Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 26
Assembly of element equations

by
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Hello, welcome to Basics of Finite Element Analysis, this is the fifth week of this particular course, and today is day two of this week. Yesterday we learned how to develop element level equations and today we will extend that discussion in context of the overall assembly. So what we will do today and possibly also tomorrow is figure out that how do we assemble all these different element level equations and bring them up to an assembly level. So with that intent in mind.

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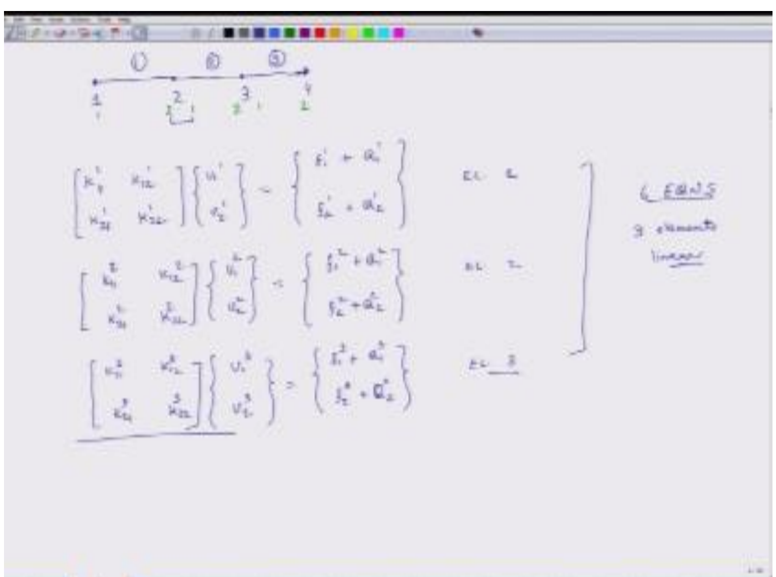

We will first define the domain. So let us say so this entire discussion is going to happen in context of the bar under x scale compression or tension problem. So let us say this is my domain and I am breaking this domain into three elements, element number one, element number two, element number three okay. Now for, so the global node numbers are node one, node two, node three and node four, the local element numbers are one unknown numbers are n2 for the first element, then this is one and two for the second element, and this is one and two for the third element okay.

Now at this stage when we have to, so what I will do is we will first write down the element level equations, so for the first element it is k_{11}, k_{12}, k_{21}, k_{22}, and we realize that these k matrices are symmetric atleast in context of our problem, u_1, u_2 is equal to F_1+Q_1, F_2+Q_2 and this is for element one, so everywhere I have this superscript one. So this is for element one, for element two I have similar equations but the superscript simply changes equations, so the superscript simply changes. So this is these are the equations for element, the second element and finally I have the equations for the third element so our aim is, so these are six equations.

Why do I have six equations, because I have three elements and each element has two nodes, if it was a quadratic element it would have three nodes then we would have nine equations okay? So if there are three linear elements, then we get six equations. If there are three quadratic then we get nine equations, and so on and so forth. But ultimately we want only four equations because there are four actual independent nodes okay.

So how do we do that, so we do that by understanding two important principles. The first principle is that because in element one and in element two these nodes are physically connected, it means that the displacement of u_2 is same as displacement of node two of first elements and node one of second element okay. So this is based on, now suppose there was a crack here then we could not have made this assumption okay, but because these are physically connected it means that u_2 for the first element equals u_1 of the second element. If this was a case of temperature heat conduction problem, there are u corresponds to temperature.

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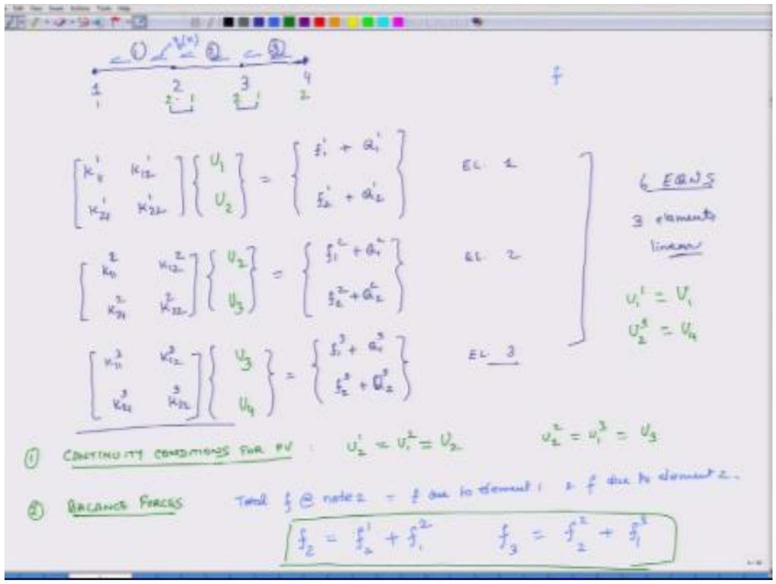

