Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 20
Different types of Weighted Residual Methods
Part - I

by
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Hello, welcome to basics of finite element analysis, today is the second day of our lecture for the fourth in this particular fourth week and in the last class we were discussing weighted residual method and we had devolved an expression for weighted residual method which was this.

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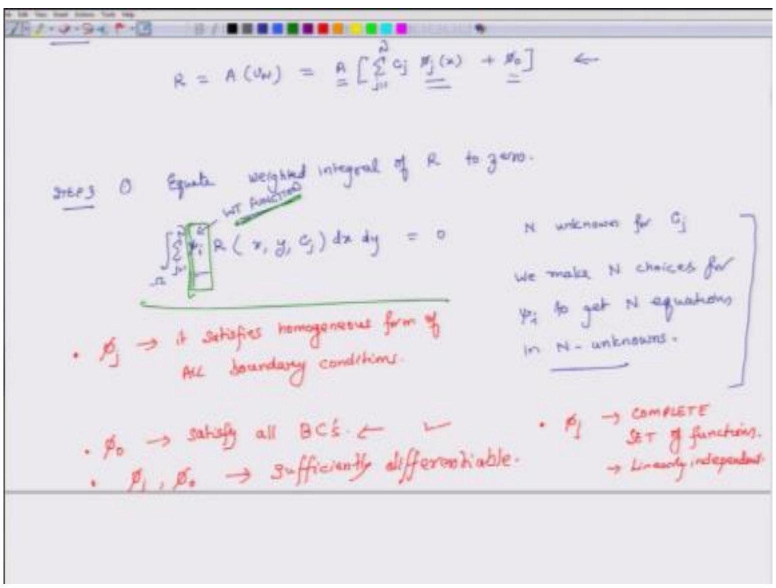

Where it is integral of the weight functions I times residue when you integrated to other domain it could be dx ty that is zero, and then I had mentioned that based on this particular weighted residual method it has different flavors and depending on the choice of our weight function it is a particular type of weighted residual method.

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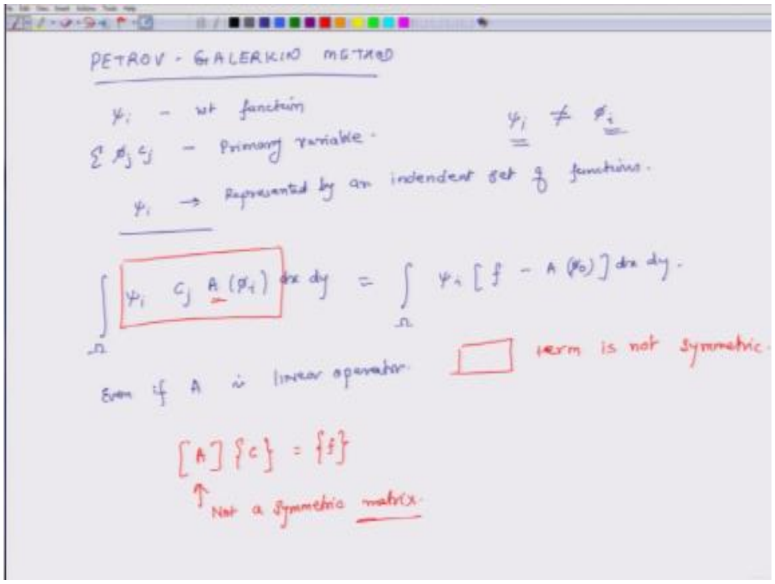

So today we will discuss some of these methods and the first method we are going to talk about is called Petrov Galerkin method, Petrov Galerkin okay, so in this the only restriction on the choice of weight function so this my weight function ψ_i and my so this is the weight function and then my u is represented as $Ø_j$ times c_j this is my primary variable, so in Petrov Galerkin method there is only one restriction on the choice of ψ and that restriction is that ψ_i should not be equal to Ø okay should not be equal to $Ø_i$.

