**Indian Institute of Technology Kanpur**
**National Programme on Technology Enhanced Learning (NPTEL)**
**Course Title**
**Basics of Finite Element Analysis**

**Lecture – 21**
**Different types of Weighted Residual methods**
**PART-II**

**by**
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Hello, welcome to basics of finite element analysis

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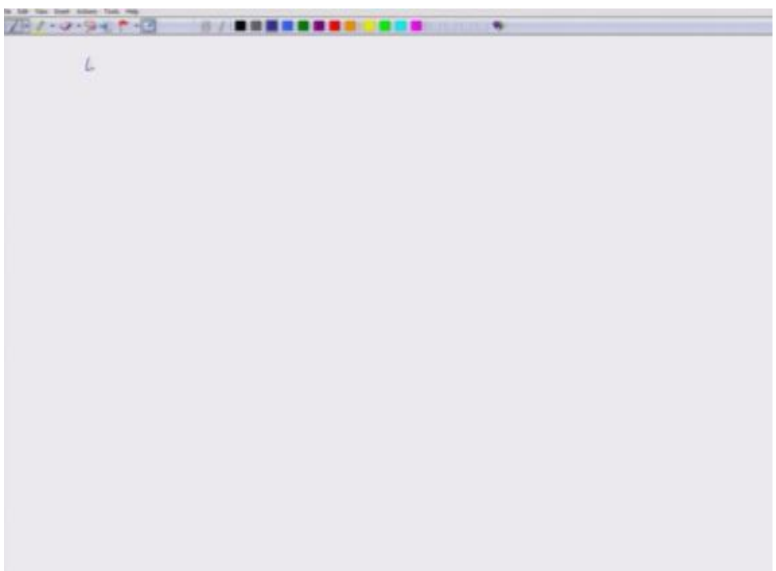

Now what we will do in next 10, 15 minutes is we will actually solve a problem so that you become clear about all these four methods and then you will see the limitations of even these four approaches.

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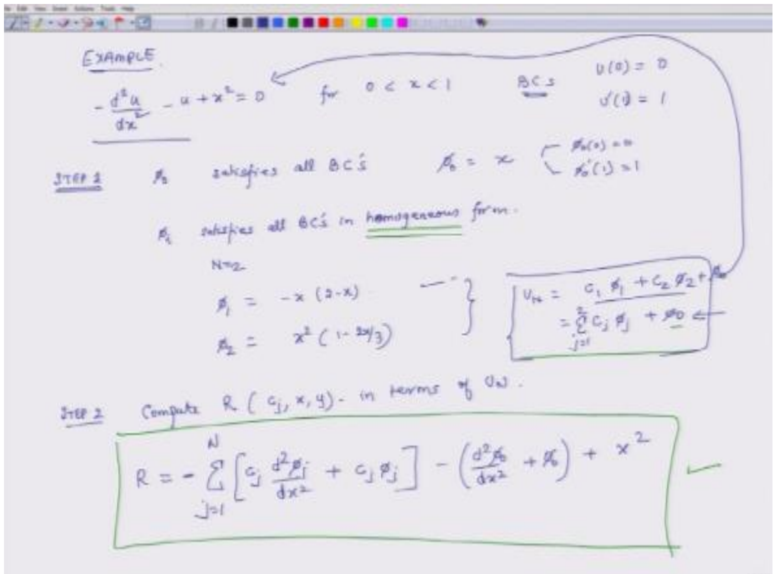


Example, so what we  are going to do is we are going to solve this differential equation and this equation is valid for 0 to between 0 and 1 $x^2$ okay and my boundary conditions are u at 0 equals 0 and u´ at 1 equals 1, these are the boundary conditions, so the first step, step one so we will solve this equation  using Petrov Galerkin, Galerkin, colloactaion and least square, all these four methods okay. So so the first step is that we choose $Ø_1$ so that it satisfies all boundary conditions okay so one boundary condition is at x=0 u is 0, other boundary condition x=1 its differential is 1, so we choose $Ø_0$ =x it meets both the boundary conditions.

