Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 28
Assembly process and the connectivity matrix

by
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Hello welcome to basics of finite element analysis today is the fourth lecture of this week and what we will do is we will continue our discussion on the assembly process and we will introduce a new term called connectivity matrix which is used to help us assemble complex assemblies on a computer but before we discuss this I wanted to make one specific point related to implementation of boundary conditions in the last lecture or the lecture before that we had seen that if we have.

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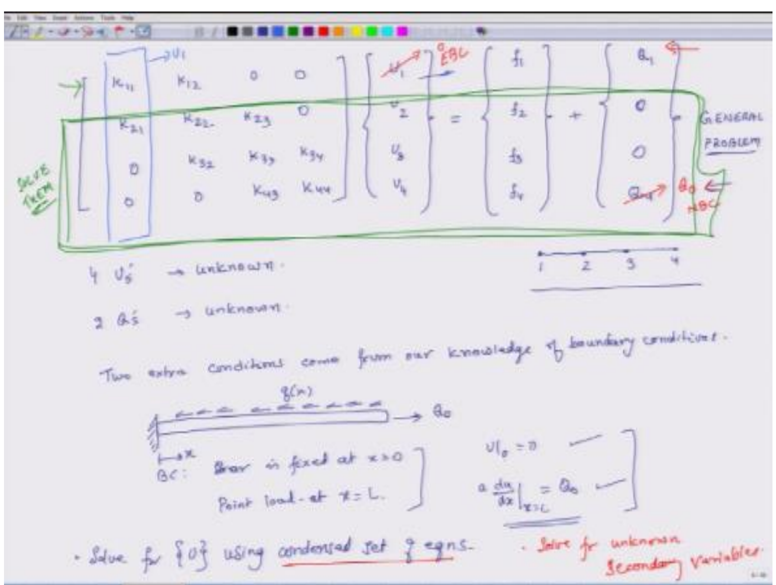

A bar which is broken into three elements then at the assembly level we got four equations and the K matrix is four by four type and then looking at the specifics of the problem we had seen that u was has specified a zero at x equals zero and that x is equal to l external force was applied or which was equal to Q_0 so we had applied in that case we had made u_1 to be 0 in the first equation and that was the enforcement of the essential boundary condition and the second boundary condition which was as specified was that force at x equals L is Q_0 and that is what we had applied here in the last equation.

And at that point I had mentioned that we look at these assembled equations and we look at only those equations for which u is not known and then we will only use those equations to solve for the unknown values of use but I should have additionally mentioned that when we look at these equations those equations which is which are captured in this green box we have eliminated basically one row from the whole system of equations and we should also eliminate one column from the system of equations so that are reduced K matrix is still of a square type.

So in this case we will eliminate the first column because the first column is associated with u_1 which is the boundary condition which we know okay so not only should remove a certain number of rows but an equal number of columns from the system of equations so that we get an n/n K matrix and using this method then we actually solve for unknown primary variables so this is something I wanted to mention because I think I forgot it in that class and you may be getting confused so just wanted to bring it to your notice. So today what we are going to discuss is

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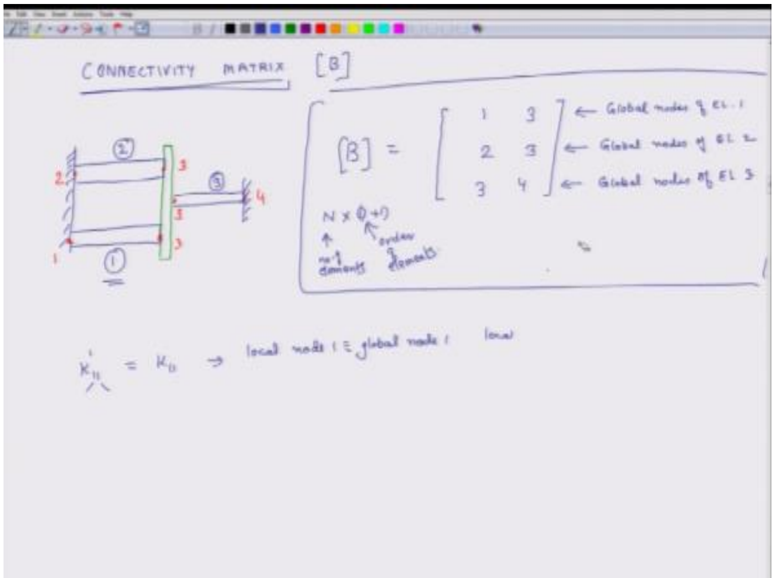

A matrix called connectivity matrix, connectivity matrix and in context of this we will look at the Multi bad problem which we had analyzed in yesterday's lecture, so the bar that problem was something like this so I have two parallel bars connected by a rigid plate and then I have another bar at the middle which is rigidly fixed and I had assign some nodes to each bar is represented as one element, so these numbers in red are the global node numbers okay and we know how to straight away calculate the stiffness are the K matrix for each element using the methods we have discussed so our final lasts several lectures what we are going to do is develop a matrix which will help us in the assembly process.

So and that matrix is known as connectivity matrix and it is designated by this letter B so let us look at this, so my connectivity matrix B equals and first I will write down the matrix metrics and then I will explain it is 132334 okay the first row of this matrix represents global nodes of element 1 okay, so I should level my element this is element number one the element, this is element number two and this is my number three, so my first element is this guy its first global node is one second global node is three so it is like that the second row represents Globe nodes of element number two.