So it should not so what that means is that these ψ is represented by an independent set of functions and there is no restriction in terms whether it has to meet the boundary conditions or not okay, so you may want to pick a function which meets the boundary condition that is fine or if it does not meet that is fine also, but it should not represent, be from this set of Ø so that is the Petrov Galerkin method okay. Now suppose our residue was expressed as I am integrating it over the domain ψ_i so c_j a($Ø_i$) dx, dy and this equals some forcing function so I am integrating it times ψ_i times some function of x-a times $Ø_0$ dx dy.

Now even if a is linear operator, even if a is linear suppose I assume that a is linear it need be, but even if a is linear this term, this term is not symmetric, symmetric okay so when I construct my matrix I will get some A matrix times some constants, unknown constant c will equal some forcing vector f this will not be a symmetric matrix, so in Petrov Galerkin method the A matrix need not necessarily be symmetric okay.

If we want this matrix to be symmetric and if we see that A is an operator which is of even order, second order, fourth order then we can weaken the differentiability

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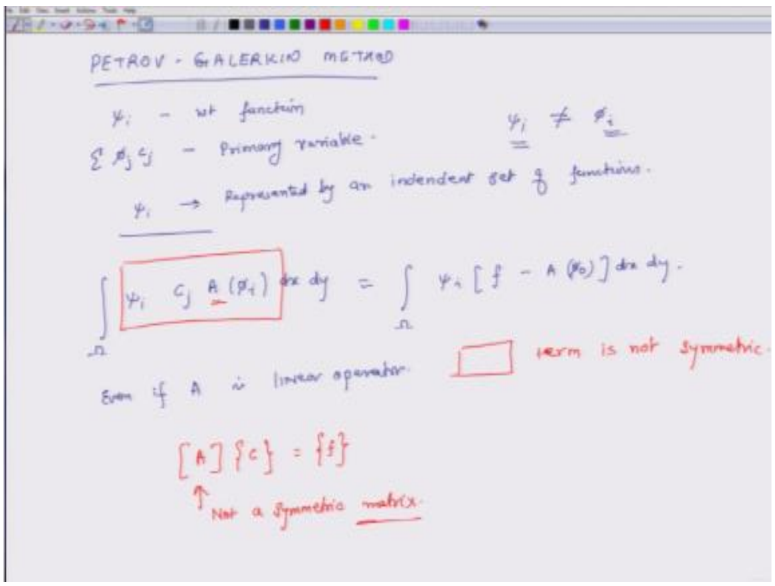

And then get a symmetric matrix but if we keep it as is then A matrix will not be symmetric, so this is all about Petrov Galerkin method

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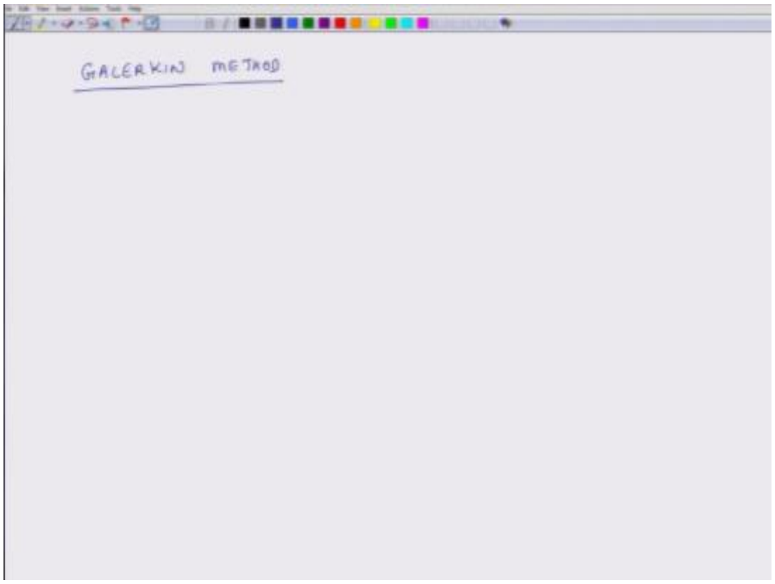