The second boundary condition u´ at 1=1 is a non homogenous boundary condition, the first boundary condition is a homogeneous boundary condition okay, so we see that $Ø_0$ at 0 equals 0 and $Ø_0'$ at 1 equals 1okay. Then what we do is we choose $Ø_i$ and $Ø_i$ has to satisfy all the boundary conditions but they are homogeneous form okay, and what we will do is that we will develop a two term solution, n is equal to 2 so we take n is equal to 2, so we will take $Ø_1$ and $Ø_2$ hence such that $Ø_1$ = -x times -2-x and $Ø_2$ is $x^2$ times 1-2x/3 okay.

So let us see $Ø_1$ at x is equal to 0 is 0 it satisfies that boundary condition, then $Ø_1$ its first differential is -2+2x right and that is equal to 0 at the other end so this satisfies both the boundary condition in homogeneous form this is very important in homogeneous form, and the same thing we see is true for $Ø_2$ also, so $Ø_1$ has to solve the actual boundary conditions $Ø_0$ $Ø_1$ $Ø_2$ $Ø_3$ $Ø_4$ they have to solve the homogeneous form of the boundary conditions so this is important.

So this is there, then our next step is compute R so R is the residue and this is a function of $c_j$ x and y, so what I do is I plug in so for this we plug in $Ø_0$ and one second so here my $u_N$ is equal to two term solution $u_2$ I will not is equal to $c_1$ $Ø_1$ + $c_2$ $Ø_2$ excuse me three, so I am sorry I have messed it up so this is my definition of $u_N$ , so we compute R in terms of $u_N$ which is defined by this equation so basically I plug in this equation in this differential equation. ठीक है, पांच, छह मिनट extra जायेंगे

In this differential equation we plug in and we get the value of R okay, so my residue is Σ j is equal to 1 to N $c_j$ second derivative of $Ø_j$ by $dx^2$ and there is a minus sign here + $c_j$ $Ø_j$ okay – d2 $Ø_0$ over $dx^2$ + $Ø_0$ + $x^2$, this is my residue okay, so this is my residue this is the statement for residue and one thing I wanted to correct here is that in this definition of $u_N$ there should have been a $Ø_0$ also, because in the second statement $Ø_0$ is already included there okay so that is my definition of residue and what I do is.

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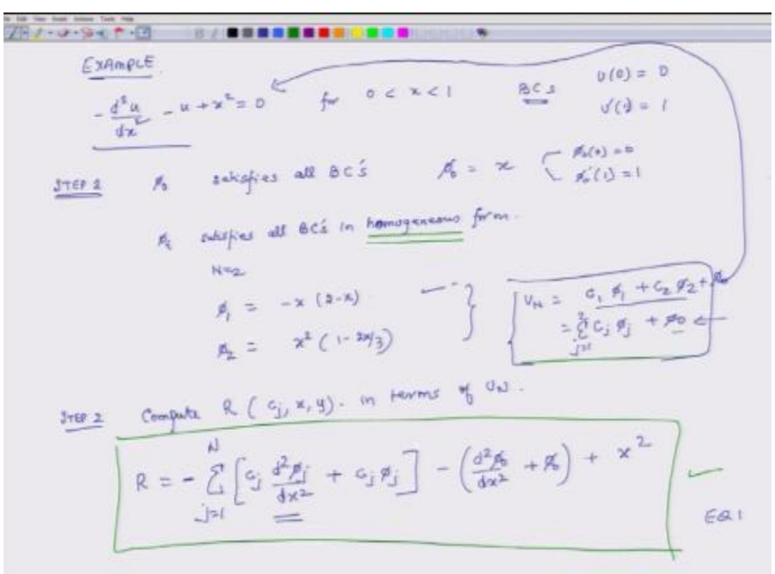

At this stage I compute the value of second derivative of $Ø_0$ and $Ø_0$ was x so it is second derivative is 0 then I compute the second derivative of $Ø_1$ because here I have to have secondary derivative terms so that is I why I am doing this, so this is equal to 2 and then the second derivative of $Ø_2$ is equal to 2-4x, so I can again plug these things back in my expression for R and I do all the math and what I get is that my R, if I plug all these things in let us call this equation 1 so if I plug $Ø_0''$ $Ø_1''$ $Ø_2''$ in the in first equation.

Then my R it comes out as $2-2x+x^2$ times $c_1$ + $-2+ 4x -x^2 +^{2x2/3}$ times $c_2$ $-x+x^2$ so this is a simplified expression for residue R okay, and I what is my goal I have to find $c_1$ and $c_2$ if I know $c_1$ and $c_2$ then I can calculate using this relation u at all locations in my domain so my problem right now is that I do not know $c_1$ and $c_2$ so I have to find $c_1$ and $c_2$.