So that is two and three and the third row represents global nodes of element three okay so this is what is the what are other dimensions of this matrix it is n times o where n is the number of elements and this is for the line elements only and o is so it is o +1 o is the order of elements so if it is a linear element then how many columns we are going to have 2 because o is 1. 1+ 1 is 2 if it is a quadratic element then I will have three columns in this matrix okay and the first row, first column see this is c_1 column one this corresponds to first local load number right.

so far global element 1 fist local load number is 1 which is what c_1 and this second column represents second local load number, okay so if I there is a number in third, second column that corresponds to the second node of the element now this matrix servers as a very powerful tool to develop our overall K matrix we will explain that, so for this system so first to compute the element level K matrices all element level K matrix and then you assemble them how do you do that.

So you are k_{11} for the first element is equal to global K_{11} why because local node 1 this is local node right this is local node 1 this is local node 2 so local node 1 corresponds to global node 1 and local node and the second indices again one local, local actually that will just create confusion I will erase this okay first one first vector corresponds to the first node second vector corresponds to the second node, third vector corresponds to third node like so on and so forth okay.

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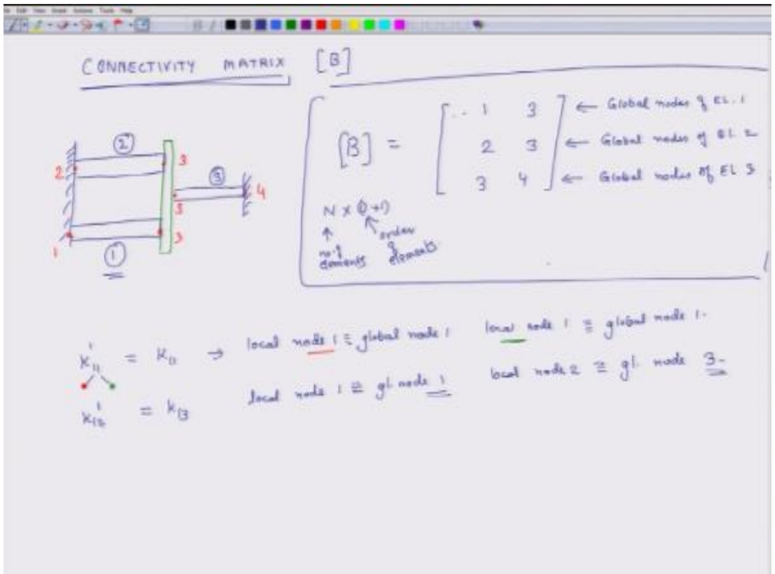

so this first indices is one and that is the local node number which is this second indices is again one which is again the local node number and we know that local node number of local node one corresponded to global node one, so K_{11}' it goes into calculation of K_{11} similarly K_{12} not prime K_{121} which is the first element it corresponds to K_{13} global K_{13} why because local node one corresponds to global Node 1 right.

So because the superscript is 1 only we are lonely looking at the first row okay then local node 2 corresponds to global node 3 right the superscript is 1 so I only look at the first row, in now the first subscribe is one there is that corresponds to global node1 the second subscript is 2 that is corresponds to global node 3 I am only looking at the first okay now we also see that term number three it occurs in three places which means that when we are calculating k_{33} it will have 3 components okay.

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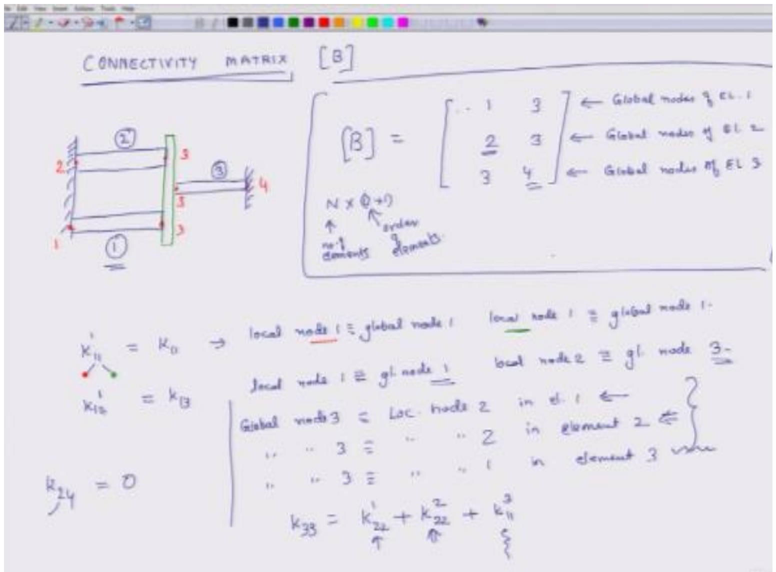

So what does this mean global node 3 corresponds to local node 2 in element 1 agreed again global node 3 corresponds to local node 3 in element 2 oh sorry 2 in element 2 and global node 3 corresponds to local node 1 in element 3 so I can say that k_{33} is equal to k^1_{22} this is coming from here + K^2_{22} this is coming from here + k^3_{11} this is coming from okay so once you have defined the B matrix you have actually captured the geometry of the overall system how different elements are connected with each other.

