

**Indian Institute of Technology Kanpur**  
**National Programme on Technology Enhanced Learning (NPTEL)**  
**Course Title**  
**Basics of Finite Element Analysis**

**Lecture – 23**  
**FEA formulation for 2nd order BVP**  
**Part - II**

by  
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Hello, welcome to basics of finite element analysis, in the last lecture we have developed expressions for approximation functions corresponding to a two noded linear element  $\phi_1$  and  $\phi_2$ . And these approximation functions look like these relations.

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Handwritten notes on a whiteboard showing the derivation of linear shape functions for a two-noded element. The text includes:

- Only possible if  $\psi_1(x_B) = 0 \Rightarrow 0 = a + bx_B$  (3)
- $\psi_2(x_B) = 1 \Rightarrow 1 = c + dx_B$  (4)
- $\psi_1(x) = \frac{x_B - x}{x_B - x_A}$
- $\psi_2(x) = \frac{x - x_A}{x_B - x_A}$
- $u^e(x) = u_1^e \psi_1^e(x) + u_2^e \psi_2^e(x)$

$\phi_1$  equals  $x_B - x / x_B - x_A$ , where  $x_B$  and  $x_A$  are the coordinates of the for the  $e^{\text{th}}$  element.

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only possible if  $\psi_1^e(x_A) = 0 \Rightarrow 0 = c + dx_A$  (8)

$\psi_2^e(x_B) = 1 \Rightarrow 1 = c + dx_B$  (9)

$$\psi_1^e(x) = \frac{x_B - x}{x_B - x_A}$$

$$\psi_2^e(x) = \frac{x - x_A}{x_B - x_A}$$

$$\psi^e(x) = \psi_1^e(x) + \psi_2^e(x)$$

And  $\psi_2$  is equal to  $x - x_A / x_B - x_A$ , I would like to make couple of comments about these approximation functions. The first thing is that these functions.

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The image shows a handwritten derivation on a digital whiteboard. At the top, two shape functions are defined:  $\psi_1^e(x) = \frac{x_B - x}{x_B - x_A}$  and  $\psi_2^e(x) = \frac{x - x_A}{x_B - x_A}$ . These are enclosed in a hand-drawn box. To the right of the box, the text "LINEAR 2-NODED ELEMENTS" is written. Below the box, the general element function is given as  $u^e(x) = U_1^e \psi_1^e(x) + U_2^e \psi_2^e(x)$ .

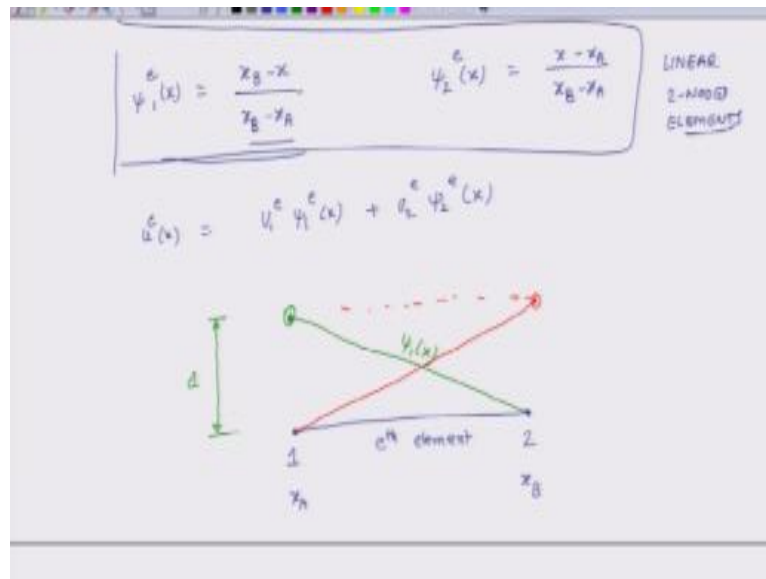
$$\psi_1^e(x) = \frac{x_B - x}{x_B - x_A} \quad \psi_2^e(x) = \frac{x - x_A}{x_B - x_A}$$

LINEAR  
2-NODED  
ELEMENTS

$$u^e(x) = U_1^e \psi_1^e(x) + U_2^e \psi_2^e(x)$$

Are good for linear elements, two noded linear elements. If it is a quadratic function then our element has three points. So we will need a  $\psi_1, \psi_2, \psi_3$ , but using similar development process we can calculate those functions as well. If it is a cubic element then we will have four such functions. But the process is same, so that is one thing.

