Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title

Basics of Finite Element Analysis

Lecture – 02

Philosophy of FEA, Nodes, Elements & Shape Functions

by

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Hello again, welcome to basics of finite element analysis part 1, this is the first week and today is the second lecture of this course. In the last lecture we had covered a very broad-based overview of the course and today we will further elaborate on the concept of finite element analysis by a very simple like, you know numerical experiment or an example. So what we are going to do today is, calculate the area under a curve.

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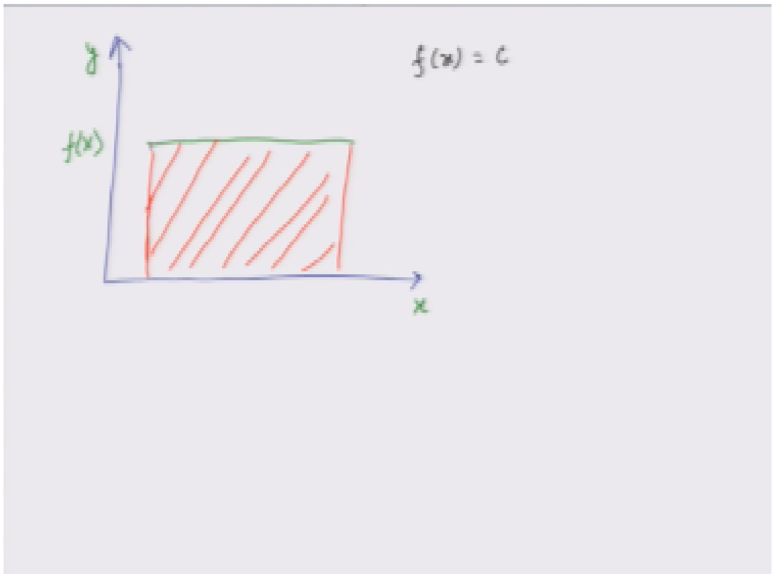

And somehow apply what we learn through this example to finite element analysis method. So suppose I have a function, so this is my x-axis that is my y-axis and this is my y equals some function of x, ok. And in this case f(x) is equal to constant some C. So the value of this f(x) is C as X is varying. And what I am interested in is to compute this area under the function, okay. So because f(x) is equal to C we know that if I integrate f(x) with respect to DX.

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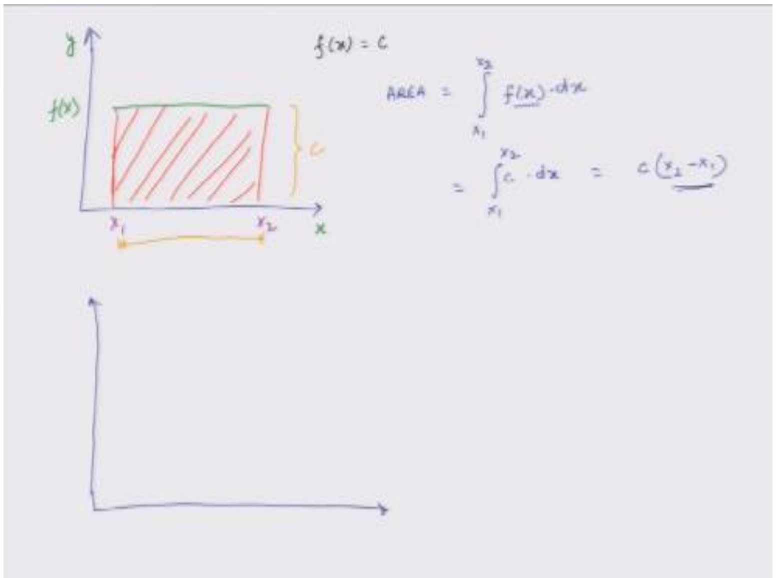

And if I evaluate the values of that integral between the limits x_1 and x_2 I will be able to compute this area. So that is what we will do we will do so what it comes down to is area. So this area is equal to integral from x_1 to x_2 f(x) dx and that because f(x) is C, so it is c times dx, x_1 to x_2 and this is equal to C times x_2-x_1, okay. So this x_2- x_1 is the width of this rectangle.

And C is the height of this rectangle and this is a very trivial example but this is how I wanted to begin with. So what I have explained is that if you have a simple function you can integrate it in a closed form way and you can calculate this area, okay. And your calculation will be accurate and the error will be 0. So this is an approximate this is a exact solution when we are trying to calculate the area.

Now let us look at another example. Oh by the way before I go to the second example, a couple of things. So here the region of our interest in which under which we wanted to calculate area was what x_1 to x_2.

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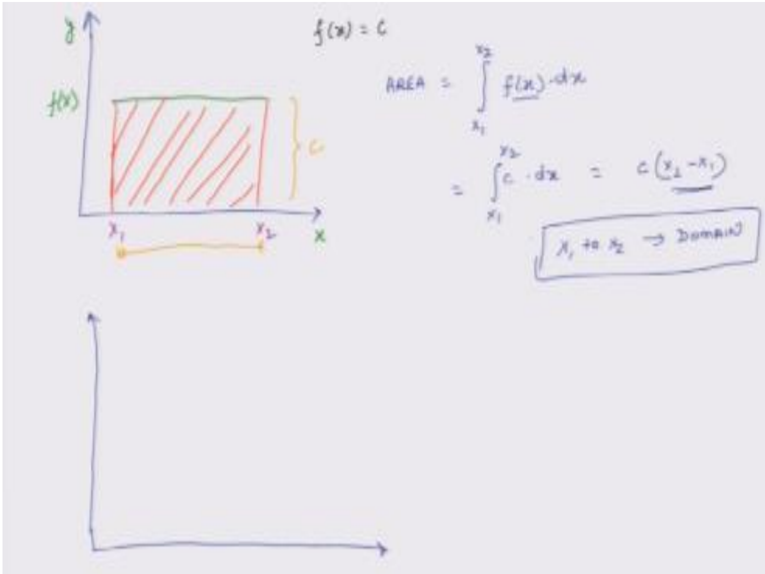

So this x_1 to x_2 is the domain. In this case domain is one dimensional in nature because it involves only variation of X. There could be situations where domain could be two dimensional in nature. Suppose I want to calculate the volume, which is covered by let us say my hand and this baseline surface. So there the domain could be that is a two-dimensional surface. And so, so there the Domain is a 2D thing, here domain is 1D.

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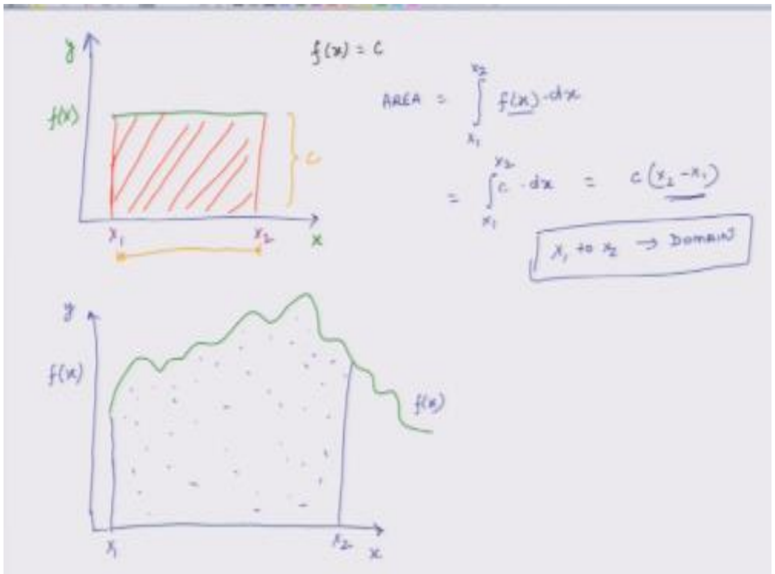

Similarly I could have a three dimensional domain, so that is there. So now moving on I am going back, I have another function f(x). So this is my y-axis, x-axis, but it is very complicated, okay. It is very complicated and I have to find the area, between x_1 and let us say X_2. So this is the area I am interested in. Now if this function was easy to integrate I would have an expression.