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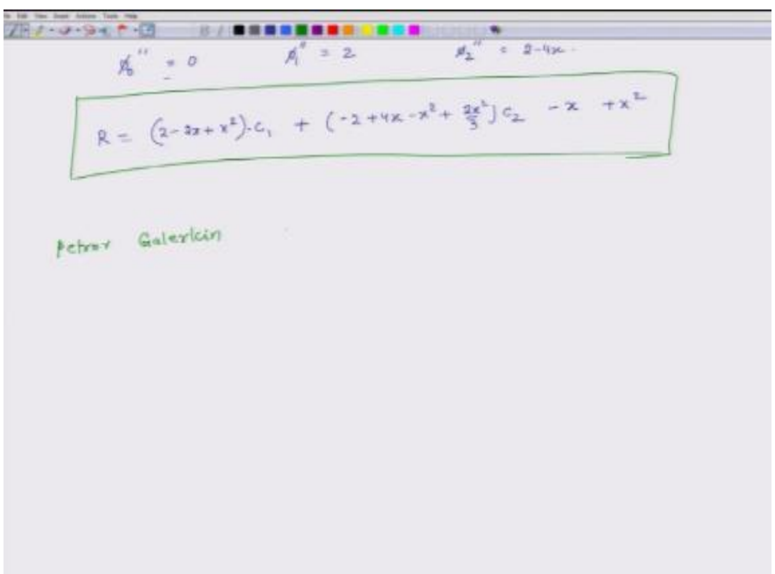


So to find $c_1$ and $c_2$ now we will apply different weighted residual methods okay and for different residual methods we have to make different choices of weight functions, so our first method is Petrov Galerkin, here the weight function should not be same as the approximation function for the primary variable.

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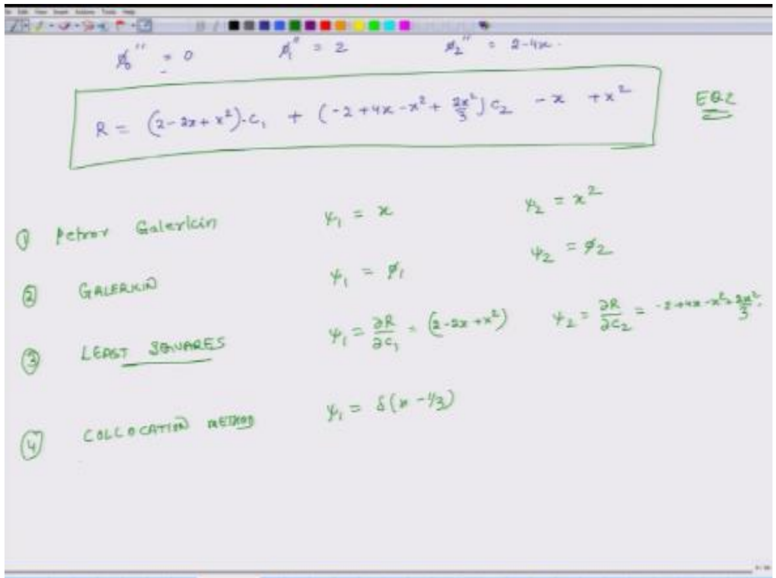


So I choose say $\psi$ is the weight function as x and $\psi_2$ is the second weight function as $x^2$ okay so this is one, then for Galerkin method the requirement is different where $\psi 1$ size should be the weight function should be same as the approximation functions so here $\psi_1 = Ø_1$ and $\psi 2 = Ø2$, then in the third case I use least squares approach, I use the method of least squares and in this method of least squares $\psi_1 = \partial R$ with respect to $C_1$ so this is equal to $2-2x + x^2$ how did I get this from equation 2 okay.