Then the same thing would hold, there temperature of node two for the first element would be same as temperature of node one of the second element okay. So first thing is that we write down these equalities, so these are known as continuity conditions. Continuity conditions for primary variable, we assume that throughout the domain of the problem the primary variable which in this case is u is continuous.

So this is u_2 for first element is equal to u_1 of second element and that is this global u okay, global u. So this is the first continuity condition right, likewise I have also another continuity condition for at node three, so the equation for that is u_2 of second element equals u_1 of third element, and that equals global degree of freedom which is u_3 okay. So what I do is, the first step so, so this is my first step so what I do is that I make these modifications to my element level equations and I replace them by u_2, and this is also u_2 agreed.

Similarly, because u_{22} and u_{13} are same I replace this by u_3, and then of course I realize, I realized that u_{11} is same as u_1 and u_{23} is same as u_4, agreed. So I replace this by u_1, and I replace u_{23} by u_4. So what I have done is, so at this stage that I have replaced element degrees of freedom with global degrees of freedom. See all these individual displacements had nodes, they are degrees of freedom so I have replaced element level degrees of freedom with global level, global degrees of freedom.

This is the first thing, the second thing I do is balance forces, second thing is that I balance forces, what does that mean okay. Now I had said that this is what distributed force which is Q, and this distributed force which is Q it appears in my element level equation says F right. So what I know is that the total force which is at let us say node two is partially due to forces on elements one and also due to forces on element two.

It is not due to forces on element three right, so total f, f at node two, total f and this f is only due to this distributed force, I am not talking about point loads at this time okay. So total f at node two is equal to f due to element one, and f due to element two, this is again because node two is connected, if node two was not connected if we had split, then this would not be the case right. Then this will not be the case, if there was a crack then this will not be the case. So what that means is that the overall force at node two, f_2 is equal to f two one plus f one two okay. Similarly I can say that f at node three is equal to its contribution from the second element which is f 22 plus contribution from the third element, which is f one three, agreed. So this is the second thing which we note, the third thing which we note okay.

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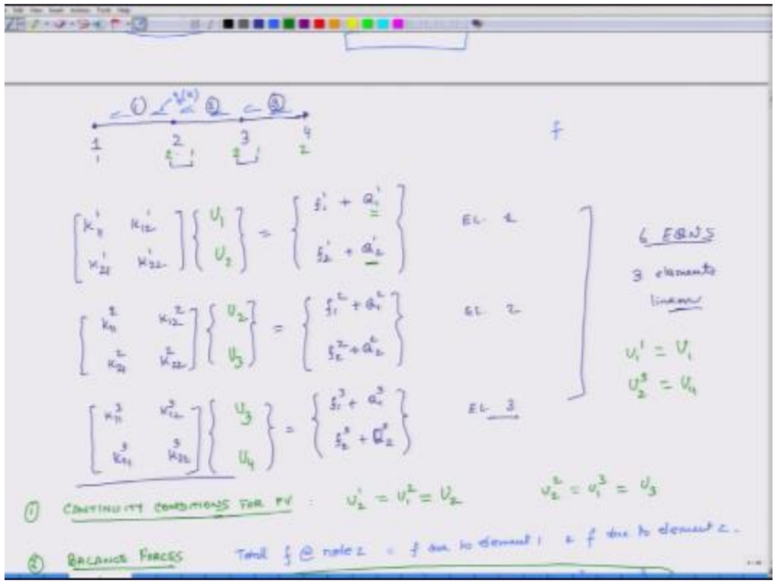

The third thing which we note is, that so this Q, these Q’s are because of a times du over dx right, so from the first element.