So this is my function f(x), if this function was easy to integrate, then I would just straight away integrate it and I will do a integration under limits x_1 to x_2 and I will get a exact solution. But here first thing is I do not know what is the, expression the for this function and the second thing is even if I knew it was it may have been away it may be a very complicated function. So I may not be able to integrate it.

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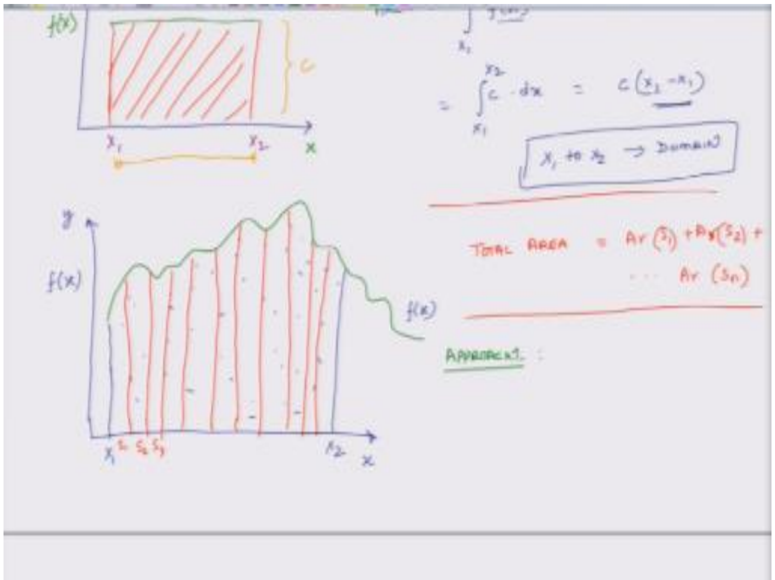

So then I use a numerical approach, okay. So what I do is, I break this area into small, I break it into small slices. And this is exactly what we do when we are trying to integrate differential equations also. It is a more complicated process but the philosophy is same. So I break this overall area which we are trying to calculate into small slices, okay. And let us say this slice is S1, this slice is s2, this slice is s3 and so on and so forth, okay.

So my total area, here also our domain is X_1 to X_2, it is a 1 dimensional domain. Total area will be area of (s_1) plus area of (s_2) plus area of (s_n). So suppose I break it up into n slices, then the total area will be the area of each individual slices. So now if I have a way to figure out how, what is the area of each slice, then I can calculate this. So what is the area of a slice?

Now I can calculate the area of a slice in several ways. So approach one, approach one here what I do is.

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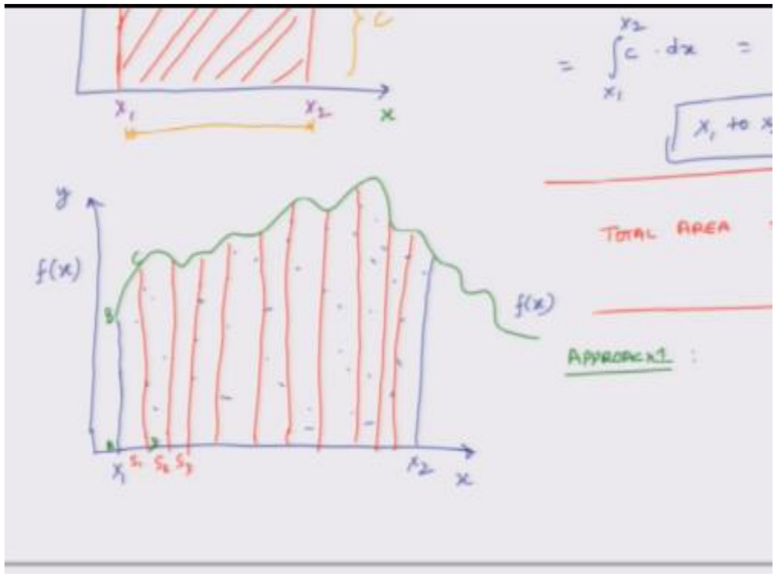

I assume that the height of this, so let us put some a, b, c, d. So what we are trying to figure out is area of ABCD. And our first approach is that the height of this ABCD is constant, okay.

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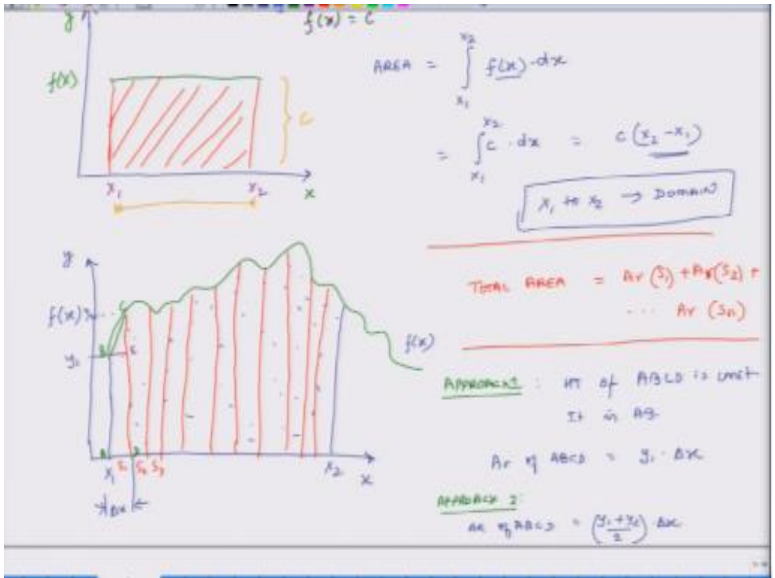

So in that case height of ABCD is constant. Suppose I assume, if that is the case and it is, it is ab. So let us say this value of ab is y1. Then area of ABCD is equal to what? Y1 times X. So let us say this distance is Δ X then Y 1 times ΔX, okay. If the height of this ABCD is constant height means, the distance in the y-direction, okay. Distance in the y-direction is constant then it will be y1 times ΔX.

Is this clear? So basically what I am saying is this, that this complicated shape ABCD is not that but I can approximate it as something like this, right? So effectively what I have calculated is not ABCD but the area a, b, e, d, you understand, okay. And in this case what is the error? The error will be b, e and c area of b, e and c that will be the error, okay.

This is my first approach, then I will say okay maybe I can have a better approach, first two. So second approach is, that I do not treat ABCD as a rectangle but I treat it as a trapezoid. So if I treat it as a trapezoid, then I draw a straight line between B and C, right? And then my in this case suppose the height at point c is y2, then area of ABCD is y_1 plus y_2/ 2 times Δx, okay.

So this is the second approach, I can have a third approach. I will say the third approach could be that I will say, okay. So first approach is that I treat this complicated shape as just a simple rectangle area, error is large and it equals the area of b, e and c. Second approach is I treat it as a trapezoid, in that case the area is the error is this region between the true two green lines, you know.