And $\psi_2$ is equal to $\partial R/C_2$ and this is equal to this entire thing $– 2 +4 x -x^2+ 2+4x –x^2 + 2 x^2/3$ okay, and in the fourth case I am going to use collocation method okay, so my $\psi_1$ is direct $\delta$ function located at x-1/3 so I am just picking up these locations so here if it is a two term solution I have to make sure that the residue goes to 0 at 2 points, so those two points are

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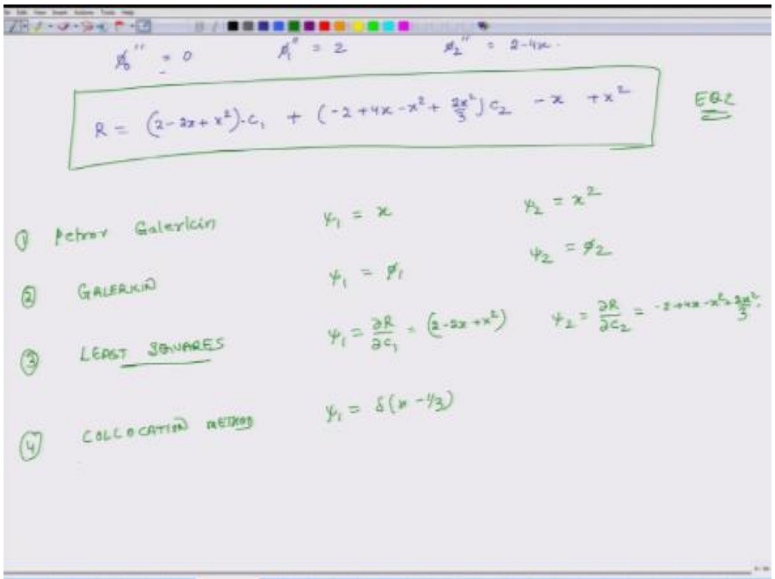


I have made that choice it is not that some special mathematics I am picking up that first point is x=1/3 and second location is x=2/3 okay, so in a domain of 0 to 1 these two points are the places where residue is 0 and all the points in the domain then are equally spaced, but I can pick some other points also so this is direct $\delta$ at x=1/3 and the other one is direct $\delta$ at x*-2/3* okay*,* so these are different weight functions and the choice of weight functions depends on different flavors of weighted residual method.

Now for each of these weighted residual methods I have two write to linear equations because my goal is to find $c_1$and $c_2$

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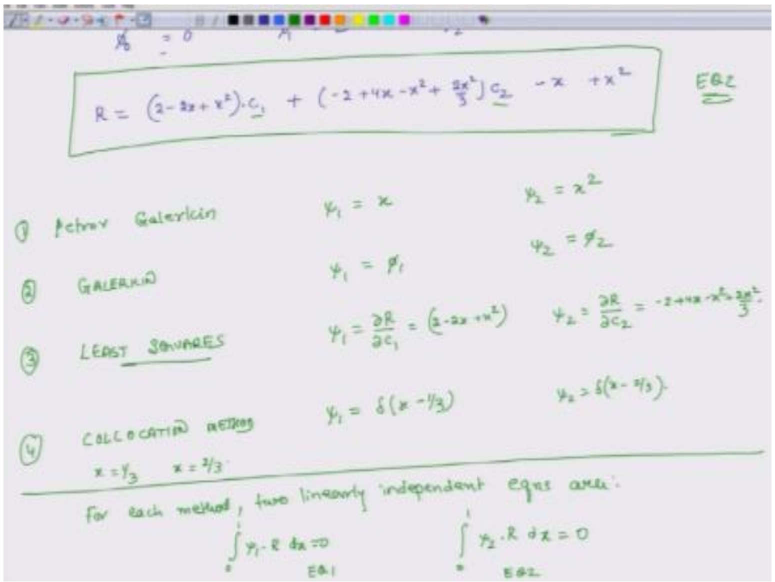


So the first linear equation is each method two linearly independent, what are these two linearly independent equations, the first equation is $\psi_1$ so I can pick which type of $\psi_1$ I am interested in right based on the method times residue, residue is defined here okay 0 to 1 which is the domain of our interest this is equal to 0, so this is equation one and the second equation is 0 to 1 $\psi_2$ times the R d Ω excuse me, so this is dx is equal to 0 and here also I will write dx =0 so this is my equation 2 okay.