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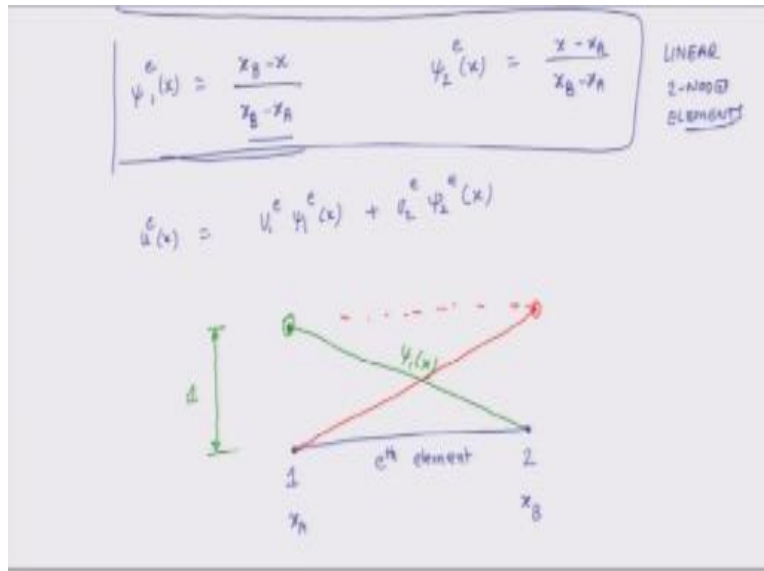


The second thing is let us consider this as my  $e^{\text{th}}$  element, okay. The coordinator of the first node. So this is node one, this is node two, the first node, its coordinator is  $X_A$ , the second node its coordinator  $x_B$ . If I plot  $\phi_1$ , what you will see is at  $x = 0$ , Ah I am sorry, at  $X = X_A$  which is the first node the value of this  $\phi$  function is one. So this is one. And at the second node  $x_B$ , this value it becomes zero, right?

When  $x$  is equal to  $x_v$ ,  $\phi_1$  becomes zero. So this looks like this and this height of this function is one. So this is my  $\phi_1$ , the second function is similar but it moves in a different way. So it is zero at the first node and it is one at the second node, okay. And this feature is something you will see you again and again in quadratic elements you will have three functions  $\phi_1$ ,  $\phi_2$  and  $\phi_3$ ,  $\phi_1$  will be having a value of 1.

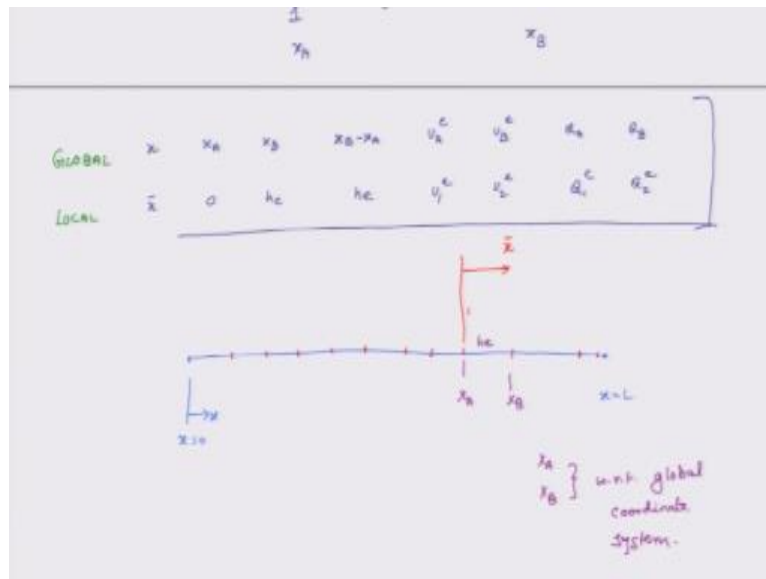
At the first node and it will have a value of zero at second and third nodes.  $\phi_2$  will have a value of 1, at the second node and it will have a value of zero at first and third nodes. And  $\phi_3$  for a quadratic element will be one at the third node and it will have a value of zero at node number one and node number 2. We will actually see that later also in detail.

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So, so this is important to note, at this point of time I will also introduce two different coordinate systems.

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So one is global coordinate system and the other one is local coordinate system. So suppose this is the length 0 to 1, okay. So here  $x$  is equal to 0 and I am breaking and this is  $X$  equals  $L$  and let us say I am breaking this domain into lot of elements and let us say the length of  $e^{\text{th}}$  element is  $h_e$  and it is at this location is  $x_A$  and let us this location is  $X_B$ . So these are  $x_A$  and  $X_B$  they are with reference to global coordinate system.

My local coordinate system it has a origin, my local coordinate system its origin is at the first node of the  $e^{\text{th}}$  element and I will call that coordinator as  $x$ -bar just to differentiate  $x$ -bar, okay. So my global coordinate system is  $x$ , my local coordinate system is  $x$ -bar, in my global coordinate system when  $x$  is equal to  $x_A$ , in my local coordinate system  $x$  is equal to 0, okay.