I do not want to shade it right now because I will draw another curve there. And I can have a third approach and I will say, okay. In the third approach this transition of height between B and C, see right now, in approach to the transition of height between B and C is linear. I will say okay it is not linear but it is some quadratic in nature.

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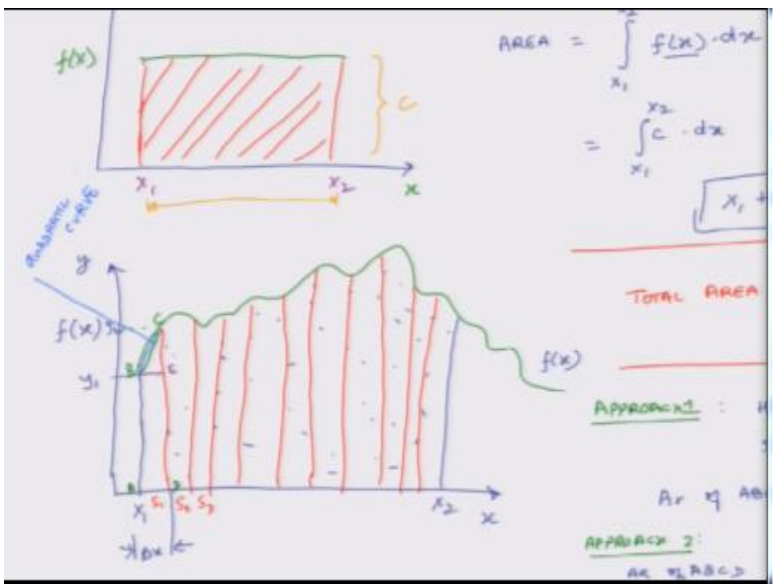

So my line will be like this, okay. And I say that this blue line is a quadratic curve, okay.

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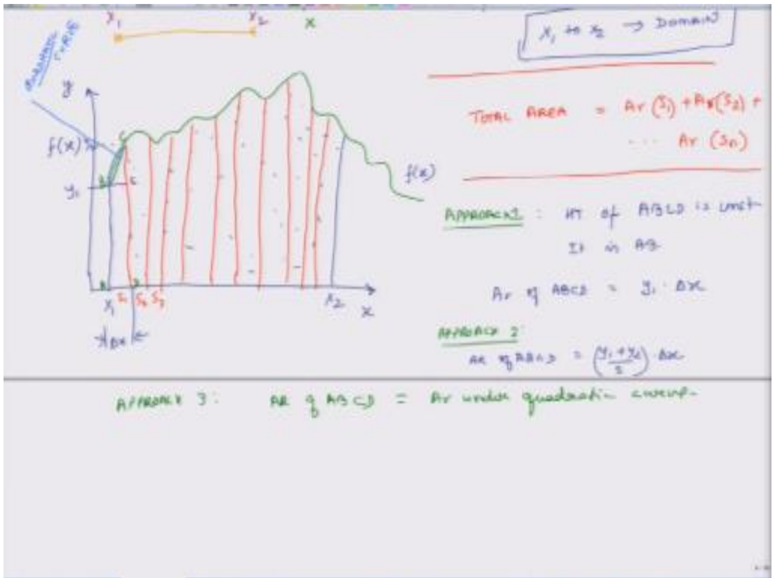

So now if I know that this is a quadratic curve, then I know the equation for the quadratic curve, right? And I can again calculate. So basically if at the moment I say it is a quadratic curve it is not a straight. See in first case it was a horizontal line, in second case it was a straight inclined line, in third case it is a quadratic curve. So in approach three, area of ABCD is area under quadratic curve.

So using principles of geometry I can develop the equation for this quadratic curve easily, right? And then I can interact, integrate that quadratic equation between limits x_1 and $x_1+\Delta x$ and I can get that area.

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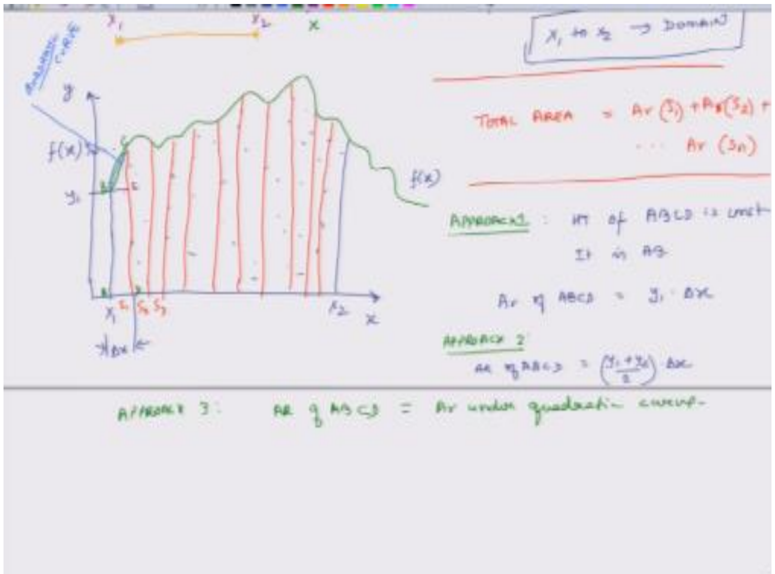

So what happened to my error? It went down further. Then I will say that okay.

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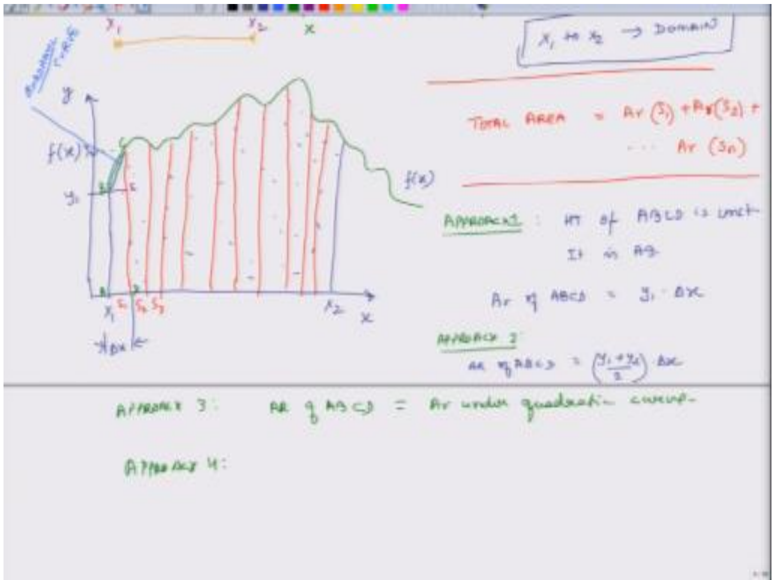

Approach four could be that it is not a quadratic curve; it is a cubic club curve. So I can make this transition between B and C more and more accurate or closer to reality or closer to reality. If I keep on increasing the number of polynomials which capture the transition from B to C more and more accurately, okay.