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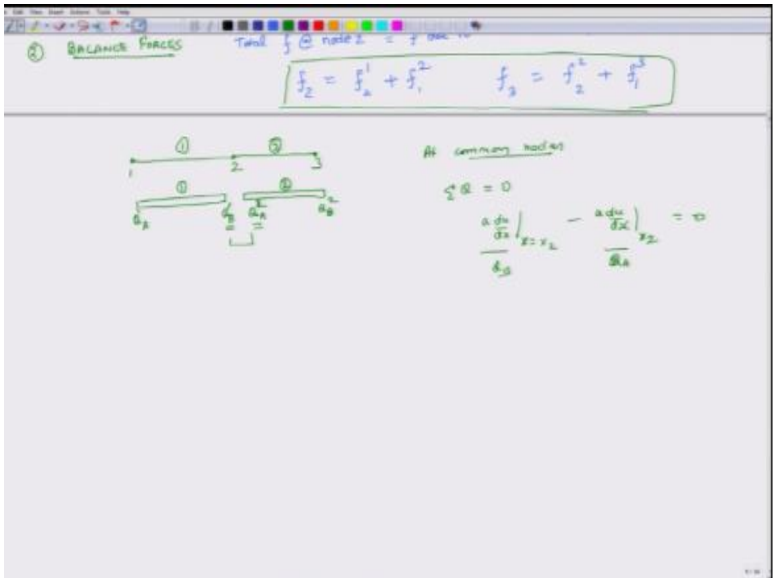

Suppose this is element 1 and this is 3, third node, then when I look at this element by itself I have Q_A for the first element, and Q_B for the first element and for this node, for this element this is, this is element one, this is element two, this is Q_A for the first element, and this is Q_B for the, oh for the second element and this is Q_B for the second element right, I have written two okay. And these forces Q's, if there is no external force they only appear because I have cut it here.

And in reality there is no partition here so these two they have to cancel out and they have to become zero, so the other condition for balancing forces is that at common nodes sum of $\sum Q$is0 okay, and that is what we also see through the definition, because what does this mean, a du over dx at x is equal to you know x_2 so this is what Q_B minus a du over dx at x is equal toX_2, this is Q_A and I am computing both the things there and this will be 0, okay so mathematically also this is there so what

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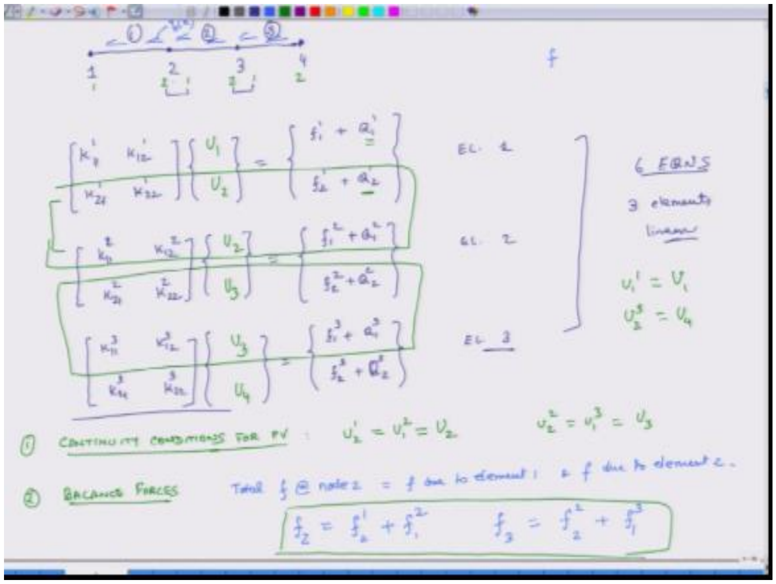

It means is that when I do my assembly level equations I have to do two things, one is that I have to replace local degrees of freedom with global degrees of freedom and also I simultaneously ensure the continuity conditions for u, the second thing is that at interfaces I have to add up the forces right so I have to add up F, and I also have to add up Q's so the way I add up these F and Q's is basically this is, these two equations I add them up these two equations right, when I add them up what do I get, I get $\sum f1^2$ and f1and $Q^1{}_2$ and $Q^2{}_1$ right so I add them up. Similarly I add them these two I add these equations okay, so first stage is I employ the continuity conditions and replace local degrees of freedom by global dus and then second I add up forces at nodes which are common

