect to θ then with respect to z. So my limits are r_A to r_B, 0 TO 2Π and 0 to 1, okay.

I have this extra r term because of this geometry and this is how the cylindrical geometry which for a Cartesian system it would have been just dx times dy time dz.

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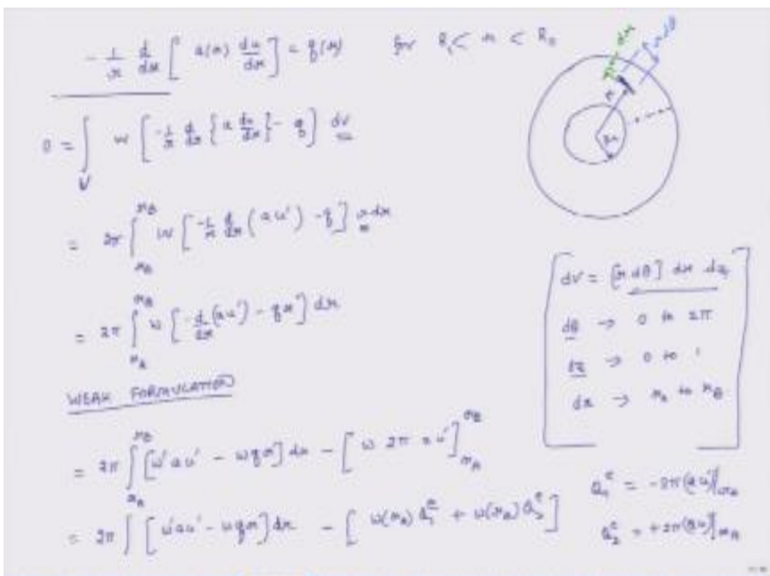

So there is extra dimension does not come but here we have this thing. So that is why I wanted to cover this. For a cylindrical system which is radially symmetric we have this type of a problem or formulation. So once I integrate it from on z all the parameters in the parenthesis are independent of z and θ. So this dz is just goes away, right? if it was 0 to l then it would have been replaced by l, okay.

Same thing with θ, so this goes away and I get a 2Π here. So this is equal to 2Π, r_A to $r_{B,}$ w and I multiply R to the terms inside the bracket. So I get this R, R cancels, so I get -du/dr au' – qr dr at this stage I do my weak formulation so I shift and yeah so I shift the differentiability operator from this au' to w okay. So what I get is, 2Π r_A to r_B w' au' – wqr dr - I get some boundary terms W times 2Π au' evaluated at r_A and r_B.

And this, so I get the same expression here and on the boundary term I get W evaluated at r_A times Q_1^e + w evaluated at r_B times Q_2^e, where Q_1^e equals -2Π au' evaluated at r_A and Q_2^e equals +2Π au' evaluated r_B, okay.

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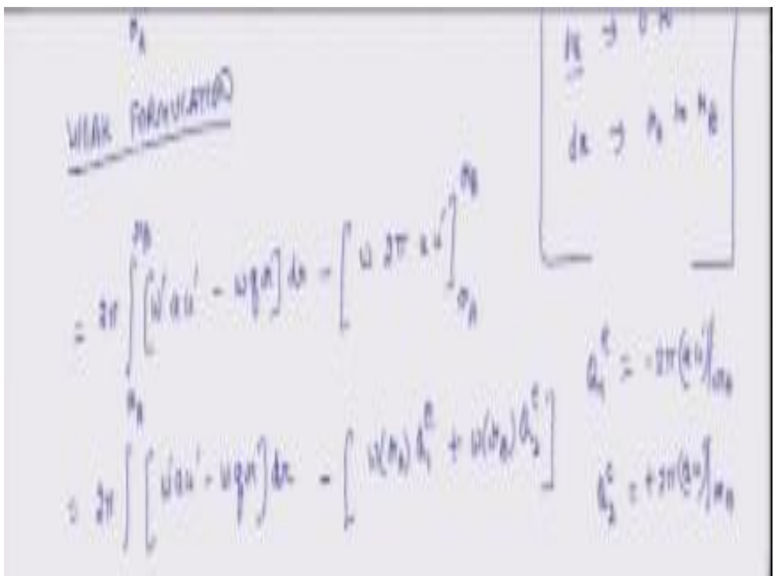

So that is my weak formulation, so now I have to add to this stage I assume some interpolation functions and what I will do is I will assume and using those interpolation functions I can construct the k and F all these mattresses okay so at this stage I assume