The next method is Galerkin method okay, so in Petrov Galerkin method

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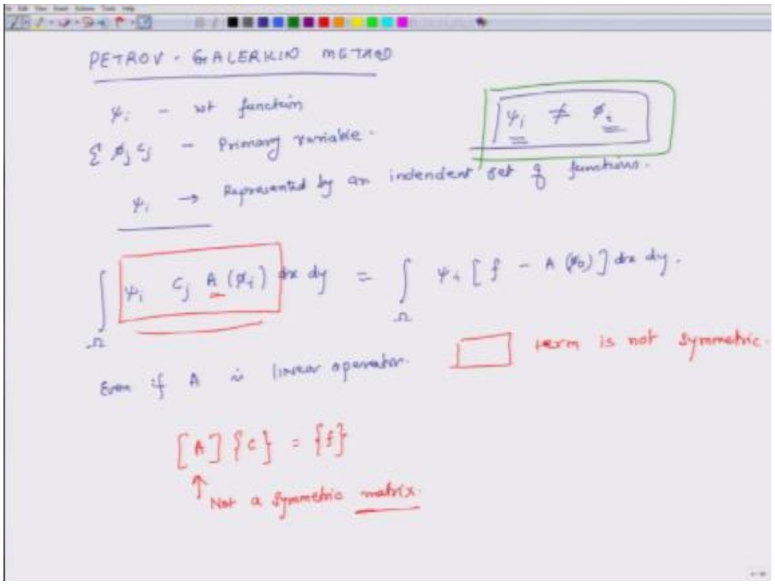

We had this restriction that $Ø_i$ should not be equal to the weight function should not be equal to the approximation further function for the primary variable.

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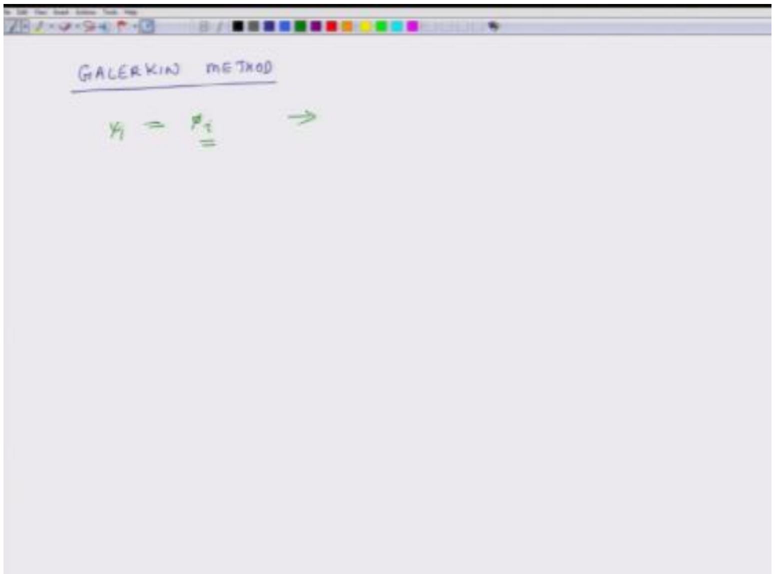

Here we say that ψ_i should be equal to $\varnothing_i$, should be equal to $\varnothing_i$, so the good thing about this is that we do not have to guess a new set of functions if we have figured out some functions which are $\varnothing_i$ which we meet all the natural boundary conditions and primary, essential boundary conditions we can use same functions