In my global coordinate system when  $x$  is equal to  $x_B$ , locally it is  $h_e$ , also  $X_B - X_A$  equals  $h_e$ , in my global coordinate system the displacement of first node of  $e^{\text{th}}$  element is  $u_1 u_A^e$ , right? If it is not there, here I just call it  $u_1^e$  this is  $u$  of point B for  $e^{\text{th}}$  element this is  $u_2^e$ . Same thing for  $q$ 's,  $q_A, q_B$  here I call it  $q_1^e, q_2^e$ .

And so on and so forth, most of the competitions when we do at element level, we do not use the global coordinate system, we use the local coordinate system, okay.

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GLOBAL  $\phi_1^e = \frac{x_B - x}{x_B - x_A}$   $\phi_2^e = \frac{x - x_A}{x_B - x_A}$

LOCAL  $\phi_1^e = 1 - \frac{\bar{x}}{h_e}$   $\phi_2^e = \frac{\bar{x}}{h_e}$

$n$  elements (linear)  $\rightarrow$   $n+1$  nodes globally  $\rightarrow$   $2n$  nodes locally

$e^{th}$  element  $\left\{ \begin{array}{l} \text{Global} \rightarrow x_A = x_e, \quad x_B = x_{e+1} \\ \text{Local} \quad \bar{x} = 0, \quad \bar{x} = h_e \end{array} \right.$

In a global coordinate system,  $\phi_1^e$  we had calculated was  $X_B - X / X_B - X_A$ , in local its  $\phi_1^e$  which is the approximation function, this equals  $1 - \bar{x} / h_e$ , in global my  $\phi_2$  equals  $x - x_A / x_B - x_A$ , in local it is equal to  $\phi_2^e$  equals  $\bar{x} / h_e$ , okay. And  $e$  is the  $e^{th}$  element, if there are total number of  $n$  elements overall and there are all of these are linear elements then total number of nodes globally, globally it will be how much?  $(n+1)$

So I will make it short and small. But number of nodes locally there are  $n$  elements each element has two nodes and then there will be  $2n$  elements, okay. Also for the  $e^{th}$  element,  $X_E$  which is the global coordinate it corresponds to  $X_E$  where  $E$  is the  $e^{th}$  element and  $X_B$  is equal to  $x_E + 1$ , right? Here this is global and the local you have  $\bar{x}$  is equal to 0; here  $\bar{x}$  is equal to  $h_e$ . So you have a global coordinate system.

And this global coordinate system is comes into play when we are doing the assembly for the whole system and that element level we are restrict to local coordinate system because it is easier, we do not have to track coordinator of each element with respect to the origin that we can

save that information in our crowd code somewhere else but we when we are doing integrations and all that the only thing we are worried about is the length of the element  $h_e$ , okay. So it makes mathematics, mathematics simpler and easier and faster.

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Handwritten notes on a whiteboard showing the derivation of global and local shape functions for a linear element.

Top part: A coordinate system with nodes  $x_A$  and  $x_B$ , and a point  $x$ . The text says:  $x_A, x_B$  w.r.t. global coordinate system.

Middle part: A box containing the shape functions:

$$\begin{aligned} \text{GLOBAL: } \phi_1^e &= \frac{x_B - x}{x_B - x_A} & \phi_2^e &= \frac{x - x_A}{x_B - x_A} \\ \text{LOCAL: } \phi_1^e &= 1 - \frac{x}{h_e} & \phi_2^e &= \frac{x}{h_e} \end{aligned}$$

Bottom part: A box containing the relationship between the global and local coordinate systems:

$$\begin{aligned} n \text{ elements (linear)} & \rightarrow n+1 \text{ nodes} \\ \text{e}^{\text{th}} \text{ element} & \rightarrow \text{Global } x_A = x_e, x_B = x_{e+1} \end{aligned}$$

Couple of other points these elements, these functions whether they are in local or global we derive these functions based on what? Considerations related to displacements only, we did not derive them based on considerations related to derivatives of the displacement, we only consider displacements, right? We did not consider derivatives of displacement.



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Handwritten notes on a whiteboard illustrating the relationship between global and local coordinate systems for linear interpolation functions.

At the top, a small diagram shows a horizontal line with a point labeled  $x$  and a coordinate  $x_0$ .

Below this, a set of curly braces indicates the relationship between global coordinates  $x_A$  and  $x_B$  and the global coordinate system.

The main part of the notes is divided into two sections: GLOBAL and LOCAL, enclosed in a box.

**GLOBAL**

$$\psi_1^G = \frac{x_B - x}{x_B - x_A} \quad \psi_2^G = \frac{x - x_A}{x_B - x_A} \quad (\text{INTERPOLATION})$$

**LOCAL**

$$\psi_1^L = 1 - \frac{\bar{x}}{h_e} \quad \psi_2^L = \frac{\bar{x}}{h_e}$$

Below the box, it is noted that there are  $n$  elements (linear) and that the global coordinate system is  $n+1$ .