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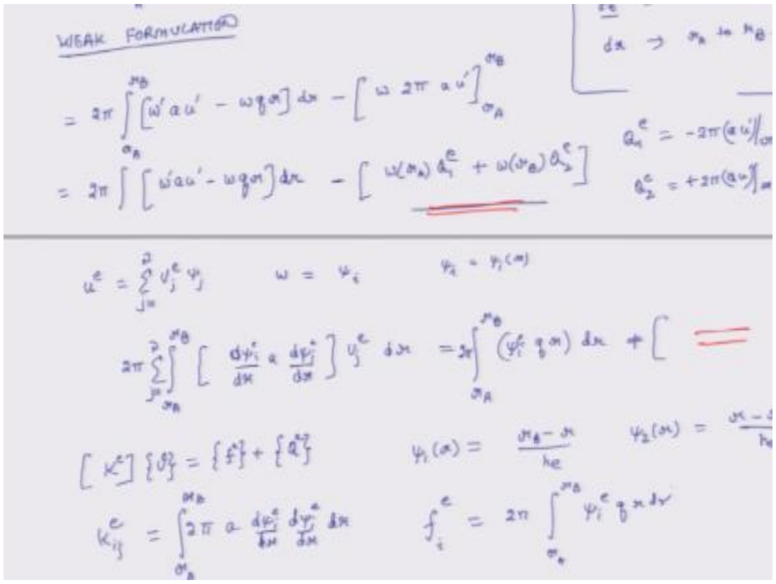

That u^e element is equal to right and w = ψ_i where ψ_i is equal ψ_i it is a function of r so using this I get 0 =2Π integral from r_A to r_B dψ_i over d_r a dψ_j over d_r times u_j^e I am Summing it at up over j . So this is one term dr-r $_A$ to r $_B$ 2Π (ψ_i qr) dr – these terms in brackets okay or what I can do is I can remove 0 from here and I put an equal sign here and this becomes a positive sign and off course this is for the e^{th} element .

So at the end of the day I get K matrix times u vector is equal to a force vector + a Q vector all at element level and what is the definition f of ψ_1 and ψ_2 So ψ_1 (r) = r_B – r divided by h_e ψ_2 (r) is equal to r- r_A divided by h_e or if I want I can go to local coordinate system and changes these things and finally K_{ij} e^{th} element equals 2Π a d ψ_i over dr dψ_j over dr if you integrated from r_A to r_B d_r for the e^{th} element and your f for the e^{th} element = 2Π integral r_A to r_B ψ_i^e times qr dr

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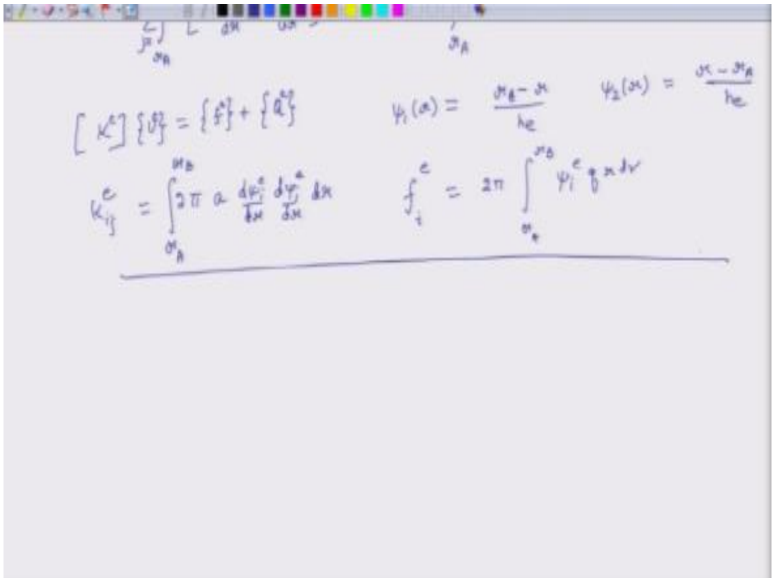

So if you have this formulation and then you can solve a lot of problems which have practical problem suppose there is current flowing in a wirer or in a heavy conductor and it is generating heat and how is that heat getting distributed through the insulating mechanism and these type program we which comes really important when there are big fat conductors running in the ground and you want to see how is your insulation going to deactivate over a period of time or flow of fluid or a diffusion in a radial direction so all these types of problems can be addressed using this formulations approach.

