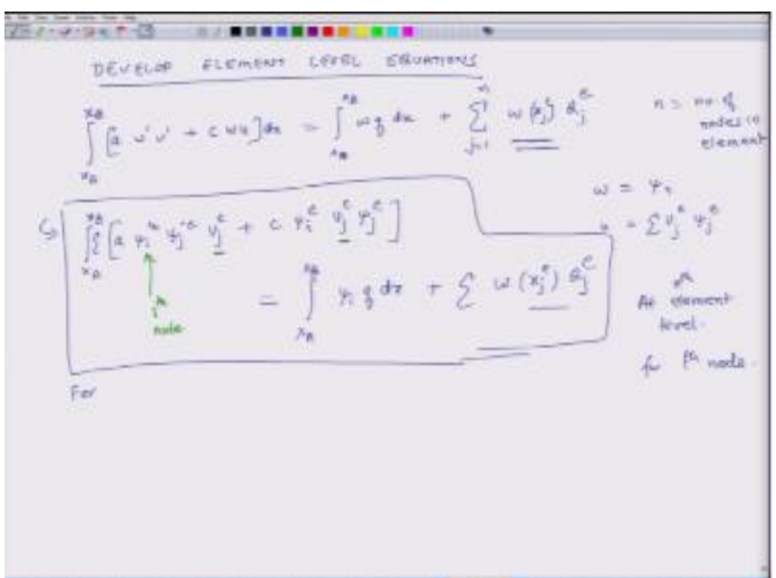
Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 24
Element Level Equation

by
Prof. Nachiketa Tiwari
Dept. of Mechanical Engineering
IIT Kanpur

Hello so last lecture we have seen how interpolation functions are developed and now what we are going to do is we are going to actually use these interpolation functions to develop a equilibrium equations at the element level, so that is going to be the focus of our discussion in today's class and if we have time then maybe we will start talking a little bit about how to assemble these equations so in today's lecture we will develop.

(Refer Slide Time: 00:46)



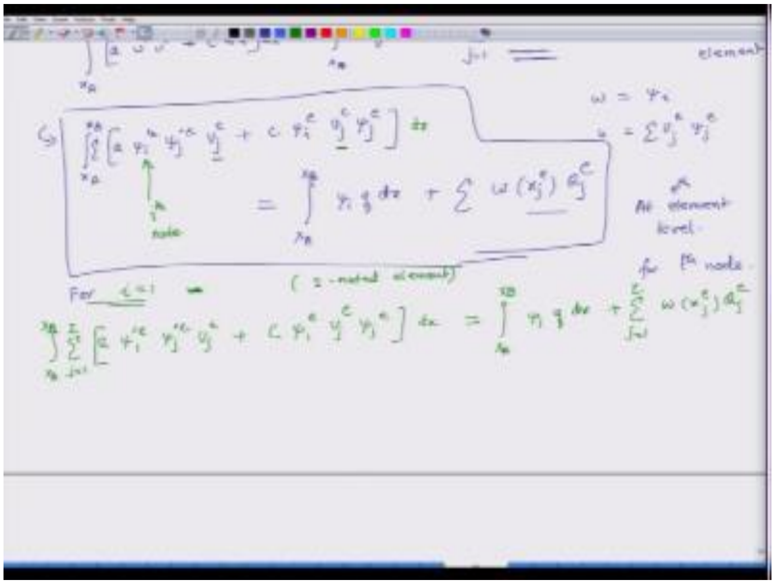
Element level equations so what we had seen was that our original equation second order boundary value, value problem equation which we are going to develop is this thing so I am going to integrate it over the length of an element and these are global coordinates a w′u′+ c wu dx = integral of x A xB w times some loading factor q and then I have boundary terms and what I am going to do here is I am going to express this boundary term in a slightly different way, so this is my shape function w in the original boundary term it was u I have replaced it by a it, know the it is w and it is this thing is being evaluated at location xj for e^{th} element q, j, e. j is equal to 1 to n, n is equal to number of nodes in element, okay now think about it and let us look at this term a little carefully so this term at intermediate nodes will be 0 right this term no okay.