The B matrix helps computer understand how different elements are connected with each other okay and that understanding can be used to develop assembly level equations you look at another term K_{24} for the global matrix okay k_{24} for the global matrix so you see that this is global node 2 global node 2 is in the second element okay and global node four is in the third element and they arte and this is the first node and this is the second node so they mutually disconnected so this will be 0 okay.

If they were in the same element then you could have created something but they are in different elements. so there is no connectivity between them there is no physical connectivity so that k^{24} is 0

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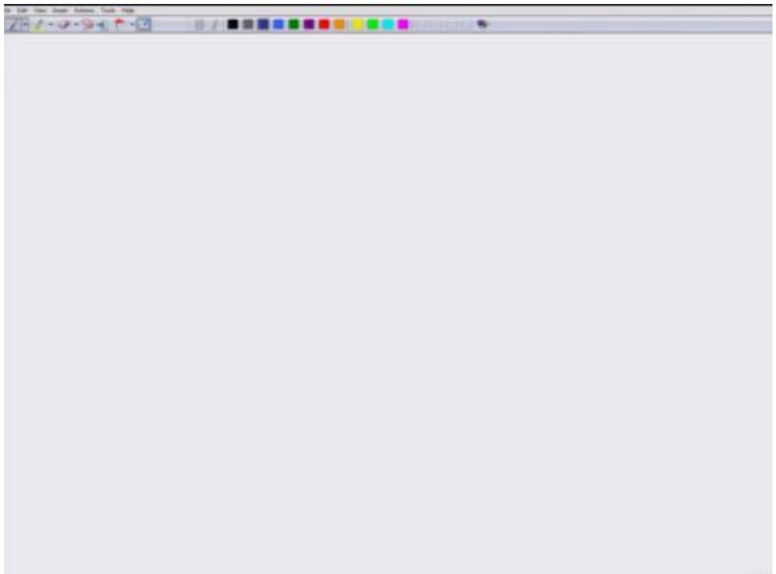

So that was about connectivity matrix now we will make a couple of other important observations which are important which are worthy of mention in context of our whole finite element method.

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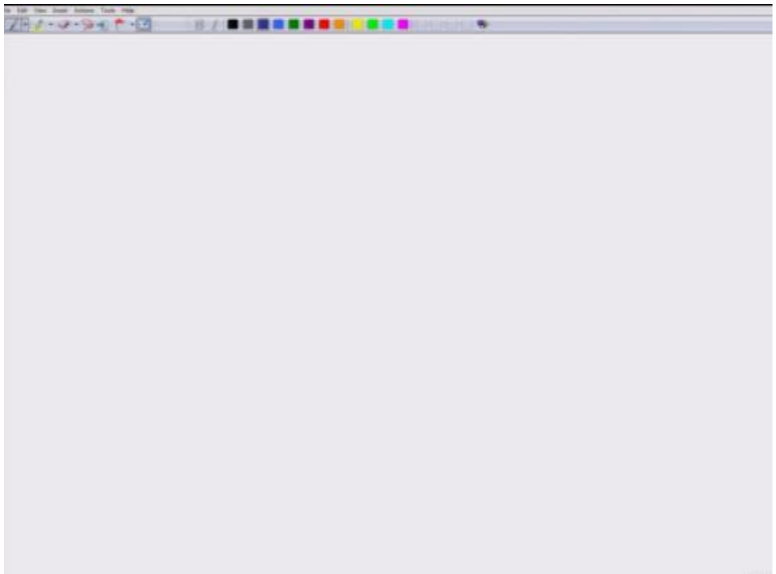

So the first comment I wanted to make is that when we enforce assembly conditions we make sure that.

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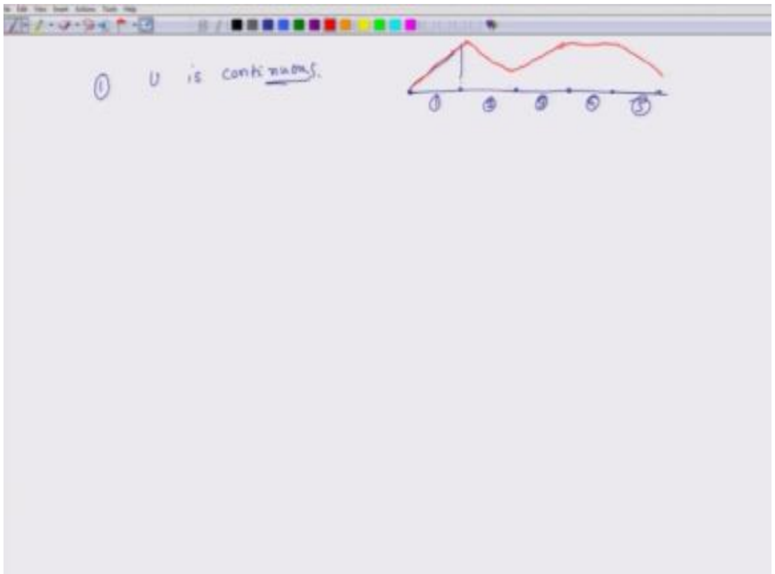

u is continuous for a one dimensional problem and problem with one variable u will be continuous if it is too dependent unknown are there u and v and then we will assume u and v in such that u and v both are continuous and so on and so forth so u is continuous so what this means is that if I have these are my different elements my over all u could be like this right so these variations between the elements between two nodes they will be linear variations if the element was linear they will be quadratic variations in the element was quadratic and so on and so forth.