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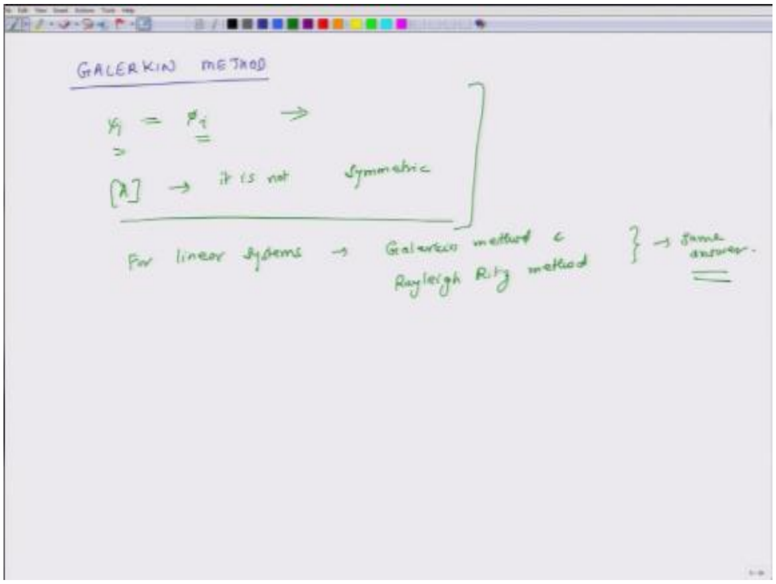

For weight functions okay, here also A matrix need not be symmetric actually is not, not need not be it is not it is not symmetric and if we want to make it symmetric again we have to see if a is a second order or fourth order differential operator then we can weaken the differentiability and then get symmetric matrix but so that is the case. For linear systems this Galerkin method and Rayleigh Ritz method both of them give the same answer okay, they give the same answer.

So we have looked at first one was Petrov Galerkin, second one was Galerkin the third method is called method of

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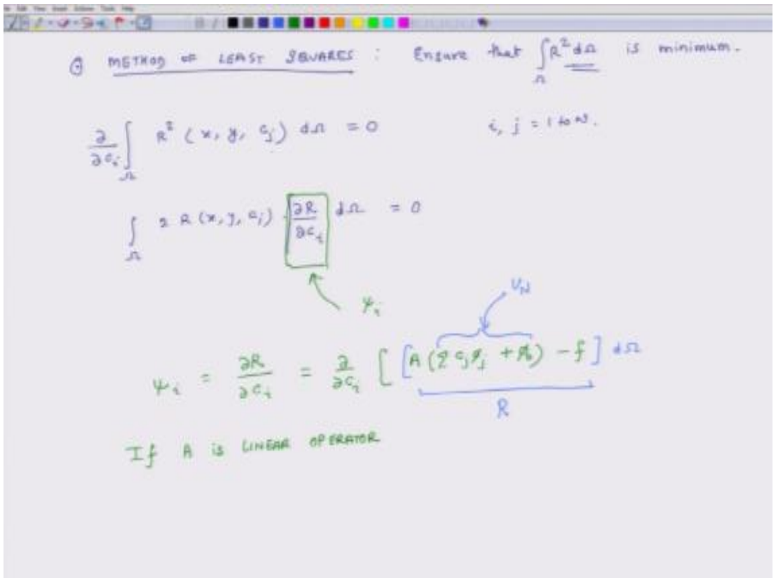

Least squares, so what do we do here we ensure that the square of residue when you integrate it over the domain, the square of residue when you integrate it over the domain is minimum, this is what we ensure so we will discuss this in detail, so this is so my residue is a function of x, y and c_j right and this residue is integrated over the domain and if this thing, if this integral is minimum if this integral is minimum for anything to be minimum what does that mean.

That it is differential and this case it will be so what is the unknown in this c_j so it is differentials with respect to this unknown c_j they should be zero okay, actually I will not write it c_j here I will say c_i because I can I will differentiate it with all, if there are n unknowns values of c_j I will differentiate with every single c_j okay, so i is equal to 1 to N so what that means is 2R times again I am sorry zero okay. What that means is, look at this term in green I can call this ψ_i okay so in method of least squares the choice of ψ_i ∂ R which is residue with respect to C_i, it is ∂ R with respect to c_i and I can write this is as ∂ over ∂ c_i and residue was what some operator, operator depends on the nature of the differential equation.