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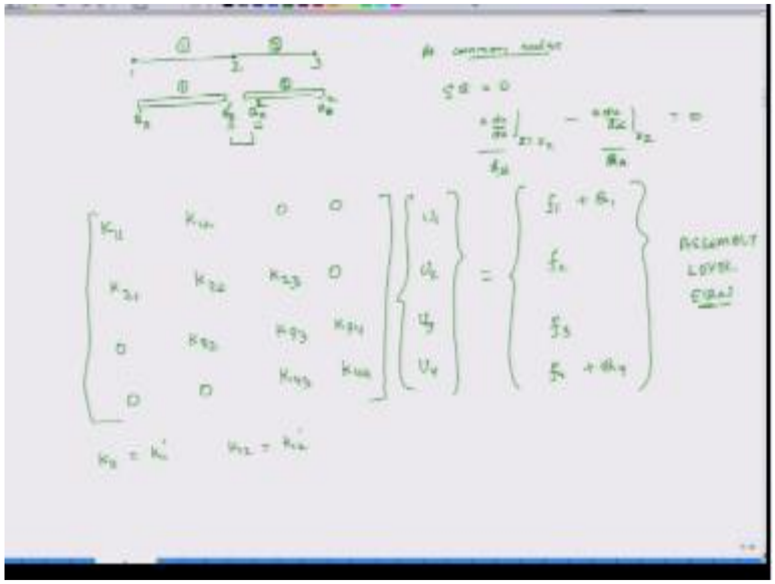

Okay so when I do that what do I get my first equation is still same, so this is my assembly level equation and I know that when I add up Q's at the interfaces their sums are 0 so this is gone, similarly $Q_{3's}$ are gone so and the definition of, so what is given, so this is global, this is my global K matrix right global K matrix so what is k_{11} it is equal to $k^1_{1,}$ what is k_{12} it is $k_{12}^{1,}$

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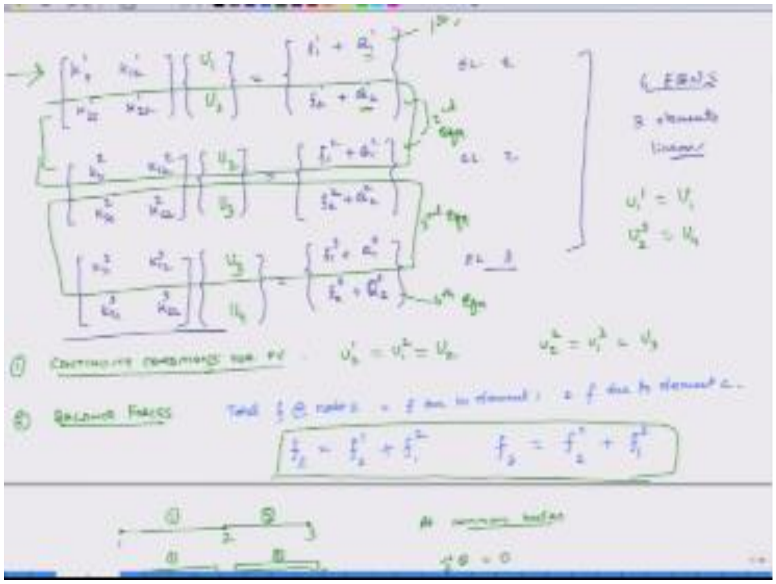

Basically I am just right, this is my equation 1, the sum of these two, the sum of these two, second equation and third equation is my second equation global assembly question right, so this gives my second equation, this gives my third equation and this is my fourth equation, and this is my first equation okay, any questions?

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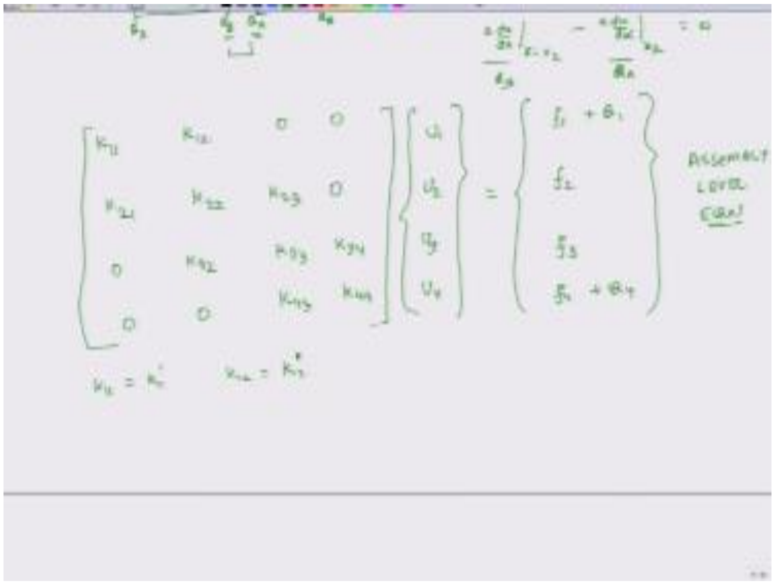