We will explain that, so just hold on to that we will come to that a little later so we will develop this further and we have said that w is the weight function and that equals ψ_i so on the left side I have a times ψ_i' and u is u equals u for the e^{th} element j, ψ_j for the e^{th} element ∑ of it over than that thing so that is ψ_j and this is for the e^{th} element times u_j + c ψ_i^e times u_j^e ψ_{je} this entire thing is going to be added up over the index j and this equals x_A to x_B ψ_i qdx + ∑ of w we will retain this till the end then it will become clear what we are doing on the this last term.

The q term okay and because our so what we see is that this whole system on the right side it is a bilinear symmetric functional to the bilinear symmetric functional so and this is this entire expression this is at element level that is the first thing which element level at e^{th} element level and this is the equation for the e^{th} element for which node i^{th} node right. This is i^{th} node and it is e^{th} element because the superscript is everywhere E okay, so this is the equation for i^{th} node belonging to e^{th} element if it is a linear element then I will take two values 1and 2 so I will have two equations in the first case I will put $\psi_i = \psi 1$ and I will get one equilibrium equation.

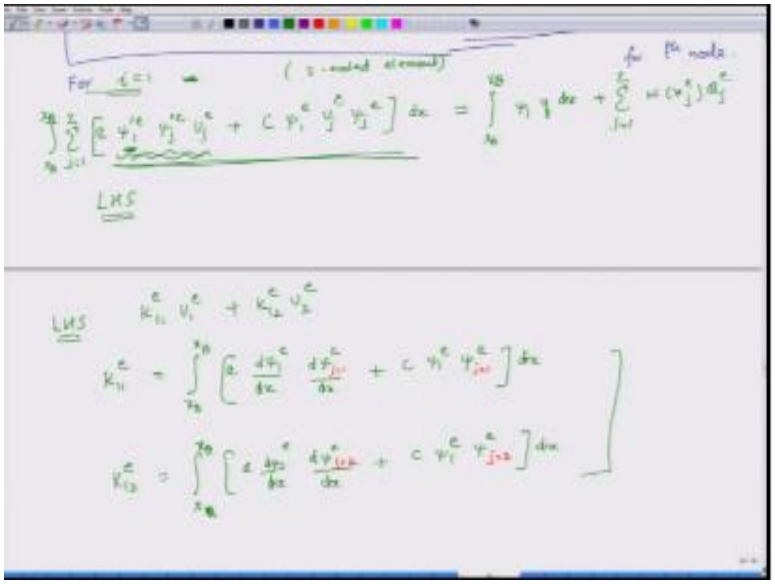
In the second case I will put $\psi_i = \psi_2$ and I will get the second equilibrium equation and I will get two equilibrium equations for a linear two nodded e^{th} element agreed, so that is what I am going to do so in, in this in each equation what is the unknown u_j is the unknown these are the unknowns ψ_i we know, we know how what are the nature of interpolation functions which we have discussed and developed early u_j is something we do not know, so I will have two equations so there will be two u_j u_1 and u_2 they will be unknowns and I will have two equations corresponding to I equals 1 and I equals 2 so, so we will make it more explicit.

(Refer Slide Time: 07:29)



For I = 1 for I = 1 equation will be x_A to x_B Σ of I am summing it to over $\psi_{1,}$ it is differential and this is for e^{th} element so it is a little crusted here times ψ_j again I mean it is differential and I am summing it over j = 1to 2,okay this is a two nodded element this is the two nodded elements so j = 1 to 2 , uj_e + $c\psi_1$ I = 1 in this case right eu_{je} $e\psi_j e$ and I have should have written dx here, dx is also here okay, and this equals x_A to x_B $\psi_1 qdx$ + Σj equal to 1 to 2 wxj^e qj^e okay.