At the bottom, a diagram shows a 1D element with nodes  $x_A$  and  $x_B$ , and a point  $x$  within the element. The element length is  $h_e$ .

So such functions, so first thing is that these are known as interpolation functions. In general we have been using the term approximation functions, here we use these functions to interpolate the value of displacement between two nodes, that is why they are known as also inter we can use the term approximation function but in context of  $f_e^a$  at element level they are more popularly known as interpolation functions.

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Handwritten notes on a slide showing the derivation of linear interpolation functions.

Diagram: A horizontal line segment with nodes  $x_A$  and  $x_B$ . A point  $x$  is marked on the segment.

Equations for Global Coordinates:

$$\begin{aligned} \eta_1 &= \frac{x_B - x}{x_B - x_A} & \eta_2 &= \frac{x - x_A}{x_B - x_A} \end{aligned}$$

Equations for Local Coordinates:

$$\begin{aligned} \xi &= 1 - \frac{x}{h_e} & \eta_2 &= \frac{x}{h_e} \end{aligned}$$

Labels:

- GLOBAL
- LOCAL
- INTERPOLATION FUNCTIONS
- $n$  elements (linear)
- $n+1$
- $e^{th}$  element
- Global  $\rightarrow x_A = x_e, x_B = x_{e+1}$

And because these functions were derived based on considerations of only the primary variable not its derivatives.

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$x_A$   
 $x_B$  } w.r.t. global coordinate system.

GLOBAL  
 $\phi_c = \frac{x_B - x}{x_B - x_A}$

LOCAL  
 $\phi_c = 1 - \frac{x}{h_c}$

$\phi_c = \frac{x - x_A}{x_B - x_A}$

$\phi_c = \frac{x}{h_c}$

INTERPOLATION FUNCTIONS  
 LAGRANGE FAMILY OF INTERPOLATION FUNCTIONS

$n$  elements (linear)  $\rightarrow n+1$   
 $x$  varies globally

$e^{\text{th}}$  element  
 Global  $\rightarrow x_A = x_c, x_B = x_{c+1}$

They are known as that they belong to Lagrange family, Lagrange, Lagrange family of interpolation functions, okay. Here the consideration was only primary this un variable not its derivatives, okay.

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The slide contains handwritten notes on a light blue background. At the top, a box contains two equations for global interpolation functions:  $\psi_1^e = \frac{x_B - x}{x_B - x_A}$  and  $\psi_2^e = \frac{x - x_A}{x_B - x_A}$ . To the left of the box, the word 'GLOBAL' is written. To the right, 'INTERPOLATION FUNCTIONS' is written. Below the box, two equations for local interpolation functions are shown:  $\psi_1^e = 1 - \frac{\bar{x}}{h_e}$  and  $\psi_2^e = \frac{\bar{x}}{h_e}$ . To the left of these, the word 'LOCAL' is written. To the right, 'LAGRANGE FAMILY OF INTERPOLATION FUNCTIONS' is written. Below the box, the text 'n elements (linear)' is written. To the left, 'No. of nodes globally' is written, followed by an arrow pointing to 'n+1'. Below that, 'No. of nodes locally' is written, followed by an arrow pointing to '2n'. To the right of these, a table is shown with two rows: 'Global' and 'Local'. The 'Global' row has  $x_A = x_e$  and  $x_B = x_{e+1}$ . The 'Local' row has  $\bar{x} = 0$  and  $\bar{x} = h_e$ . The word 'element' is written above the table.

GLOBAL

$$\psi_1^e = \frac{x_B - x}{x_B - x_A}$$

$$\psi_2^e = \frac{x - x_A}{x_B - x_A}$$

INTERPOLATION FUNCTIONS

LOCAL

$$\psi_1^e = 1 - \frac{\bar{x}}{h_e}$$

$$\psi_2^e = \frac{\bar{x}}{h_e}$$

LAGRANGE FAMILY OF INTERPOLATION FUNCTIONS

n elements (linear)

No. of nodes globally  $\rightarrow n+1$

No. of nodes locally  $\rightarrow 2n$

Global  $\rightarrow x_A = x_e$   $x_B = x_{e+1}$

Local  $\bar{x} = 0$   $\bar{x} = h_e$

element

So these could be Lagrange family of interpolation functions they could be linear in nature, quadratic, cubic and so on and so forth. So these functions are very popular in second order differential equations, okay. Because in second order differential equations when we weakened the continuity we need only first-order derivatives  $c_0$  continuity is only needed, okay.