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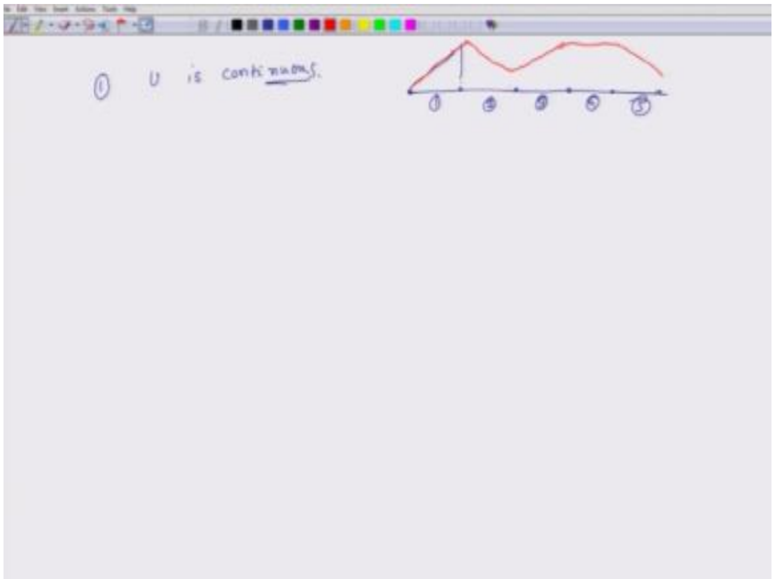

So that is why I have here made straight lines okay but at the junction of nodes if you calculate the value of u from left element or the right element the values will be same because we have mathematically enforce the continuity of u at junction points so u will be continuous second thing.

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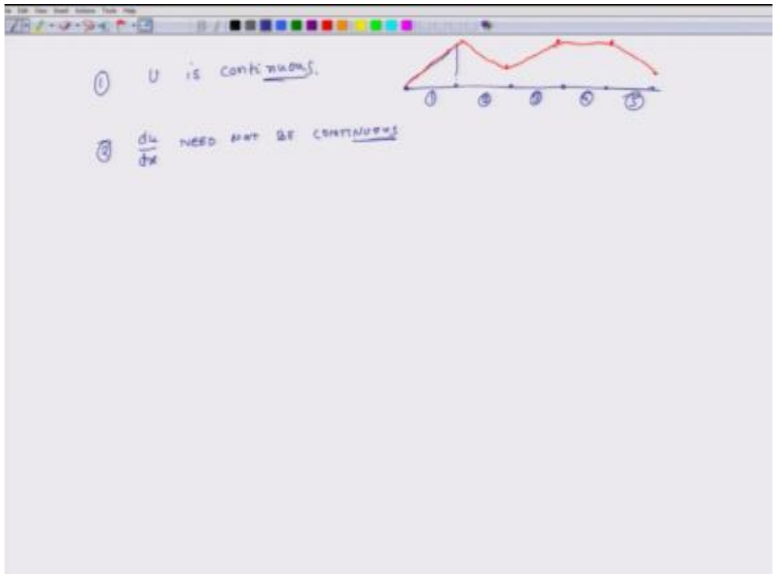

du over dx need not be continuous because okay because when we were doing the assembly process we never said that d over dx is same okay so in this case

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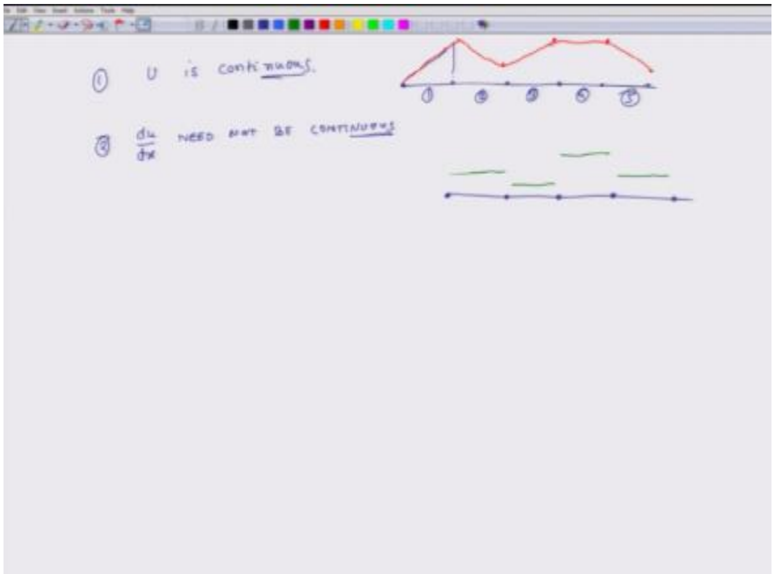

for a linear element if you have a linear number of elements if you have linear elements then du over dx will be constant over one element it may have a different constant value over another element it will be like this for linear elements for quadratic elements it will be it will be a linear it will have a linear you known constant slope but again there is no reason to think that the at the boundaries it will be the slope will be same and they will meet okay.