So some operator A which was a function of x, y and u right so x, y I do not have to worry so this is a function of sums of c_j $Ø_j + Ø_0$ this is my definition of u this is un right, and then there is also some forcing term –f okay so this entire thing is my residue and I am integrating this entire residue on the domain okay. Now I say that if A is linear operator then what will be the powers of $C_{j's,}$ they will not be 2, 3, 4 right the power of C_j will be only one, if it is a linear operator then because if it is a linear operator what does that mean that u will not come as a square or cubic terms in the equation it will only come in when its power is one if its power is one then the power of C_j will.

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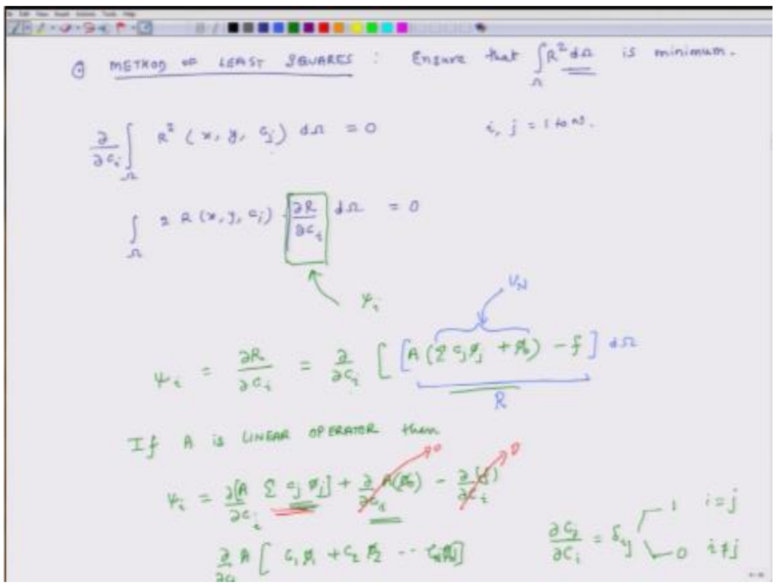

Also be never be more than one it will always be identically one, so if a is linear operator then, then if A is linear operator then ψ_i equals I have to differentiate this, this entire thing with respect to C_j right, so then $Ø_i$ equals this thing and C_j $Ø_j$ + $Ø_0$ –f so ∂ over ∂ c_j right and then again I differentiate it ∂ over, oh I am messing it up, c_i - ∂ over ∂c_i f, so if it is a linear operator first thing I can do is can separate it in c_j and $Ø_i$ if it is a linear operator, that is what we had defined several lectures back.

Now this part it does not depend on c at all right it is only it only depends on $Ø_0$ so this is 0, this is also 0, so the only thing which depends on c's is this thing right, so the next thing you should look is that in this term it will be basically ∂ over ∂c_i A of let us say c_1 $Ø_1$ + c_2 $Ø_2$ and so on so forth right. It will go on till c_n $Ø_n$ okay, now when I differentiate so what does that mean that if i equals one then only c_1 term will appear in the system because the ∂c_2 to with respective c_1 is 0 right.

So ∂ cj with respect to ∂c_i is equal to this chronicle delta i_j and its value is one when i equals j, its value is 0 when I does not equal j okay, understood? So because of this what you will get is if I am differentiating it with respect to the r^{th} constant, constant all other terms will vanish and the only term which will remain will be $A(Ø_i)$ understood