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GLOBAL

$$y_1^c = \frac{x_B - x}{x_B - x_A} \quad y_2^c = \frac{x - x_A}{x_B - x_A}$$

LOCAL

$$y_1^c = 1 - \frac{\bar{x}}{h_e} \quad y_2^c = \frac{\bar{x}}{h_e}$$

INTERPOLATION FUNCTIONS  
LAGRANGE FAMILY OF INTERPOLATION FUNCTIONS

1D element

$n$  elements (linear)

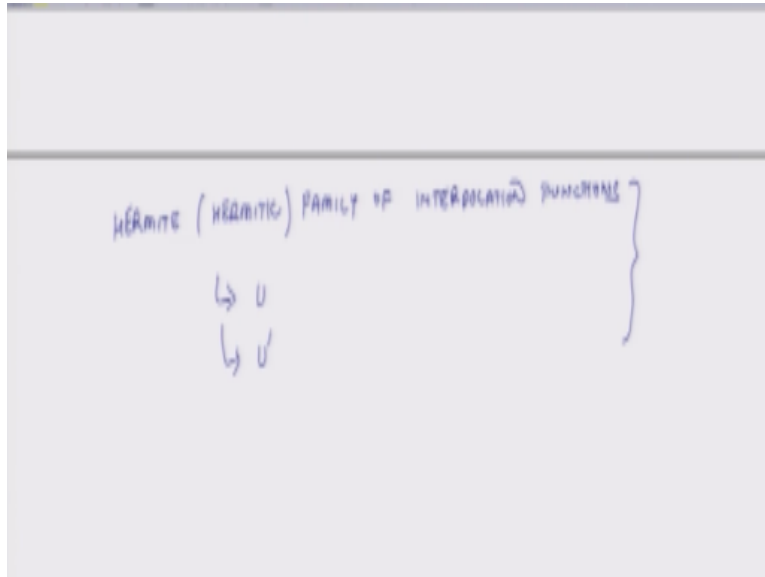
No. of nodes globally  $\rightarrow n+1$

No. of nodes locally  $\rightarrow 2n$

Global	$x_A = x_e$	$x_B = x_{e+1}$
Local	$\bar{x} = 0$	$\bar{x} = h_e$

Then there are functions, there is another class of functions known as hermetic.

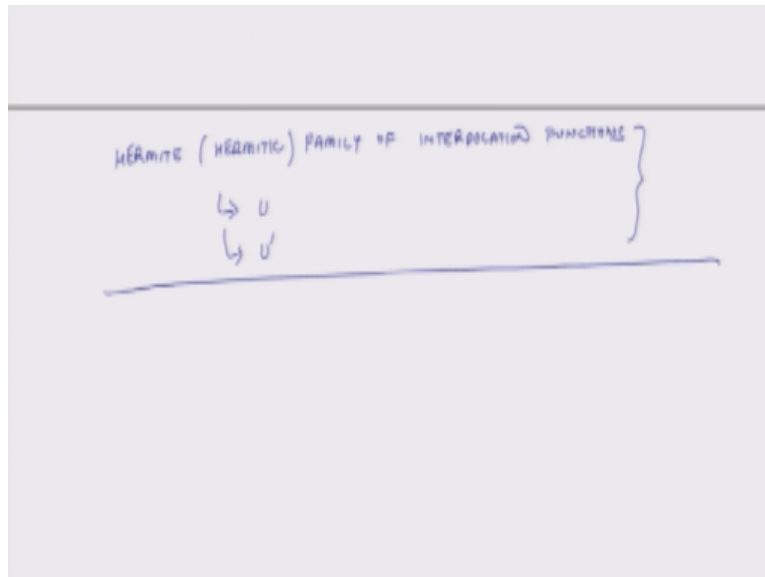
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Hermit or hermitic. Family of Hermitic family of interpolation functions and these functions are developed not only on considerations of primary variable but also its derivatives. These types of functions are very popular when you are also trying to solve fourth order equations for instance  $b$ , where you are not only worried about the deflection but also about slopes for instance slope at  $x = 0$  and a cantilever.

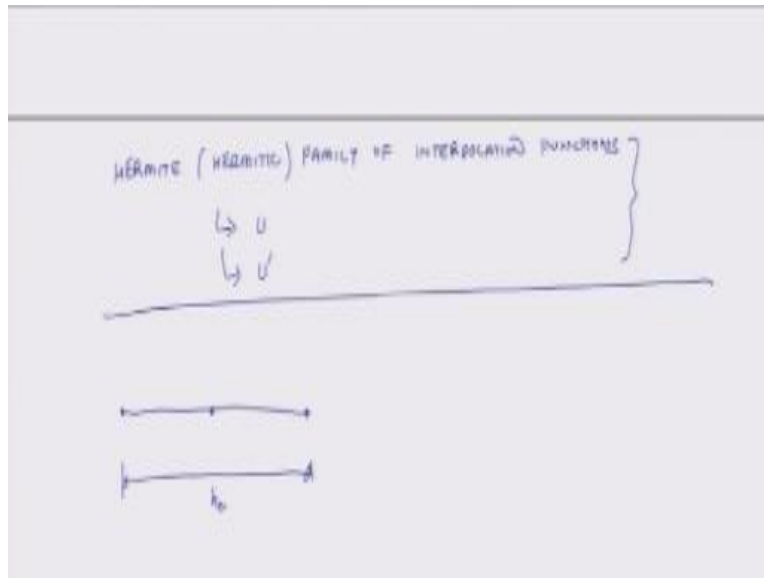
Fixed end is zero, right? So you are not only interested in ensuring that the function itself is continuous over the entire domain but you also want the slope should be continuous over the domain, okay. Slope has to be continuous over the entire domain.