Because the only thing we have ensured is that u is continues we have not mathematically imposed continuality of slopes so this distinction will be there.

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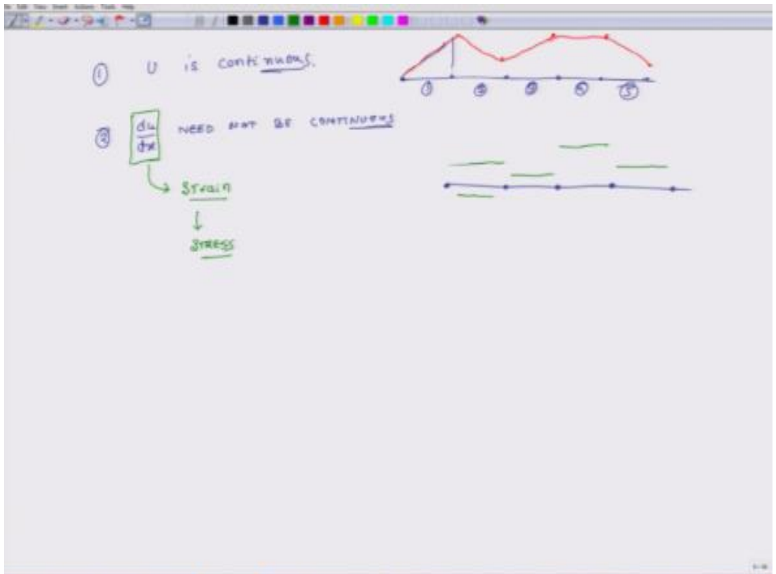

what does that mean what it means is that when you solve so du over dx in context of the bar problem is related to Strain right d over dx is his Strain from his train I can calculate his stress which means that in the bar problem Strains will jump across the nodes on left side at the same location it may be two hundred micro Strains on the other side it may be 300 micro Strains okay.

Same things is also true for stress they will jump from one value to the values across the interfaces okay, so Strains will not be continuous stresses will not be continuous thermal gradients will not be continuous heat fluxes will not heat fluxes, thermal gradients will not be continue because thermal gradient is ∂t over ∂x okay so all these derivatives of fundamental quantities they will not be continues for second order boundary value problems.

Because we never ensure there continuity third however if we keep on increasing the number of elements larger and larger, these jumps between Strains this jump

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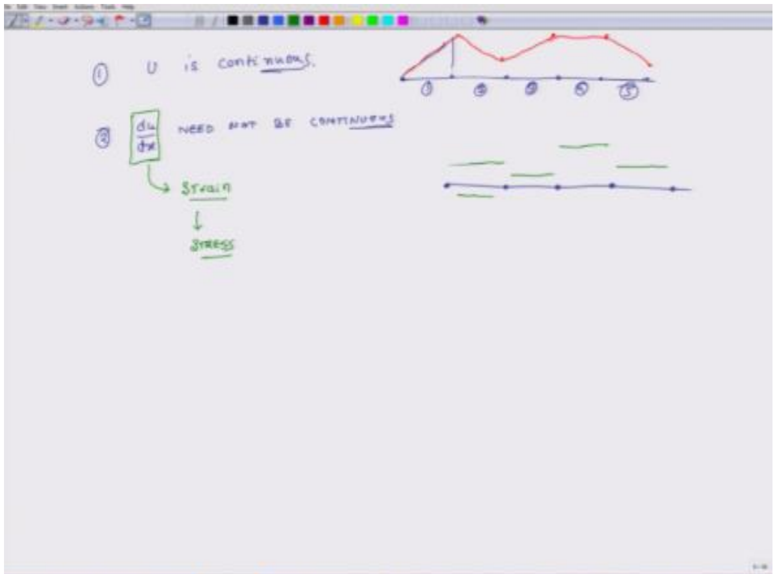

They will reduce they will never become zero mathematically they can never become zero but they will keep on reducing so if you're looking for a convergence of his stress you have to.

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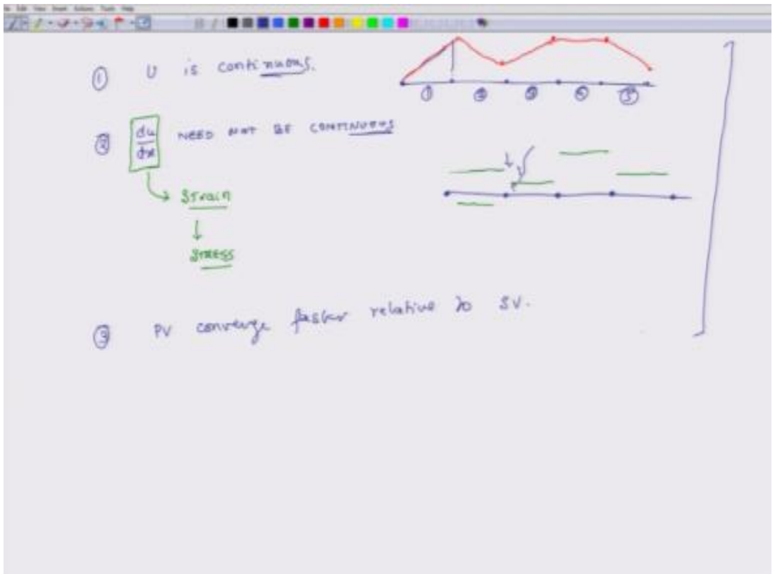

if you are looking for convergence of displacement because displacements are continuously converge faster if you are looking for convergence of his stress you need even larger number of elements you need even larger suppose you have convergence of u for twenty thousand elements that does not guarantee that your stresses will also be converged for that maybe you have to go to forty thousand elements.