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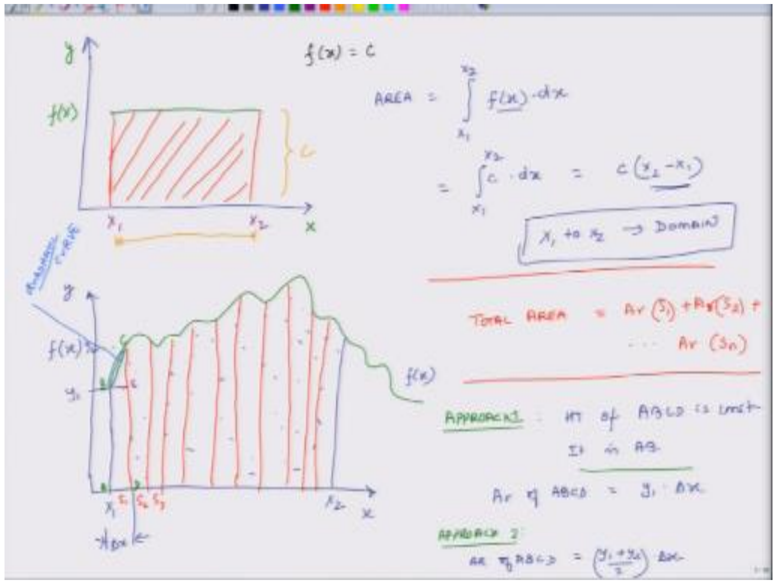

So, so this is what happens in finite element analysis also. So we can take either of those estimates, the first estimate is approach one where the height is constant, second approach is where height is linearly increasing, third is it is quadratically increasing, fourth could be it is cubically increasing. And in each step of sophistication, I am getting more and more accurate what, solution, more and more accurate solution for the area under area for this rectangle ABCD.

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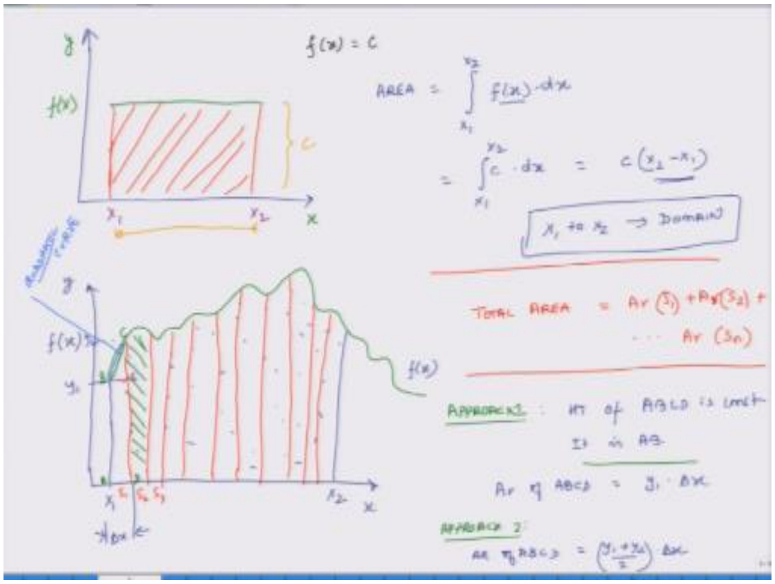

And similarly, I can get I can calculate areas for the second slice. Again I can have different approaches. Approach ABCD right?

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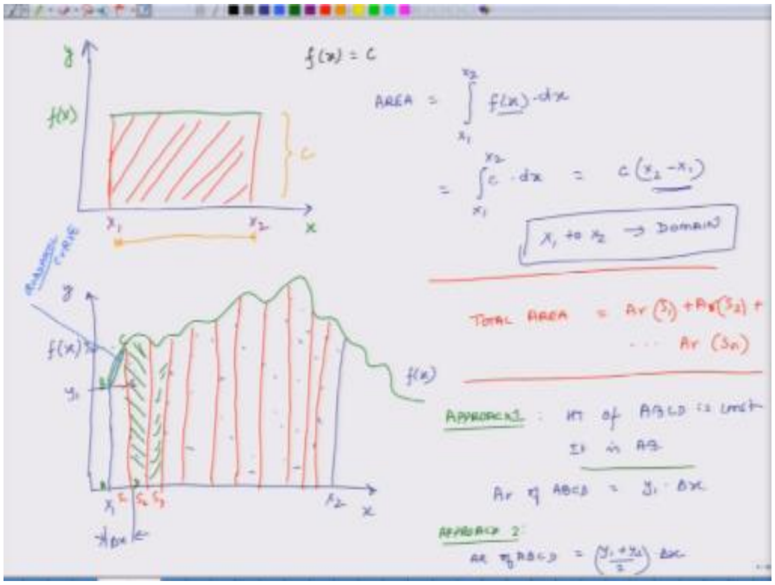

Then I can have for the third slice. So whatever approach I think is most accurate, I can pick up that approach and I can then compute area of each of these slices, add up all the areas of these slices and get the final area under the curve, okay.

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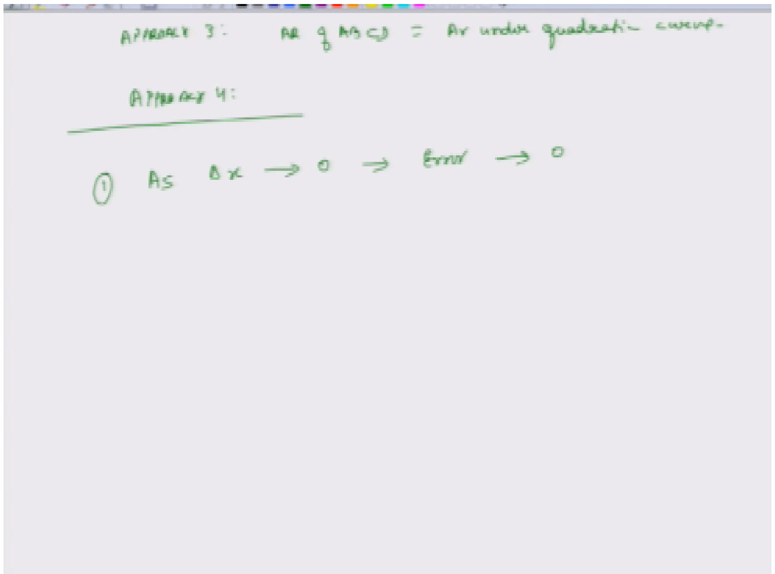

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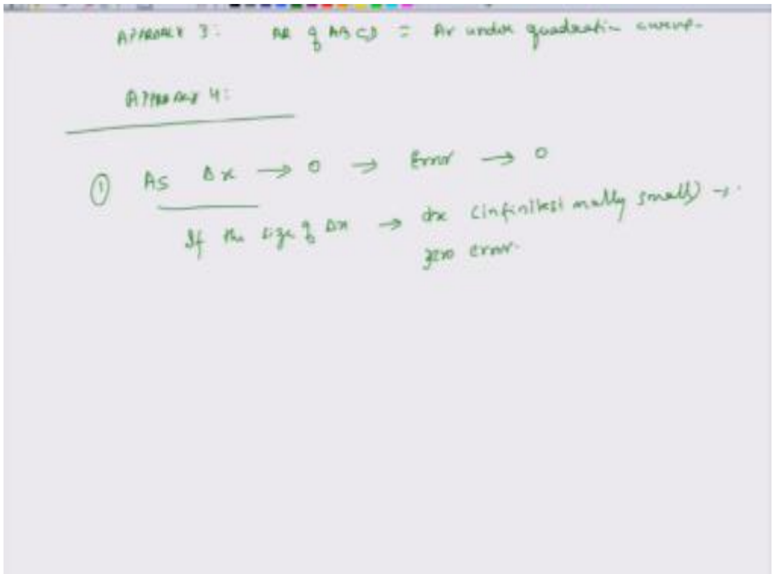

So in this context I would like to make a couple of points. First, as ∆X goes to 0, error goes to 0 also. It does not matter whether it is a constant curve or it is a incline curve or it is a, you know this is one thing. So, so this is one which means that if the size of ∆ X is dx, which is infinitesimally is small, infinitesimally small, then I will have zero error, agree?