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So those are hermetic family of interpolation functions, we will look at these functions maybe next week, but today and this week we will only work on Lagrange family of interpolation functions, okay.

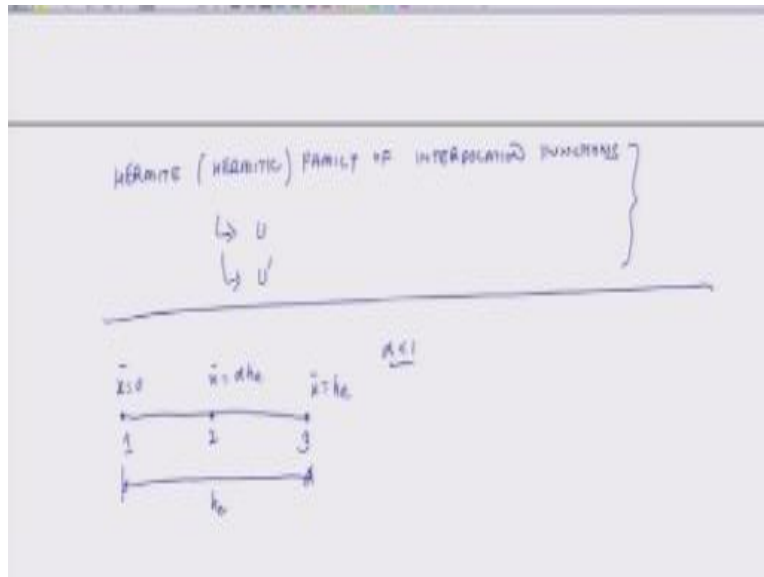
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Suppose our element was not linear but it was a quadratic element. Let us say its length is  $h_e$  of  $e^{\text{th}}$  element. So we have looked at linear elements, right? Let us say now in and in other case we want that our interpolation function or shape function is quadratic in nature which means that  $u$  varies quadratically over the length of each element. Suppose we want to make that assumption.



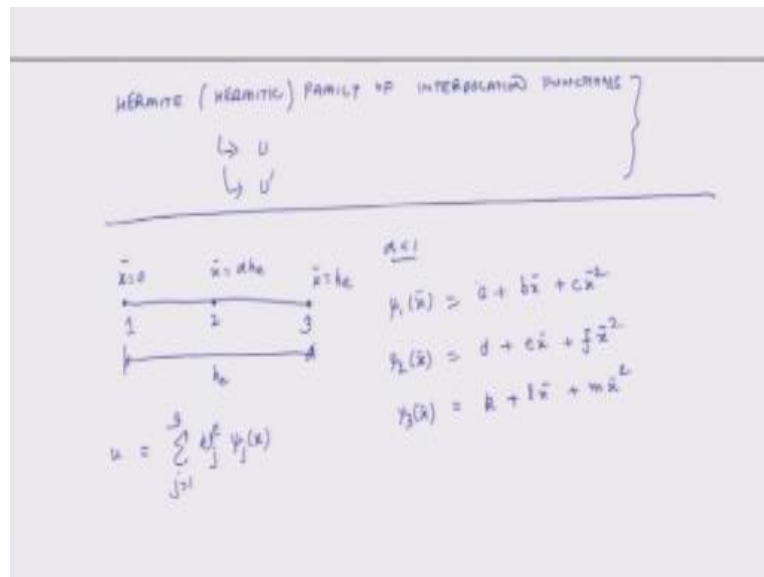
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So my first node, so it will have three nodes, first node is location in global local coordinate system will be 0,  $\bar{x}$  equals zero. So this is node one, this is node three, this is two, overall length is  $h_e$ . So this is  $\bar{x}$  is equal to  $h_e$  and let us say the location of second point is at  $\bar{x}$  equals  $\alpha h_e$  where  $\alpha$  is less than 1, some fraction, if it is perfectly located at the middle then  $\alpha$  will be half.

So then how will you derive the interpolation functions? So we will develop these interpolation functions and because this is a quadratic element it has three nodes and the interpolation functions will be three number  $\phi_1, \phi_2, \phi_3$  and we want to know what are the exact functions.