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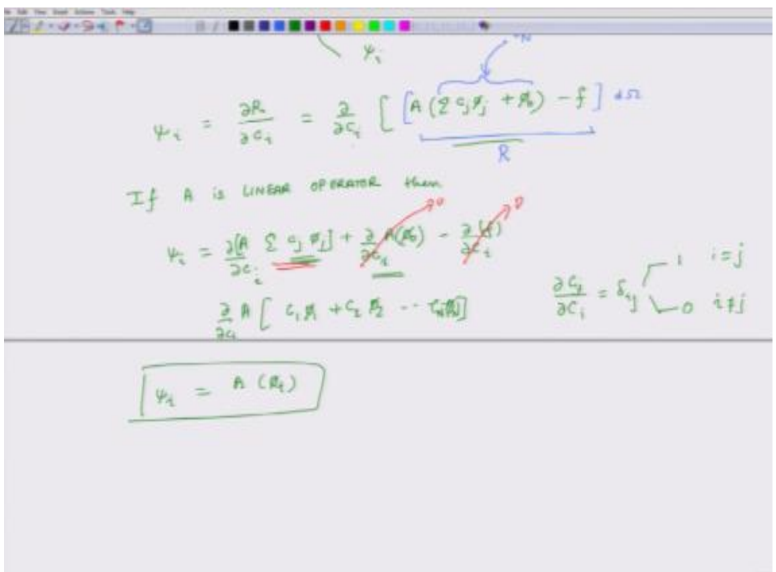

because of this ψ_i equals $A(Ø_i)$ understood, because all other terms will be 0 and the coefficient of $Ø_i$ in this case will become one because $\partial(c_i)$ with respct to c_i is one if i and j are same okay, so this the second this is the third method which is known as method of least squares okay also when you look at this because I have this what I am doing is.

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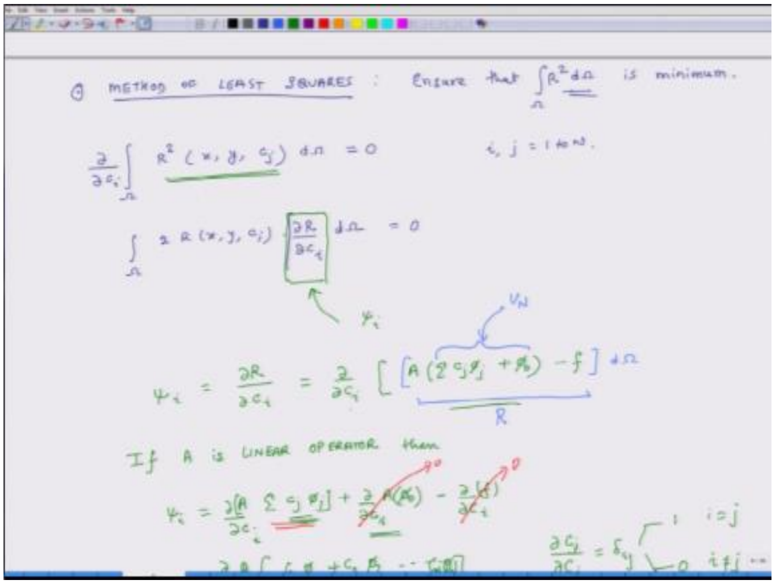

What am I doing I am minimizing the integral of R^2 so this is, mathematically it looks very similar to our first situation where we were minimizing i which was a bilinear functional, bilinear and symmetrical functional, same thing is true here also this is R^2 okay so this functional also it looks mathematically similar to the i functional which we had discussed in context of Rayleigh Ritz method so because of that reason.

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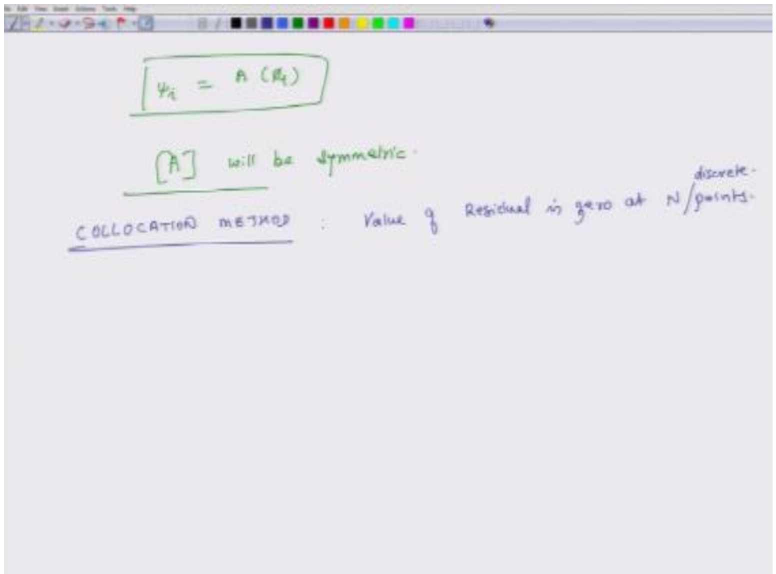