This is one thing but what does that mean? If I have infinitesimally small size of the slice, it will mean that I will have to do these computations infinite number of times, okay. So but in finite element method the size of the slice and now we will use this term called element.

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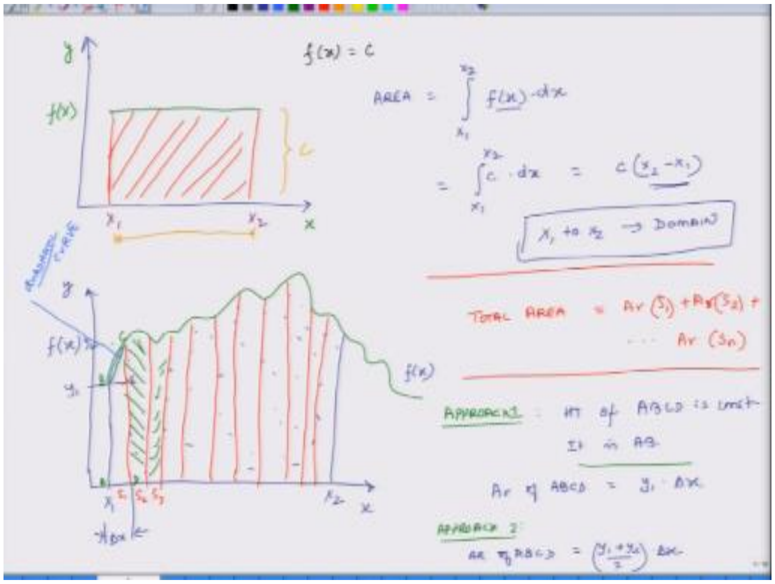

So here what we were doing is, that we have broken this line which is for f(x) into slices I can call it or I can call them as elements, okay I can call them as elements. So if the size of the element goes to 0, I have to do that computation infinite number of times, which is physically not possible.

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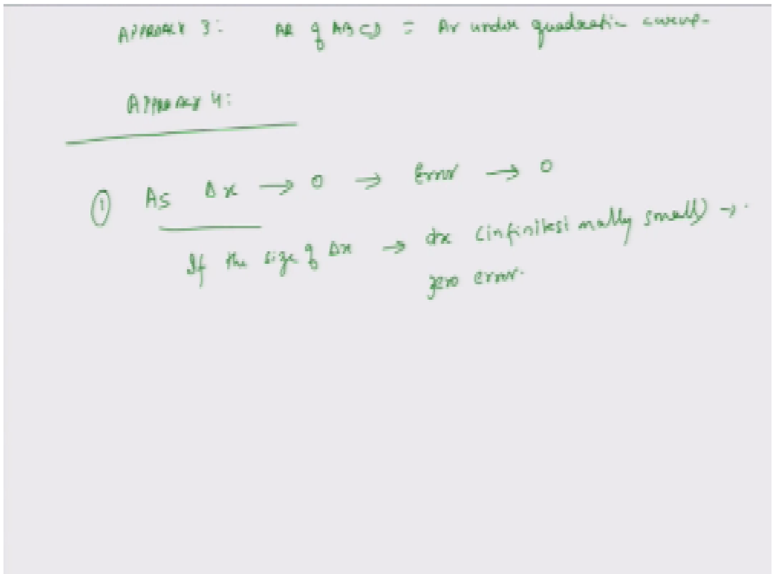

So what I do is that I reduce the size to a significantly small number but it is still not infinite, infinitesimally small. So I still have not infinite number of elements but finite number of elements, okay. That is where the term finite element comes into picture, that the size is not extremely small and vanishingly small but it is finite in size and also the number of elements is finite.

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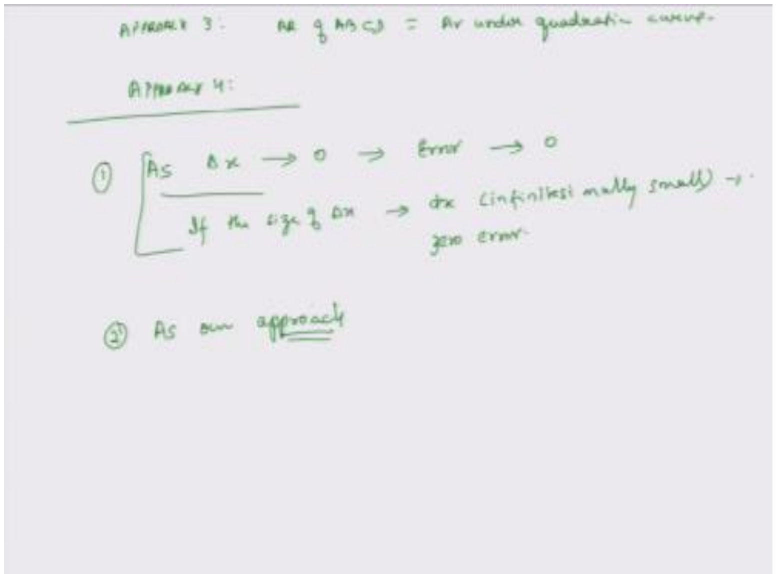

So this is 1, so as ΔX approaches 0 my error goes to 0, this is one thing. The second one is, as our approach. See we in first approach we assumed that height was constant, in second approach we said height was linearly varying, in third approach we said that height was quadratically varying.

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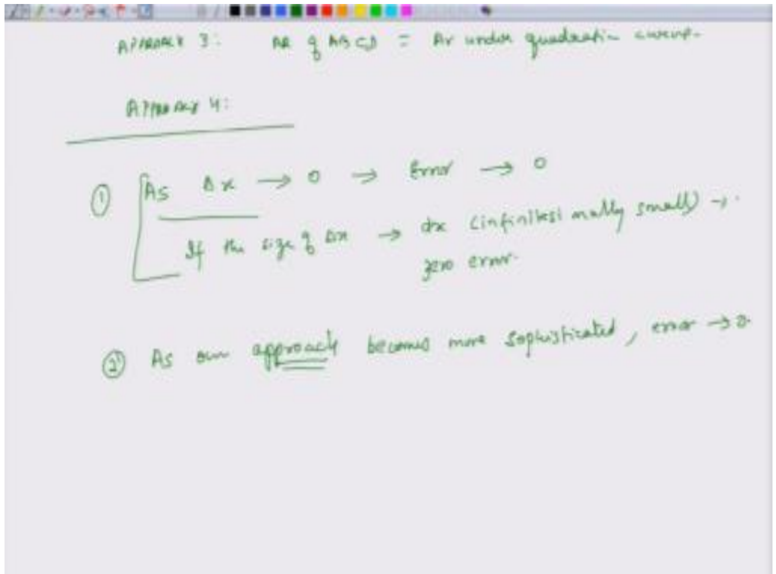

So as our approach becomes more sophisticated and this is a general thing because more sophisticated, error goes to 0, okay.