So I get 2 equations for each method and I have two unknowns $c_1$ and $c_2$ and in this case it is a linear system so they are linear equations, simultaneous equations I can solve them and I can calculate the values of c’s okay so what I will do is so once the c is there then my solution

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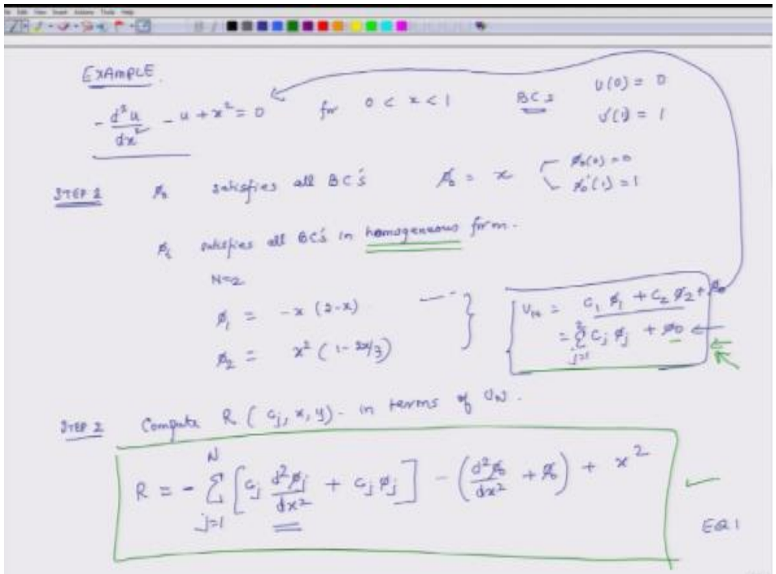


ux is equal to, I get a solution of something like this and the values of $a_1$, $a_2$, and $a_3$ can be determined by, once I have calculated c's I plug those c's in this relation and I can calculate the values of $a_1$, $a_2$ and $a_3$ right.

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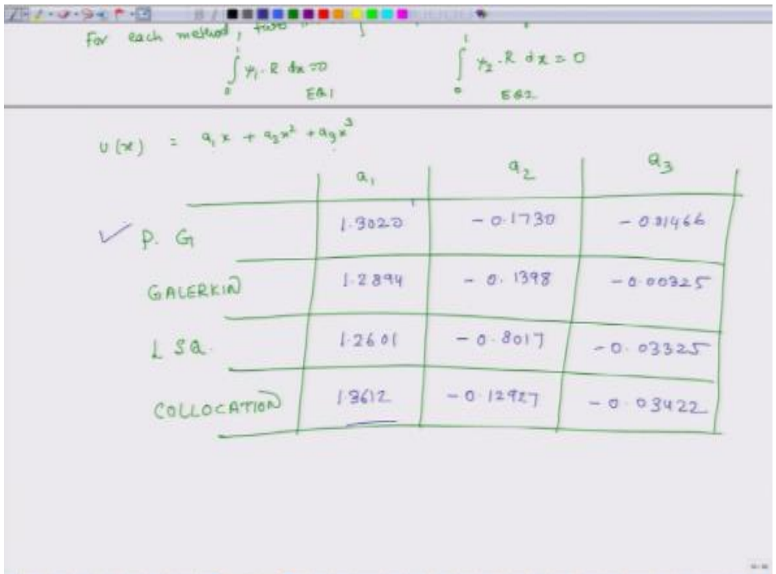


I can get the value of $a_1$, $a_2$ and $a_3x^3$ yeah and what I will do is I will actually tabulate so first solution method is Petrov Galerkin, second method is Galerkin, third method is least squares, fourth method is collocation, okay and here I am going to write $a_1$, $a_2$ and $a_3$ okay.

So this number is 1.3020- 0.1730 -0.14 excuse me 01466, for Galerkin I see 1.2894 -0.1398 -0.00325 okay, for least squares I am getting 1.2601-0.8017 -0.03325, and the collocation method gives us 1.3612 -0.12927 and this is – 0.03422, and if you compare the, so what you see is that the first term parameters and they are pretty close to each other, pretty close to each other and if you keep on increasing the number of terms in the solution all these solutions will slowly converge to the same value, they will converge.

To the same value and in this particular case if you actually compare the results with exact solution you will find that this method gives the best solutions but that may not necessarily always be the case for all problems, so this is there and this completes our discussion on weighted residual methods and what we have explored are four different weighted residual methods Petrov Galarkin, Galarkin, least squares, and collocation method and the choice of weight function decides what type of weighted residual method we are going to practice, so that concludes our lecture for the day, we will continue our new topic, we will start a new topic starting tomorrow, thank you.

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