(Refer Slide Time: 09:31)



This term has a I can this on that so this is the left-hand side and let us look at the left hand side again carefully so LHS I can re-express this LHS as k_{11} this is only for the first equation so I = 1 times $u1^e$+ k_{12} u_2^e Where k_{11} and this is for the e^{th} element so I am not going to lose my subscript, superscript e is nothing but integral x_A to x_B a time's $d\psi_1 e$ over dx this is times $d\psi$ hence j this is the j_1 you know so I am putting j = 1 over dx and again for the e^{th} element + C ψ_1^e ψ here j = 1^e.

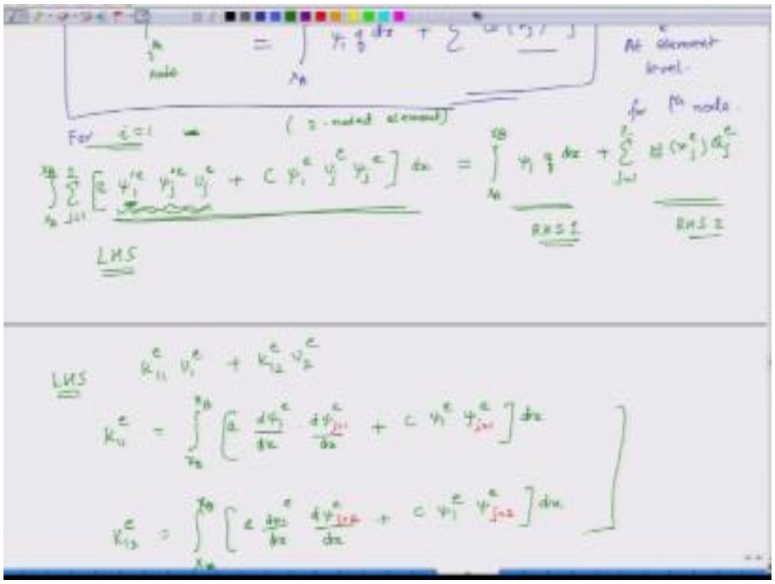
See each of these terms will have each this, this, this term will have two components one will be for j is equal to one will be for j is equal to 2 so this is k_{11}, corresponds to j is equal to 1 where j is equal to one term will be multiplied by u_1 j is equal to 2 term will be multiplied by u_2, understood similarly $k_{12}{}^e$, x_A to x_B a times du_1^e over dx or only is not $d\psi$, $d\psi$ and here j is equal to 2 dx+Cψ_1^e okay, I know everything in this integral I know the limits x_A and x_B because I know the physical boundaries of my e^{th} element it is coordinates have broken and I know what is the value of x_A and x_B I know a.

(Refer Slide Time: 13:00)

It is given in the differential equation which I am trying to solve I know ψ_1 we have calculated it the interpolation function and I also know ψ_2, so I can differentiate it so I also know it is differentials I know C so I can integrate these this these two relations to get k_{11} and k_{12}^e agreed okay, now let us look at this next function ψ_1 times qdx, so this is still the first equation so this is the first function RHS 1 we will call it this first term on the RHS side so RHS 1 is what integral of x_A to x_B ψ_1q times d_x this I call it as f_1^e okay, this is f_1^e and this term is RHS to, so this I call it as q_1^e .