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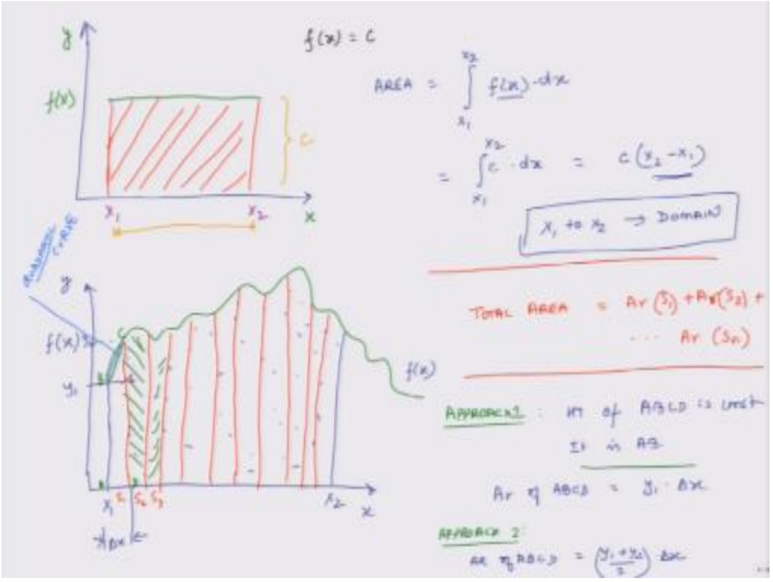

This transition from point B to C, right? In first approach there was no transition. It was constant, in second approach it was linear transition, in third approach it was quadratic transition.

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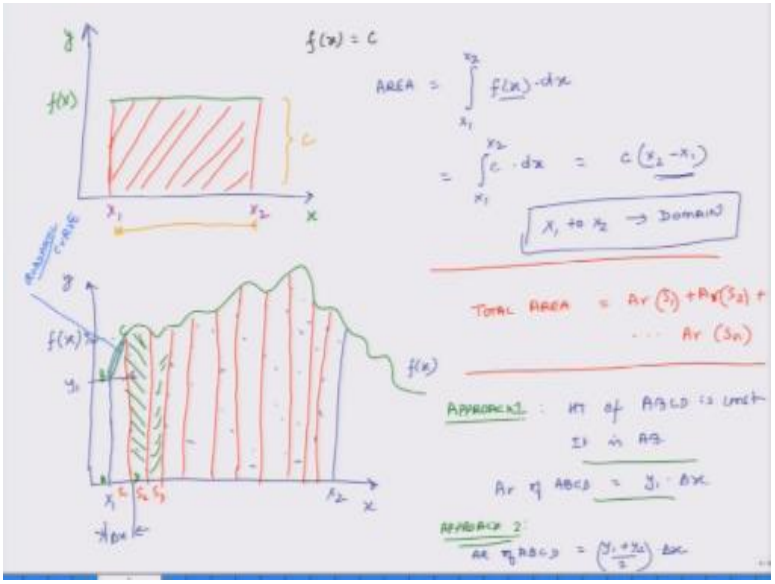

And this function which helps in understanding the transition, so here it is a constant function in first approach, right? Constant form, constant transit, constant function, here it is a linear function in approach to, in third it is a quadratic function. These functions which explain how variables are changing over an element, how variables are changing over an element, not across elements over an element, they are known as shape functions, okay.

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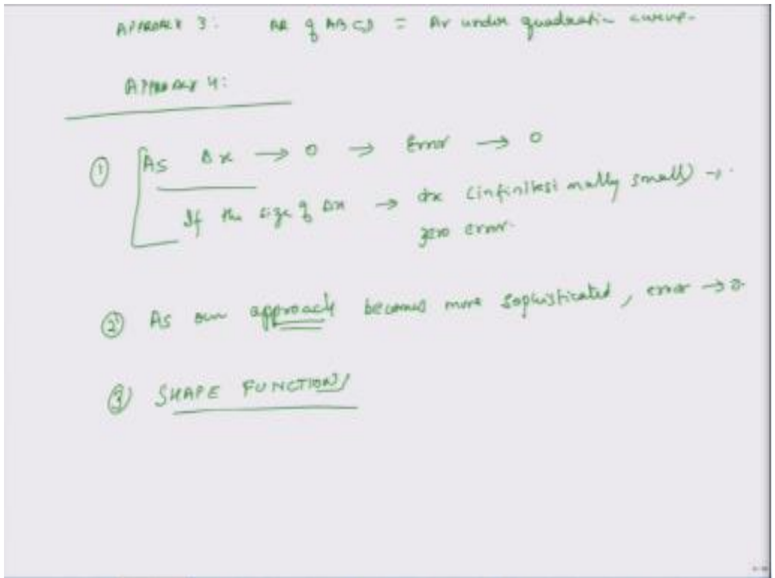

Shape functions. Because they help us understand the shape of the transitions from point A to point B on an element.

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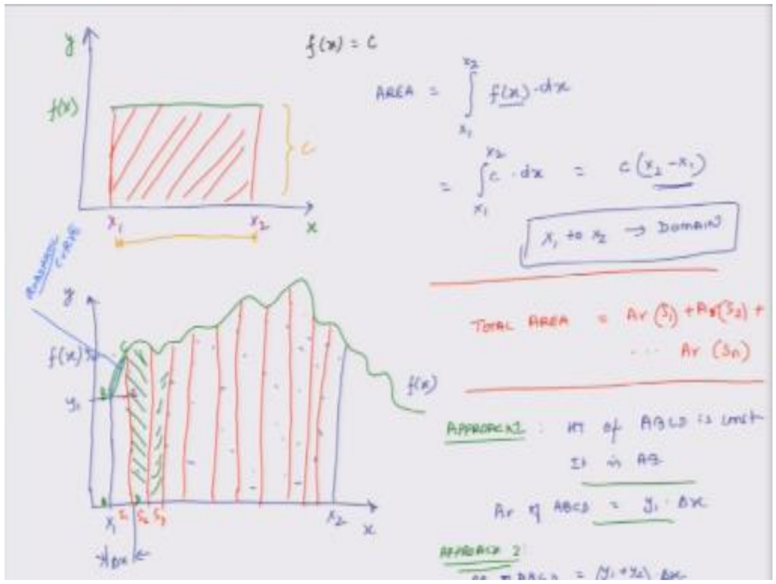

Now these functions which are plotted in this line are one-dimensional because they are only functions of X. If you have a plate or a shell it could be a two dimensional function. If you have a solid object it could be a three dimensional function, okay. So these functions could be 1D, 2D or 3D. But they help us understand that within an element how is u, suppose you are trying to capture u, how is u changing from one point to other? Is it linear transition, is it a quadratic transition, is it a constant, is it cubic, okay. So that is there.

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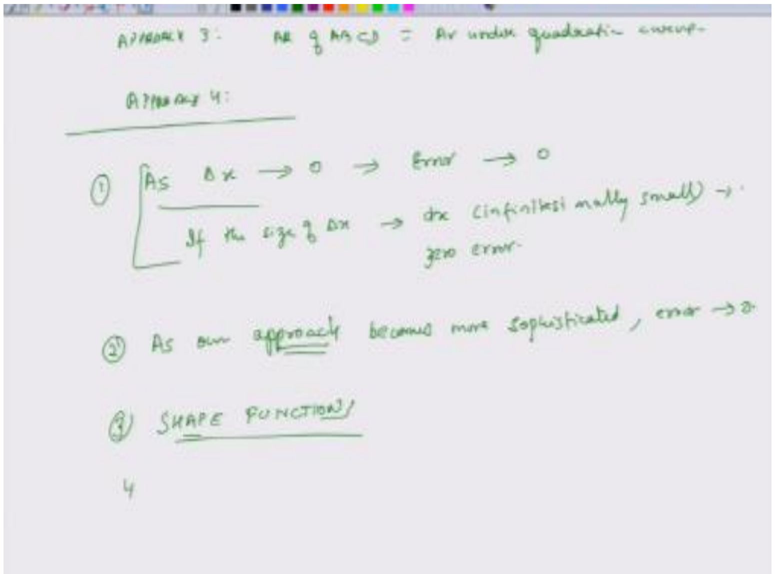

So these are known as shape functions and fourth is.