So is stresses in his strains converge slowly slower compared to the primary variable okay, so PV's converge faster related to secondary variables, so these are very important things to remember because lot of times when we do finite element analysis maybe our use have converged but if we what we are really interested in finding out Strain or stresses then we have to look at those quantities to see whether they have converged or not okay.

If we are interested in convergence of u or for instance a thermal problem we are interested in fine temperatures then that is fine but if you are interested in finding out how T there gradients are changing then we may have to do more refined analysis with the higher node numbers

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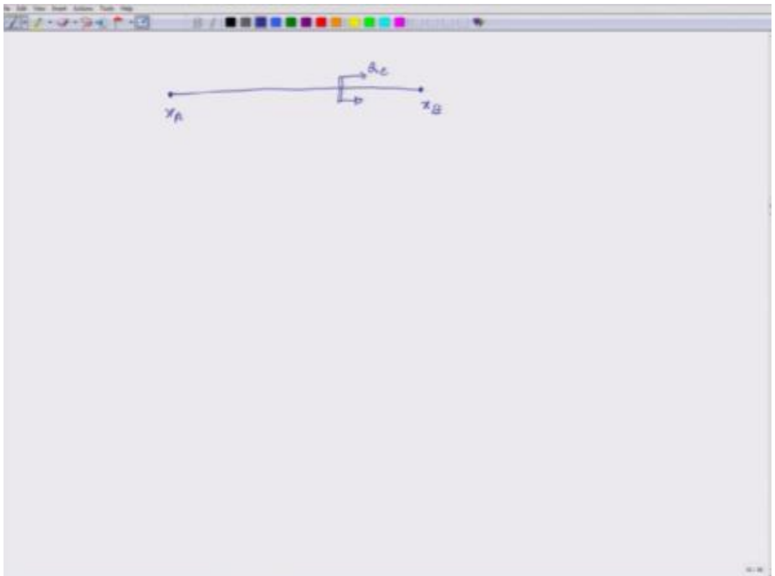

Last point I wanted to make is that suppose you have this element except to x_A to x_B and earlier we had said so there is some distributed force and we know how to calculate the value of that force at x_A and x_B we have seen that f_1 and f_2 right now suppose I have at this location a point force and let us say that value is Q_e so at location I have a point for sampling it so that is Q_e if the location of this Q

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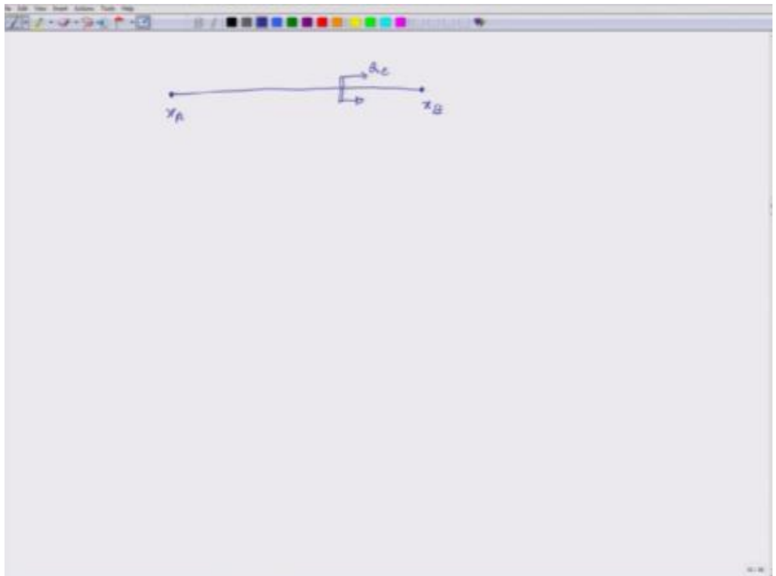

was exactly at the node then I would have when I did the assembly process then Q from the

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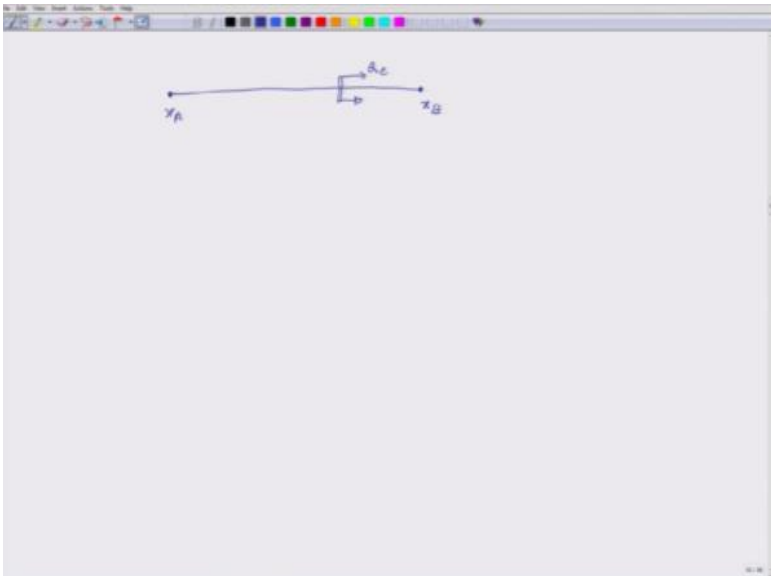

This element and Q from the next element when they add up this would have been equal to Q right but if this force is somewhere in the point force is somewhere middle then how do we distribute this force at two different locations so that is what we will discuss.