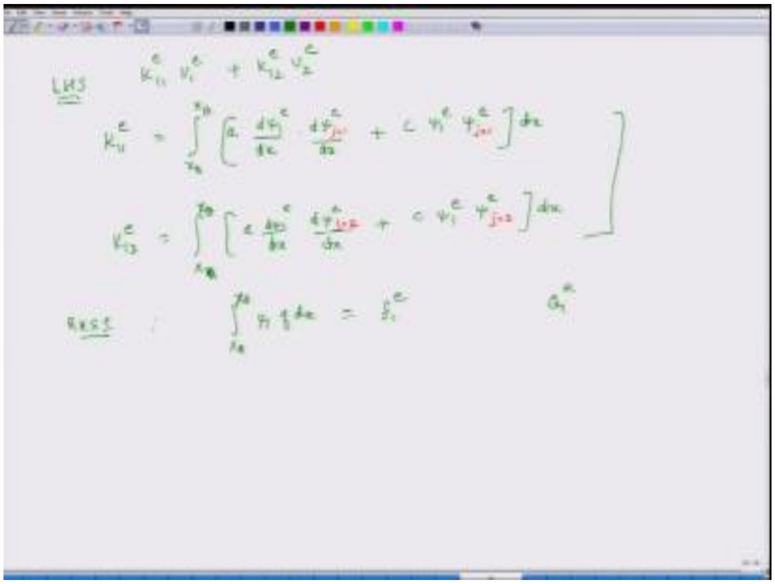
Why is it q_1^e here w I am putting it as ψ_1 first variation when I put ψ_1 w is ψ_1 right the value of w which is ψ_1 at first node is one right and at the second node it is 0 so the first node it is one and.

(Refer Slide Time: 14:49)



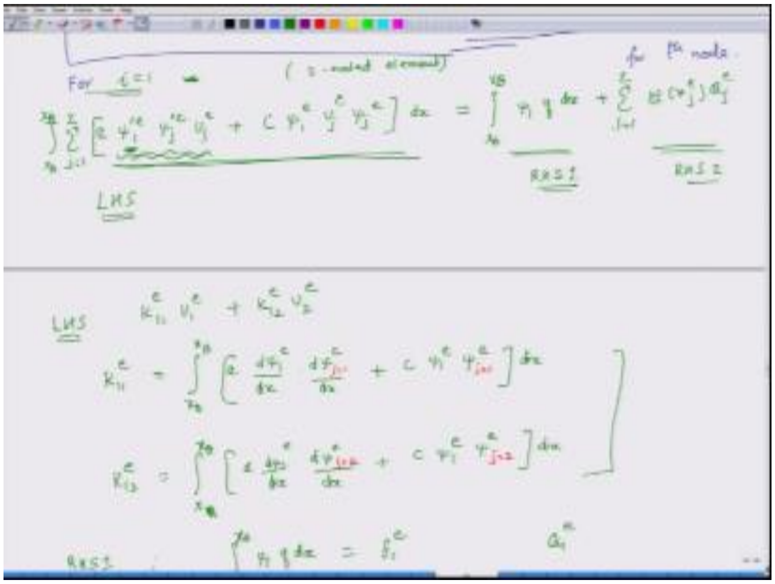
And at the first node it gets multiplied by this q so the first node what is the value of q, q_1 and this is what this is q_g.

(Refer Slide Time: 15:02)



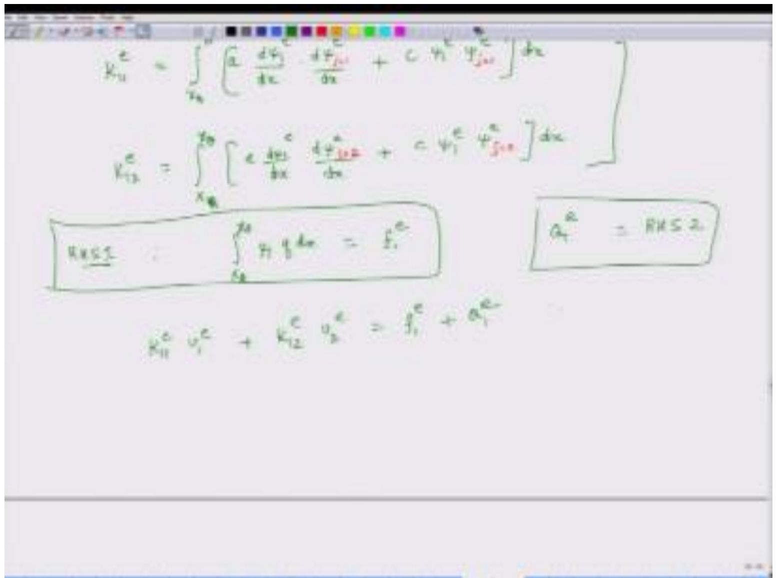
So I get q_1^e from here and the value of ψ_2.