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So we just assume a general form  $\phi_1 x$  equals  $a + bx + cx^2$  this is the first approximation function,  $\phi_2 x$  is equal to  $d + ex$ . So I have to put a bar here because now I am in local coordinate system plus  $\bar{x}$  square and  $\phi_3 x$ . So again there is bar here, this equals  $f, g, h, I$ , I will use  $k + l \bar{x} + m \bar{x}^2$ . So we do not know what are the values of  $A, B, C, D, E, F, K, L, M$ , right? And we have to figure it out.

But we also know that the value of  $u$  over the element it varies and it can be written as  $u^e_j \psi_j(x)$ ,  $j$  is equal to 1 to 3, right? Because there are three functions, so very much similar to what we did in the last case we will develop conditions.

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The diagram shows a beam element of length \$h\_e\$ with nodes 1, 2, and 3. Node 1 is at \$\bar{x}=0\$, node 2 is at \$\bar{x}=d\$, and node 3 is at \$\bar{x}=h\_e\$. The displacement \$u\$ is given by the sum of shape functions multiplied by nodal displacements:

$$u = \sum_{j=1}^3 d_j^e \psi_j(\bar{x})$$

The shape functions are:

$$\begin{aligned} \psi_1(\bar{x}) &= a + b\bar{x} + c\bar{x}^2 \\ \psi_2(\bar{x}) &= d + e\bar{x} + f\bar{x}^2 \\ \psi_3(\bar{x}) &= k + l\bar{x} + m\bar{x}^2 \end{aligned}$$

At node 1 (\$\bar{x}=0\$), the displacement is \$u(0) = u\_1^e\$. This is equal to the sum of the shape functions evaluated at 0 multiplied by their nodal displacements:

$$u(0) = u_1^e = u_1^e \psi_1(0) + u_2^e \psi_2(0) + u_3^e \psi_3(0)$$

For this to be possible, the following conditions must be satisfied:

$$\begin{aligned} \psi_1(0) &= 1 & \Rightarrow 1 &= a + 0 + 0 \\ \psi_2(0) &= 0 & 0 &= d + 0 + 0 \\ \psi_3(0) &= 0 & 0 &= k + 0 + 0 \end{aligned}$$

We have to develop nine different equations so that we can find these nine unknowns A, B, C, D, E, F, K, L, M, okay. So the first condition is that at  $x = 0$  or  $\bar{x}$  equal zero which is this location node one, what is the displacement or the what is the value of primary variable?  $u_1$ ,  $u_1^e$ ,  $u_0 = u_1^e$  and this is equal to  $u_1 \psi(0) + u_2 \psi_2(0) + u_3 \psi_3$  evaluate it at zero. And of course I have to put a superscript e.

For the  $e^{\text{th}}$  element suppose, okay? Now this relation is valid for all values of  $u$ 's it does not matter what values of  $u_1$ ,  $u_2$ ,  $u_3$  are but it is valid for all values of  $u$ 's. So I get three different relations, so this is possible only if  $\psi_1(0)$  equals one,  $\psi_2(0)$  equals zero,  $\psi_3(0)$  equals zero which means one equals A and in this case I am going to put in this equation  $\psi_1$  this is the relation for  $\psi_1$ .

So this is equal to  $A+B \bar{x}$ ,  $\bar{x}$  is,  $\bar{x}$  is local and the value of  $\bar{x}$  is zero. So  $0+0$ , second equation is zero equals  $d + 0 + 0$ , third equation is  $0 k+0 +0$ , okay. So I right away get three unknowns. Then I do the, so this is first condition. What is the second condition?

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$$\begin{aligned}
 u &= \sum_{j=1}^3 u_j^e \psi_j(x) \\
 \text{1st Condition At } \bar{x} &= 0 \quad U(0) = u_1^e = u_1^e \psi_1(0) + u_2^e \psi_2(0) + u_3^e \psi_3(0) \\
 \text{Possible only if: } & \left. \begin{aligned} \psi_1(0) &= 1 & \Rightarrow & 1 = a + 0 + 0 \\ \psi_2(0) &= 0 & & 0 = d + 0 + 0 \\ \psi_3(0) &= 0 & & 0 = k + 0 + 0 \end{aligned} \right\} 3 \text{ eqns}
 \end{aligned}$$