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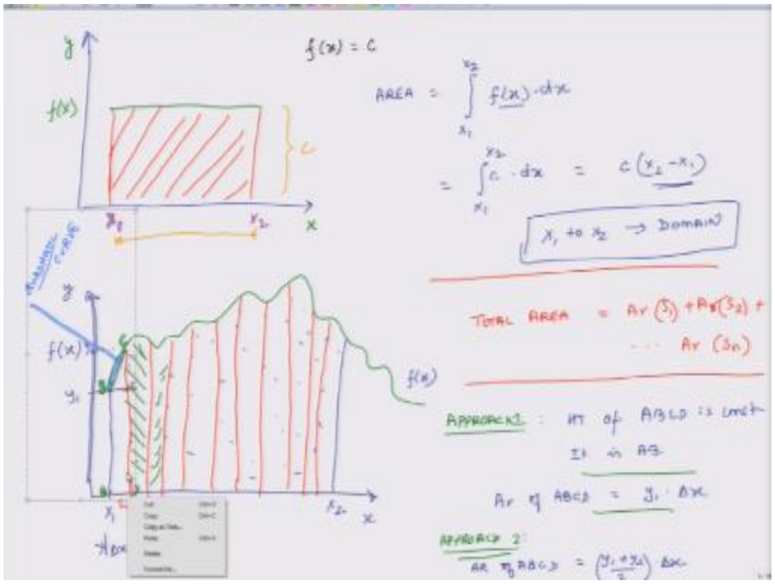

That these small slices which we called slices initially.

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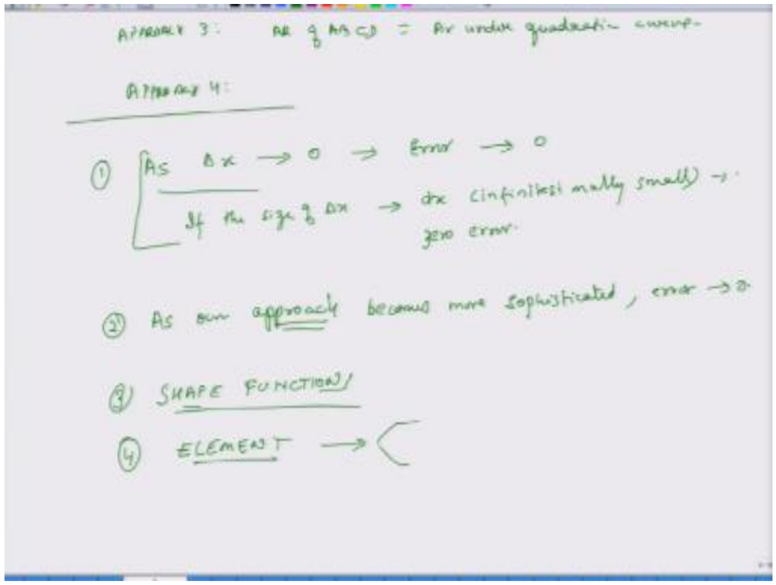

This is what is element; this is what is an element. These elements have at least in case of a linear you know when the element it has two end points, right? It has two end points but they could have internal points also, they could have internal points also.

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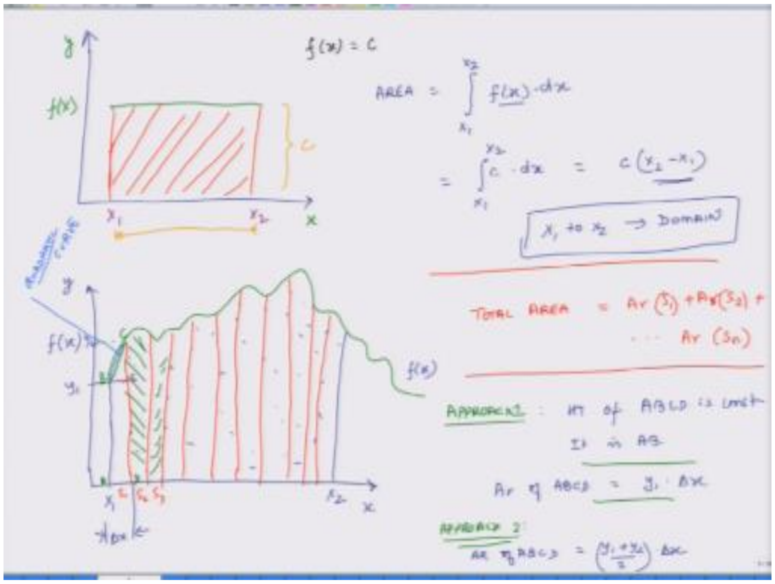

Suppose so, so suppose you have a quadratic function then to see, to see how the function is changing over an element you have to develop a quadratic expression, right?

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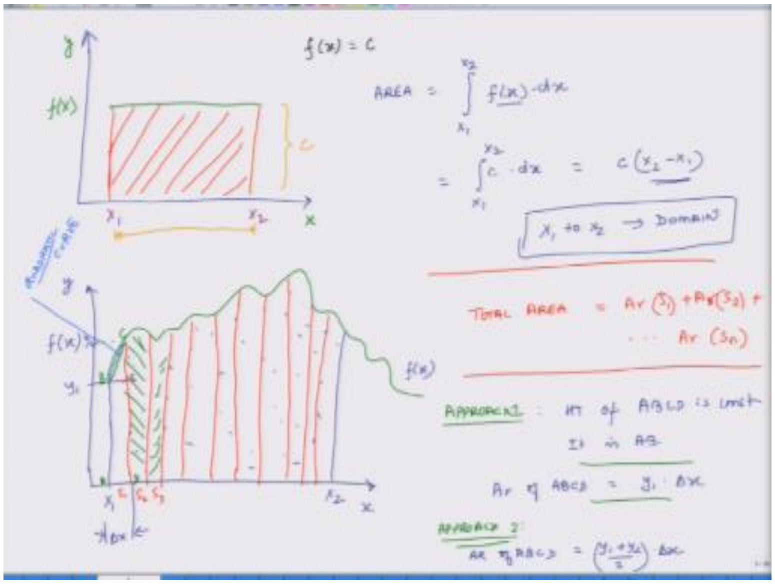

To define a line you need two points, to define a straight line you need two points. So this linear function, for this you need the values of this green line f(x).

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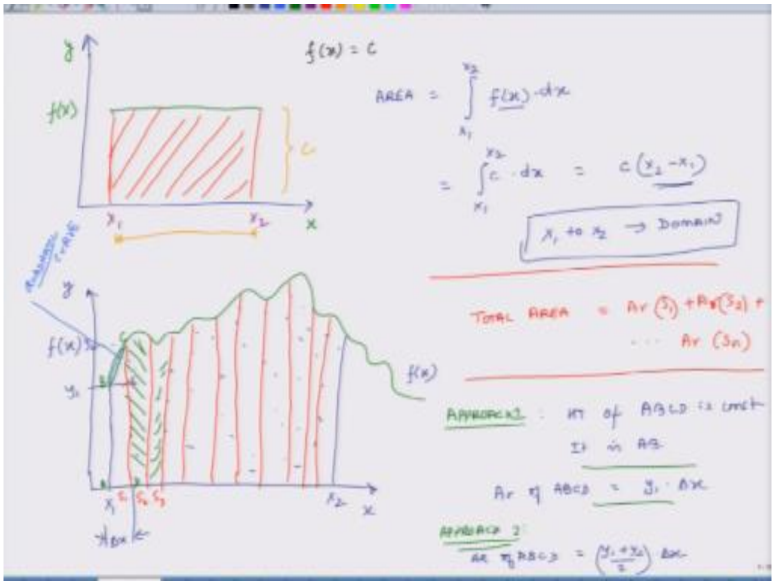

Only at point B and C, right? But to define a quadratic function which transits from b to c, you need three values, you need three values. So for a quadratic function which is one dimensional in nature you have to get value at one end point, you have to get value at other end point, and you also have to get a value at somewhere in between.