(Refer Slide Time: 15:05)



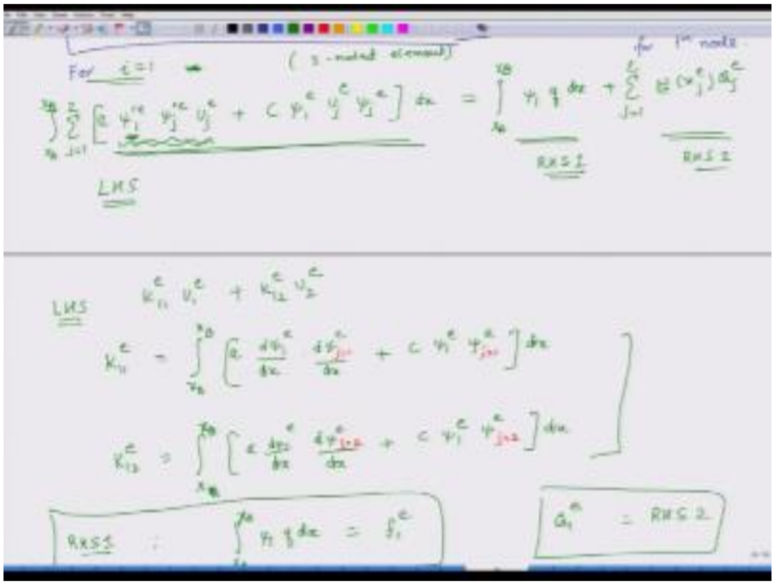
Ψ1 at the second node I am summing it over j right, this is q_j the value of ψ_1 at the second node is 0 so this term becomes the second component of this term is 0 have a

(Refer Slide Time: 15:24)



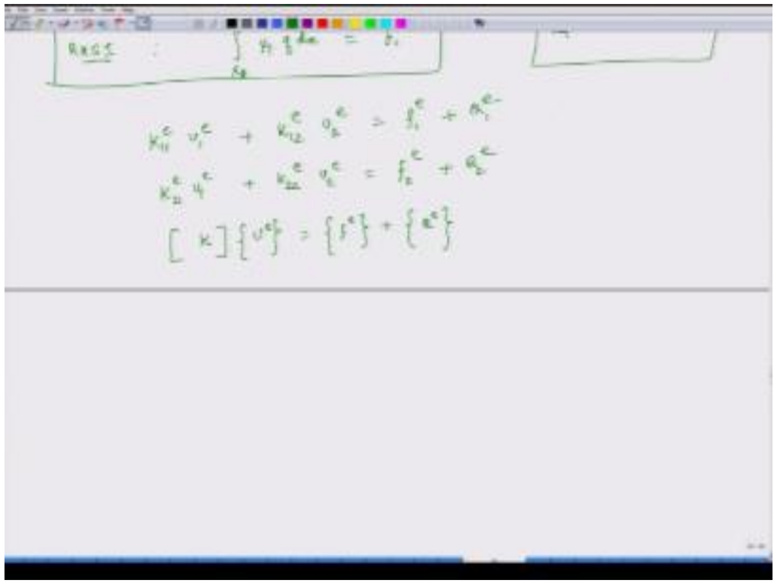
So this is q_1 so this is RHS 1 and this is equal to RHS 2, so my overall equation becomes k_{11}^e times u_1^e + k_{12}^e, u_2^e = f_1^e +Q_1^e agreed, now so this is one equation I have two nodes in the element.