Here a matrix will be symmetric, so in method of least squares your A matrix is going to be symmetric and the last method we will discuss is the called collocation method okay, so in this method what we say is that the value of residual and I will make a discreet points, so we identify in the domain and at those n number of points suppose I have end term solution so I need end term equations, so I identify end points in the domain which I think are important and at those end points I ensure that the condition that this residual is zero, I enforce that condition okay.

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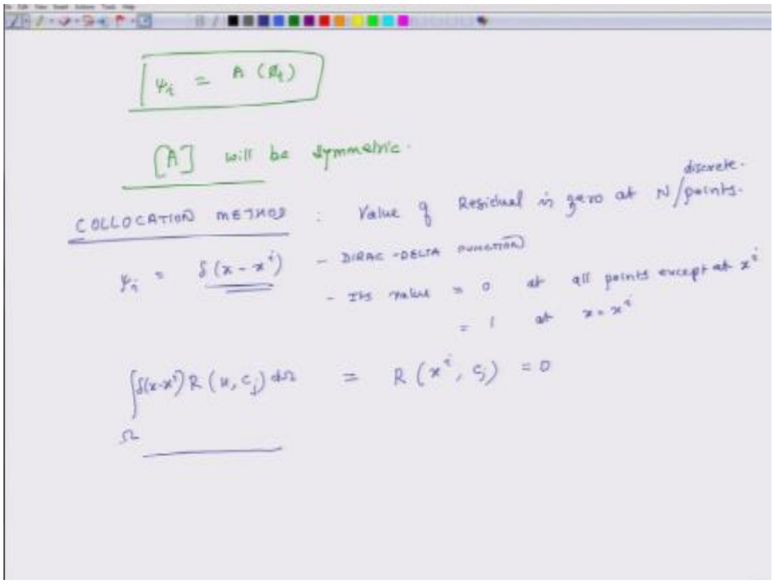

So what does that mean, so in this case my $\varnothing_i$ is what it is a direct δ function, it is a direct δ function and its value is equal to 0 at all points except at location x^i, so it is not x to the power it is just way to designate it okay and its value is one at x is equal to x_i. So in this case my equation is integral over Ω $\varnothing_i$ this was my weighted residual statement and suppose it was only one dimensional so d Ω right and this I replace $\varnothing_i$ by what, direct δ function okay.

And if you go back and check your mathematics this is nothing but integral of, oh sorry the value of this integral is equal to residue at location x^i and of course it is a function of c_j, so this is equal to 0. Whenever you multiply a direct δ function by some function and integrate it the integral of that entity is nothing but the function evaluated at that particular point okay, so this is the condition.

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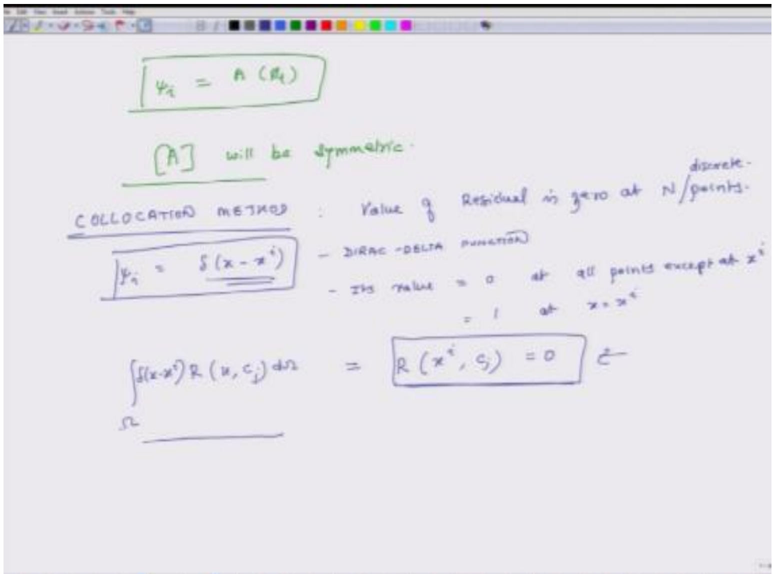