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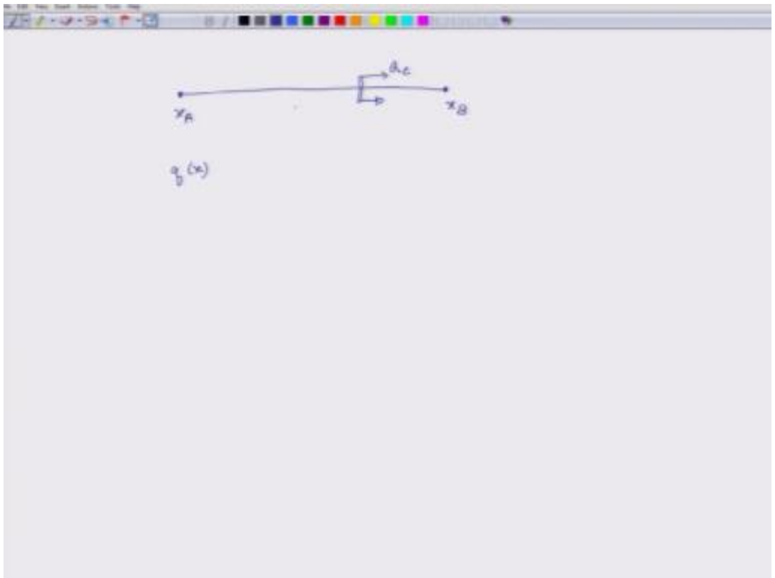

So what I can do is so this is a point load and I can represent this point load as against some q(x) because I know how to handle q is the distributed force right q is distributed force so I will express.

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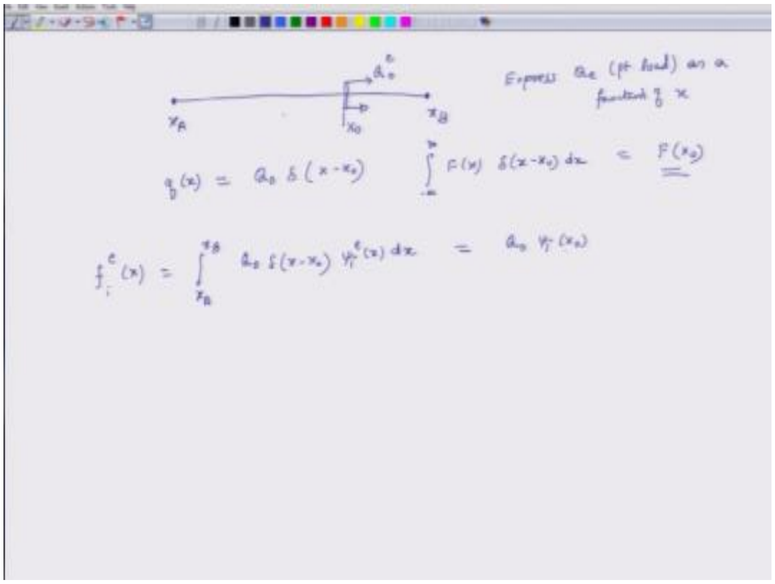

Q I will express Q_e which is a point load as a function of x let us say this coordinate is x_0 okay so I will say that it is equal to Q_x is equal to Q_0 excuse me it is Q_0 for the e^{th} element so it is equal to Q Q_0 times are direct ∂ function x at x_0 okay so what does that mean then the value of this thing will be Q_0 only x right at all other positions it will be zero now we know that if I have any function $F_{(x)}$ any function it could be linear or none linear does not matter and I multiply that by some direct ∂ function $x\text{-}x_0$ and I integrate it over the dx with respective to x and I integrated from infinity to whatever infinity.

Then this is nothing but F at x_0 okay so I use this principle to calculate the Q okay , so what does this do so what I do is that I say that f_i e^{th} element okay is integral from x_A to x_B q(x) times direct ∂ function $x\text{-}x_0$ and then I have to multiply it by weight function lied by their function $\psi^e{}_1$ dx right I have to multiply this how I calculate it by f vector if there was distributed force then there would be another component right if it was but I am just ignoring that because suppose I have to worry only about this yeah.