(Refer Slide Time: 16:08)



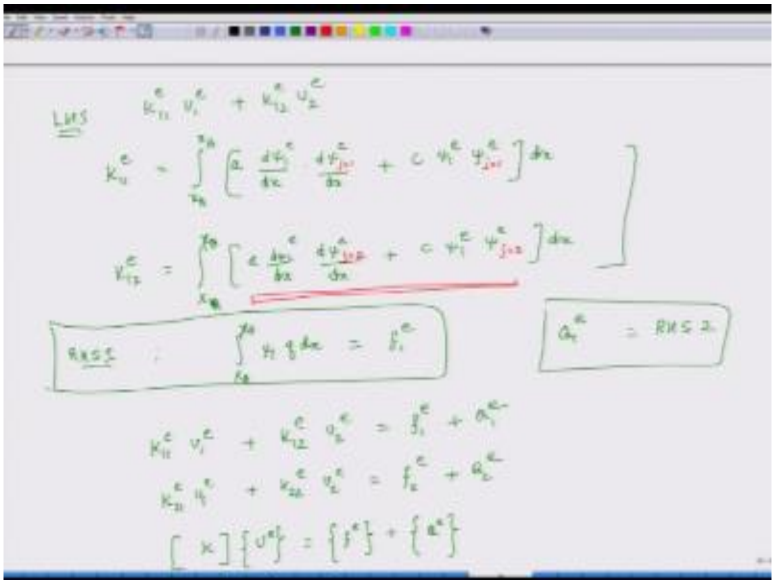
So I can take what the second value which I can take his I = 2 and I can do the same mathematics like that and I will get another equation.

(Refer Slide Time: 16:19)



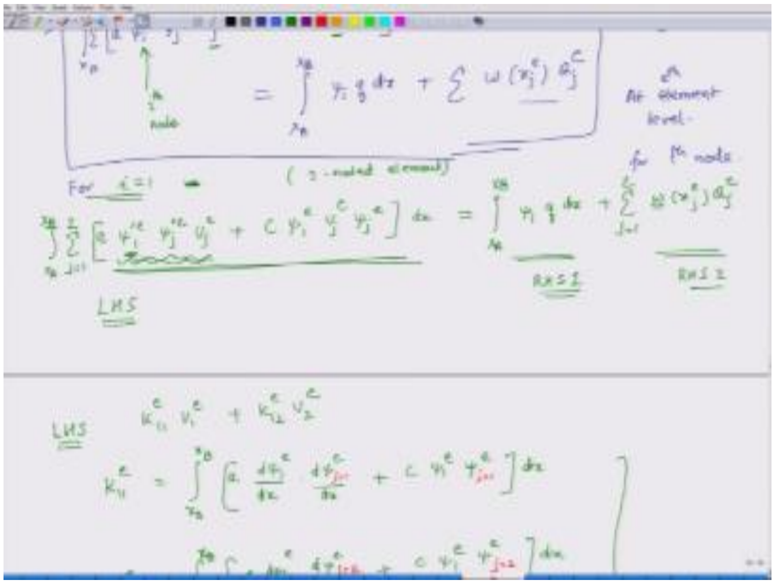
k_{21} u_1^e + k_{22}^e u_2^e = f_2e + Q_2^e agreed or I can put this in matrix form I can call this as K matrix and a lot of times this K matrix is known as that is stiffness matrix at least in solid mechanics this is my vector for e^{th} element and this equals some force vector f^e + some secondary variable Q^e okay, and oh this case also having a superscript e because this is all specific to.