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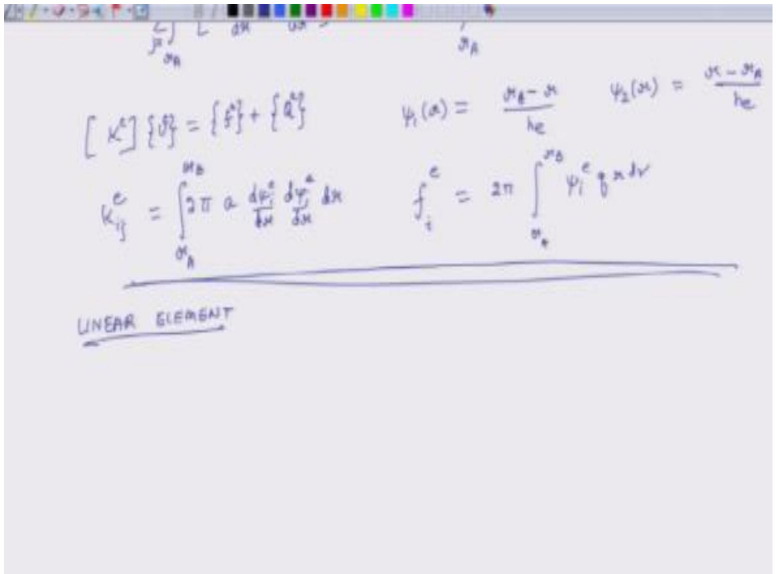

For the linear element that is a an element which is linear in the radial direction in the radial direction okay so we have to remember that.

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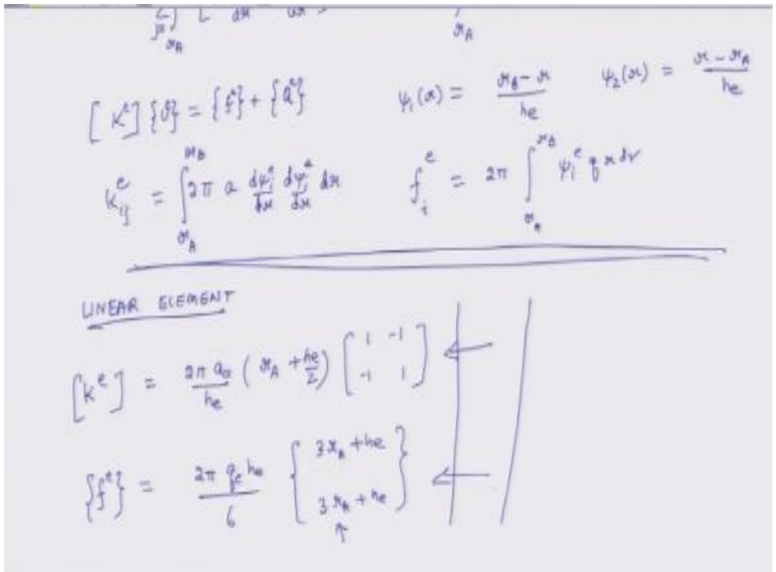

k^e = 2Π a_e over h_e times $r_A + h_e$ over 2 1-1 -1 1 so that is the definition of K matrix and the definition of e matrix vector for the e^{th} element is to 2Π q_e h_e over 6 times a vector $3r_e + h_e$ and 3 $r_e + h_e$ okay so what is r here it is the actual position of that radius, so this I just wanted you to look at the excuse me so this is actually not e this is r_A.

So the stiffness matrix properties in this case they change as I move out or move in if they are dependent on r_A okay so this is what I wanted to discuss in context of readily symmetric problems for cylindrical coordinate systems and you can use this formulations to address some of your needs this is also a one dimensional problem but it is formulations requires a little different approach because here you just do not

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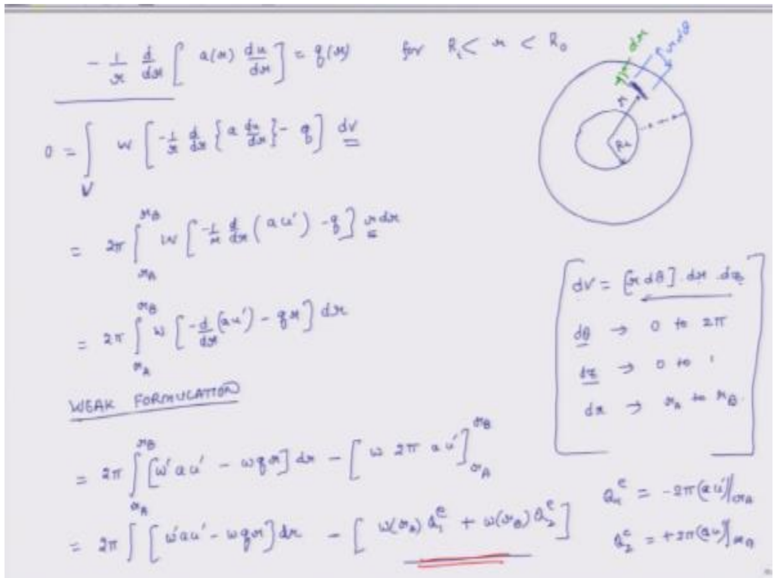

Integrate with respect to dx,dy,dz rather the definition of d_v is rdθ times dr times dz so in the formulation things become slightly different, so this is what I wanted to discussion today in context of Radially symmetric problems, thank you very much and we will continue our discussion the next class bye.

<u>Acknowledgement</u>
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