So this is how I construct my global K metrics and global f and q vectors, we will talk about that later yeah. So what I have explained since he asked that is that suppose

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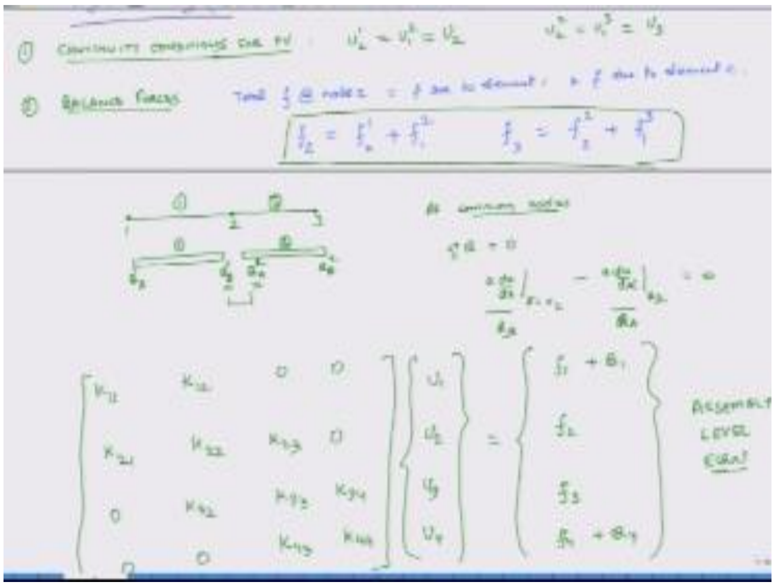

There is a point load so in this case we have assumed, not assumed we know that Q’s, qv and the interface they add up and become 0, they add up and then they become 0, but if there is an external point force then what do we do, so we will discuss that question a little later, maybe in the next lecture or something like that but this is how we construct the global K metrics.

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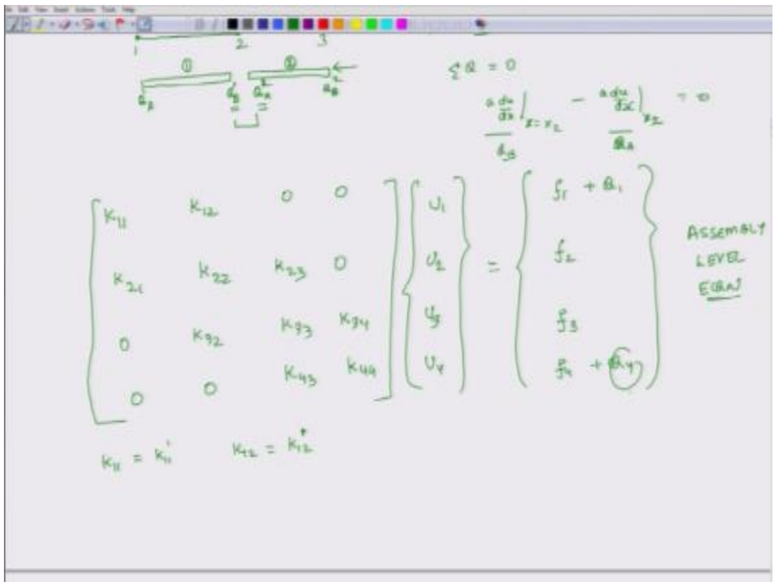

If there are no point loads, if there are only distributed loads applied on the system, if there is an n node, suppose there is an n node here then the value of this Q_4 will be that number, but suppose there is something in between that is something we will discuss later okay. So this completes our discussion for the overall assembly process, we will now continue this discussion and now we will start discussing about boundary conditions and how do we apply boundary conditions in the next class, thanks.

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