(Refer Slide Time: 17:21)



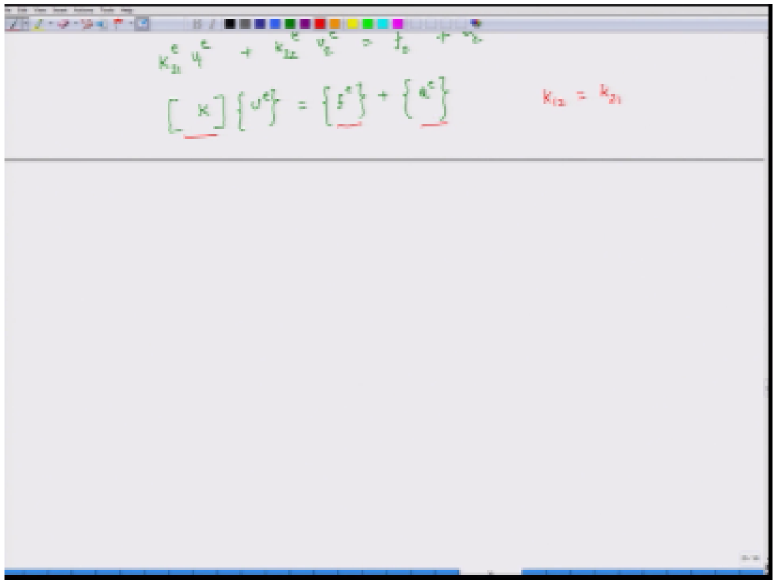
The other thing is you look at k_{12}, k_{12} when you look at it, it is symmetric in one and two if you replace ψ_1, ψ_2 and ψ_2 by ψ_1 dissymmetric so you will see.

(Refer Slide Time: 17:44)



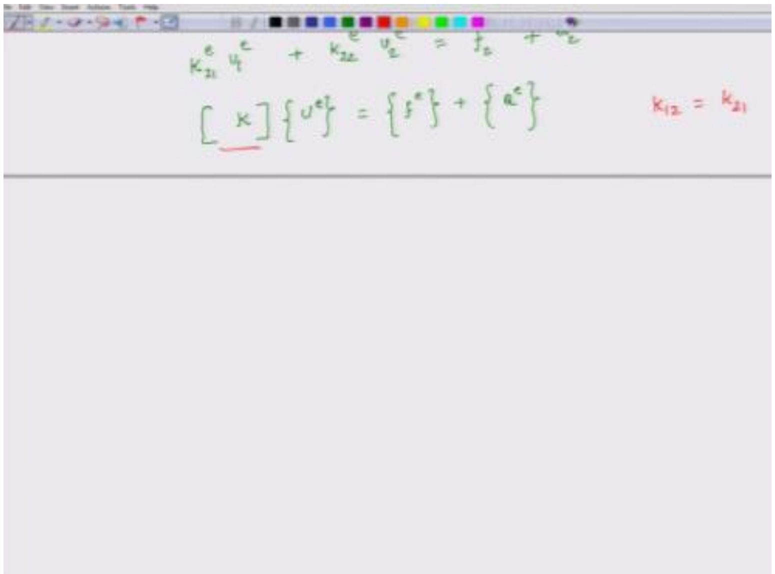
That k_{12} is equal to k_{21} for this problem okay, so you have stiffness matrix the K matrix in this case because we are using the weak form and it is a second order differential equation it the K matrix is our symmetric matrix it is a symmetric matrix so all element level equations for this equation will have K matrices and if it is a linear element what will be the size of the matrix 2/2 and they will be symmetric matrixes if it is a cubic element then the only thing you have to do is here j is equal to 1, 2, 3and I will become in first case one, second case two, third case three.

(Refer Slide Time: 18:34)



So it will be 3/3 matrix so your element matrix K matrix for a cubic element will be quadratic element will be 3/3 for a cubicle element it be 4/4 and so one and so forth but now you know how to calculate the matrices at element level both the

(Refer Slide Time: 18:50)



K matrix the f vector and the Q vector so this will conclude our discussion for today and in the next class we will learn how to assemble these equations okay, thank you.

Acknowledgement
Ministry of Human Resource & Development

Prof. Satyaki Roy
Co-ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K. K. Mishra
Aradhana Singh
Sweta

Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

an IIT Kanpur Production