For collocation method, so we, so in this case my Ø is a direct δ function in case of method of least squares my Ø was

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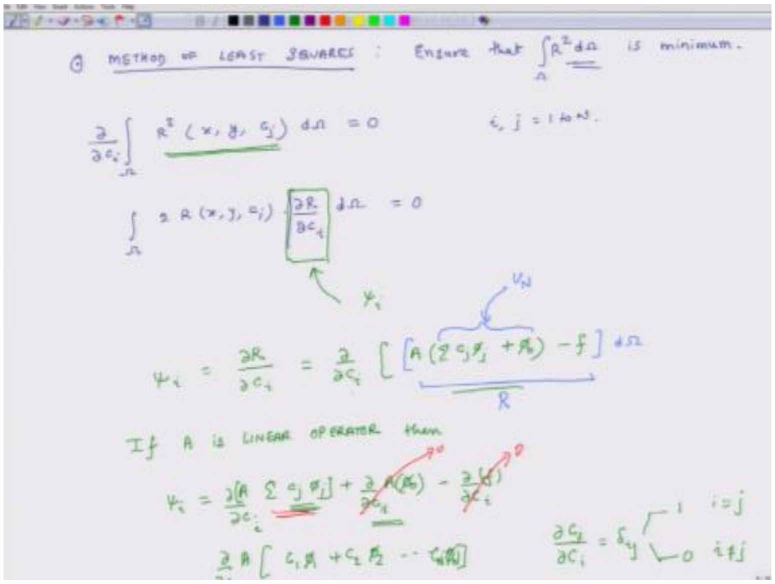

δ over δc but effectively what we had seen

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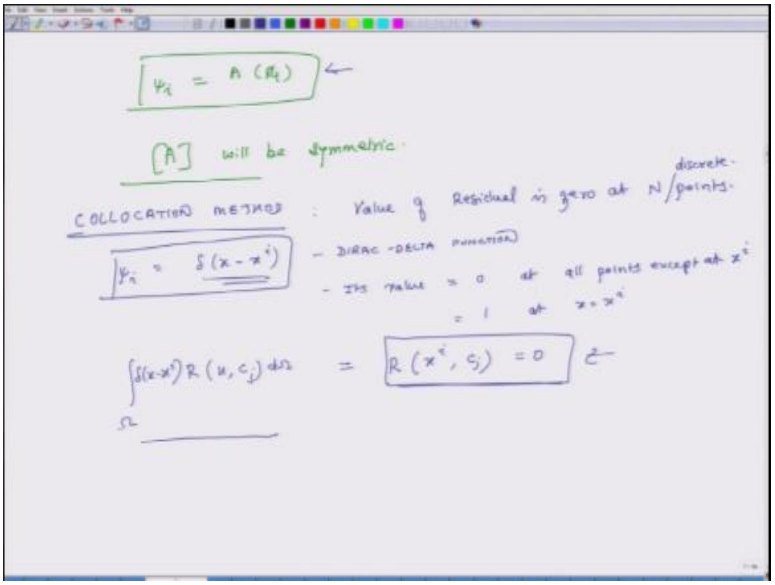

That it comes down to a(c_i), in case of Galerkin method ψ_i was same as $Ø_i$, and in case of Petrov Galerkin method ψ_i and $Ø_i$ they were different, they were required to be different. So that concludes our lecture for the today, we will continue our, a new talk, we will start the new topic starting tomorrow, thank you.

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