Q_0 yeah so this is the thing and this means that this value is Q_0 ψ times ψ_I so this is for the i^{th} shape function evaluated it x_0 okay which means that

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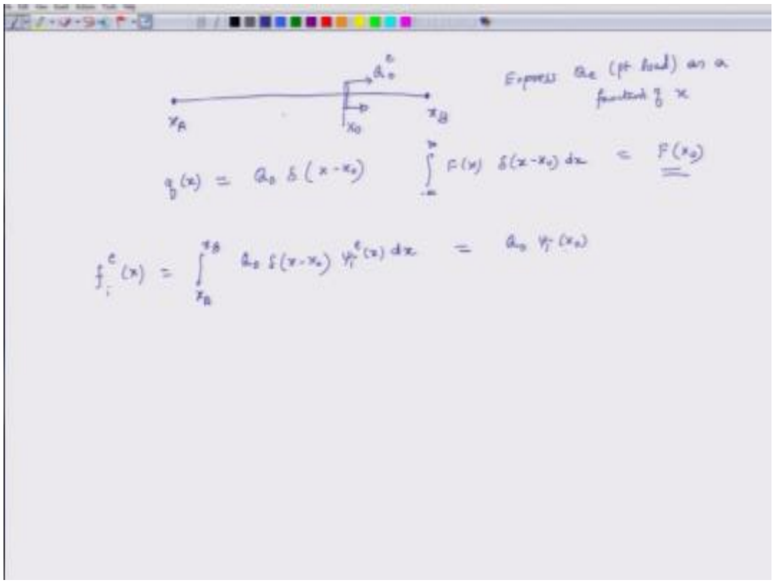

f_1 corresponding to the first approximation function.

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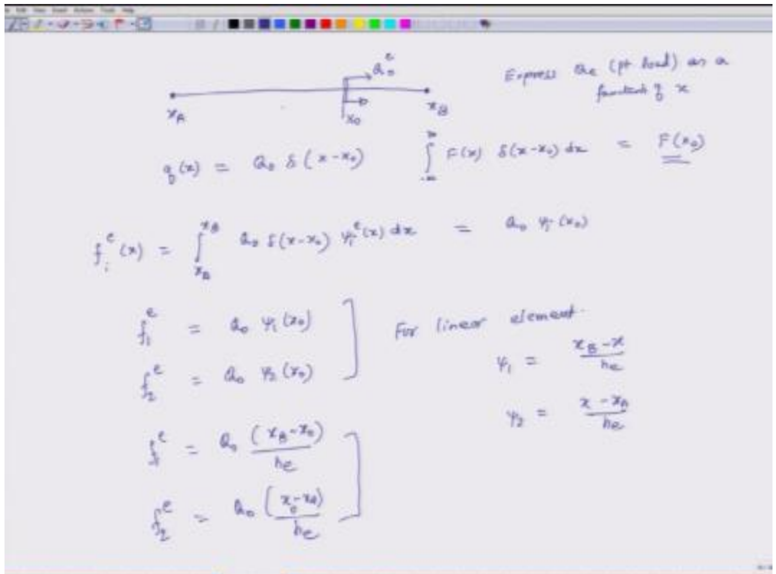

e is equal to Q_0 at $\psi_1$at x_0 and f_2^e is equal to Q_0 ψ_2 at x_0 now ψ_1 so this is for a linear element if it was a cubic element then we change ψ_1 ψ_2 ψ_3 okay now we know that ψ_1 equals x_B –x over h_e and ψ_2 is equal to x-x a over h_e so $f^e{}_1$ equalsQ_0 times x_B-x0 divided by h_e and $f^e{}_2$ is equal to Q_0 x_B – o sorry x-x_A x-x_A divided by h_a x_0 and when you add these two up you will get the sum of these f_1 and f_2 will be Q_0 okay so if you have a point load in the middle of an element you can do two things either you break that up that element further so that there is a node at that point or you use this mathematics to distribute that load accordingly between the two nodes if it is a three nodded element then you have to use ψ_1 ψ_2 ψ_3 soon on forth so this is

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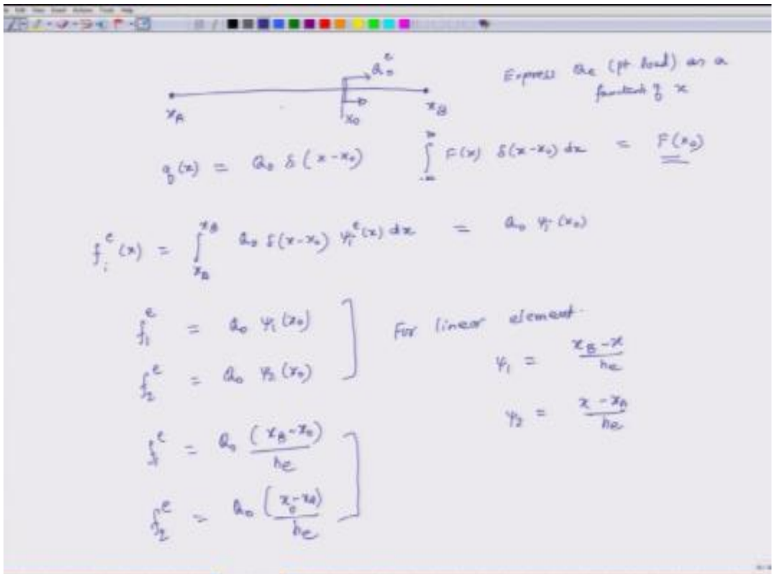

The concluding part of this lecture and we will continue our discussion tomorrow thank you.

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