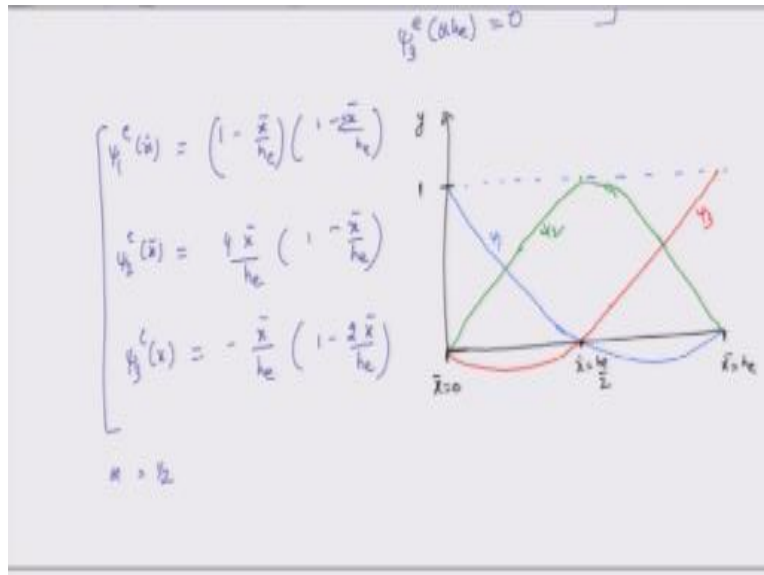

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$$\begin{aligned}
 \text{2nd Condition At } \bar{x} &= \alpha h_e \quad U(\alpha h_e) = u_2^e = u_1^e \psi_1(\alpha h_e) + u_2^e \psi_2(\alpha h_e) + u_3^e \psi_3(\alpha h_e) \\
 \text{Possible only if: } & \left. \begin{aligned} \psi_1(\alpha h_e) &= 0 \\ \psi_2(\alpha h_e) &= 1 \\ \psi_3(\alpha h_e) &= 0 \end{aligned} \right\} 3 \text{ eqns}
 \end{aligned}$$

That at  $x$  is equal to or  $x$ - bar equals  $\alpha$  time  $h_e$ , this is the second condition.  $U(\alpha h_e) = u_2^e$  and this is equal to  $u_1 \psi_1(\alpha h_e) + u_2 \psi_2(\alpha h_e) + u_3 \psi_3(\alpha h_e)$ , and again I have to put superscripts everywhere. So my second this is going to be possible only if  $\psi_1^e(\alpha h_e)$  when I calculate that is equal to 0 right? So  $\psi_2^e(\alpha h_e)$  is equal to one and  $\psi_3^e(\alpha h_e)$  equals zero.

So these are the second three conditions. So I have three equations here, I have three equations here and similarly I will have a third condition, that at the third node I will get three more equations. So in this way I get 9 equations. 9 unknowns I can solve them and I eventually what I get is.

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That  $\psi_1^e$  which is a function of  $\bar{x}$  equals  $1 - \frac{\bar{x}}{h_e}$  times  $1 - \frac{2\bar{x}}{h_e}$  ( $\alpha h_e$ ) you have to check that, okay.  $\psi_2^e$  of  $\bar{x}$  equals  $\frac{4\bar{x}}{h_e} \left(1 - \frac{\bar{x}}{h_e}\right)$  and  $\psi_3(\bar{x})$  is equal to  $-\frac{\bar{x}}{h_e} \left(1 - \frac{2\bar{x}}{h_e}\right)$ , and I think these relations are valid. So there is a 2 here when  $\alpha = 1/2$  if the midpoint is located at the centre, okay. And if you plot these, suppose here this is  $\bar{x}$  equals zero.

$\bar{x}$  equals  $h_e$  here  $\bar{x}$  equals  $h_e$  over 2 and your  $y$ -axis let us say  $y$ -axis and this is one, then your first function  $\psi_1$  varies like this,  $\psi_2$  it varies like this, and  $\psi_3$  will vary like that. So that is my  $\psi_3$ , this is my  $\psi_2$ . So once again you see that these shape functions, these shape functions are having a value of unity at specific nodes and at other nodes they are zero, at other nodes they are zero.

(Refer Slide Time: 25:02)

The image shows a handwritten slide titled "PROP. OF INTERPOLATION FUNCTIONS". It contains the following text and equations:

①  $\psi_i^c(\bar{x}_j^e) = \delta_{ij}$

Annotations on the slide:

- An arrow points from  $\psi_i^c$  to the word "function".
- An arrow points from  $\bar{x}_j^e$  to the word "node".
- A bracket on the right side of the equation indicates the value of  $\delta_{ij}$  for two cases:
  - $\delta_{ij} = 1$  when  $i = j$
  - $\delta_{ij} = 0$  when  $i \neq j$

So very quickly we will cover these properties. The first thing is that the value of  $\psi_i$  at location  $j$ ,  $j^{\text{th}}$  node. So this is the coordinate of  $j^{\text{th}}$  node. This is  $i^{\text{th}}$  function, okay. So the value of  $i^{\text{th}}$  function at  $j^{\text{th}}$  node is equal to  $\delta_{ij}$  where  $\delta$  equals 1 if  $i = j$  and it is zero if  $i$  does not equal  $j$ . This is the important thing. So this concludes our discussion of interpolation functions and in the next class what we will do is we will use these interpolation functions.

To develop element level equations and once we have developed those element level equations then we will actually assemble those equations. So now we will start looking at all the nitty gritty details of  $F_A$ . So thank you very much, and we look forward to meeting tomorrow.

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