Because then because what is a quadratic function? It is $ax^2 + b x + c$. So to find a, b and c you need values of that function at three points. Similarly for a line, linear line it is $y = ax + b$, to calculate values of a and b you need values at the end points.

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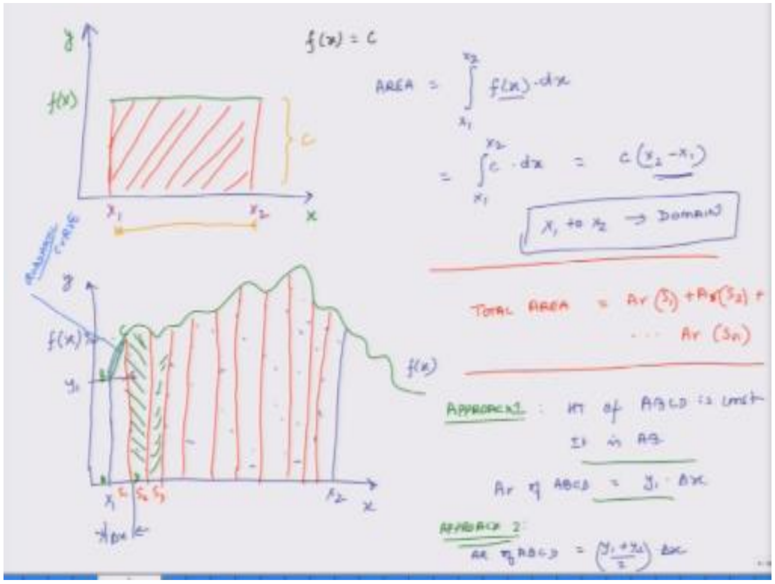

But for quadratic you not only need the value of the function at end point but also the value at maybe midpoint or somewhere in between, it is somewhere in between, okay.

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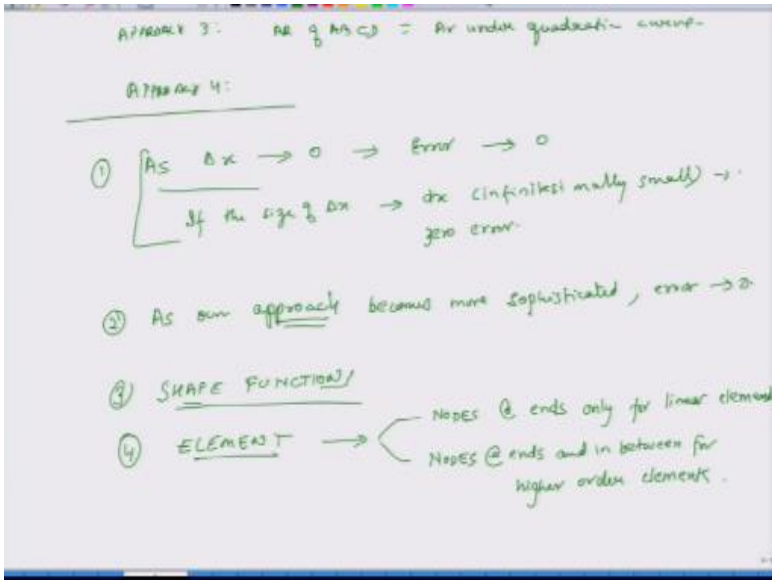

So, so elements have nodes where we have to, where we are interested in finding the values at ends only for linear elements, and they also have nodes at ends and in between for higher order elements, okay. Is this clear? Now how do we all, we use all of this study, we will explain it in the next lecture. But what I wanted to impress upon you is.

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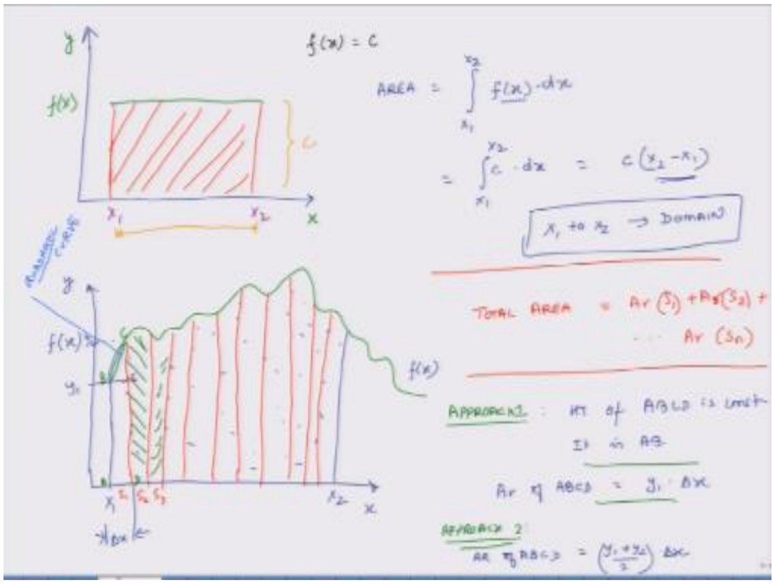

That the accuracy of our, when we are trying to calculate the area it, we have seen in this case it depends on two parameters. The first parameter is how thin each slice is or how small each element is, the more small we make the better we get in terms of reducing the error. The second way of reducing the error is that not only we make the slice small, but we also pick up a better transition between two endpoints of the unknown variable, okay.

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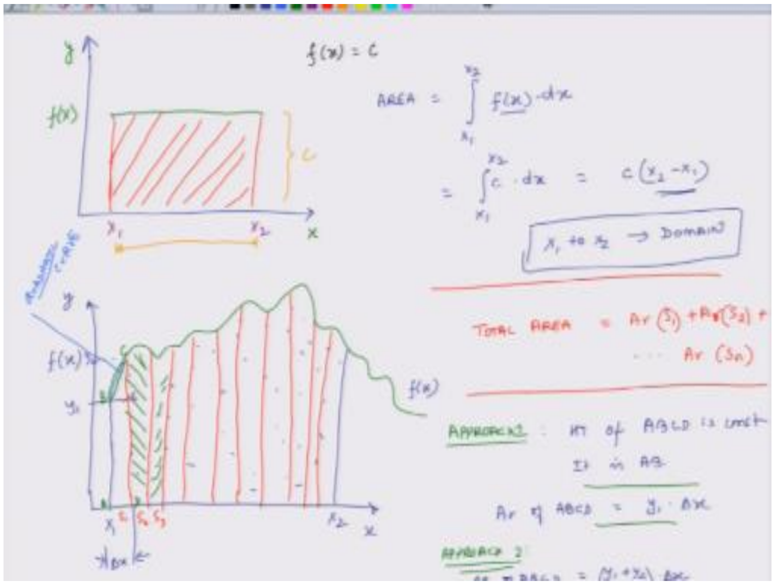

So, so a better, a higher order shape function, another term for this shape function is interpolation function.

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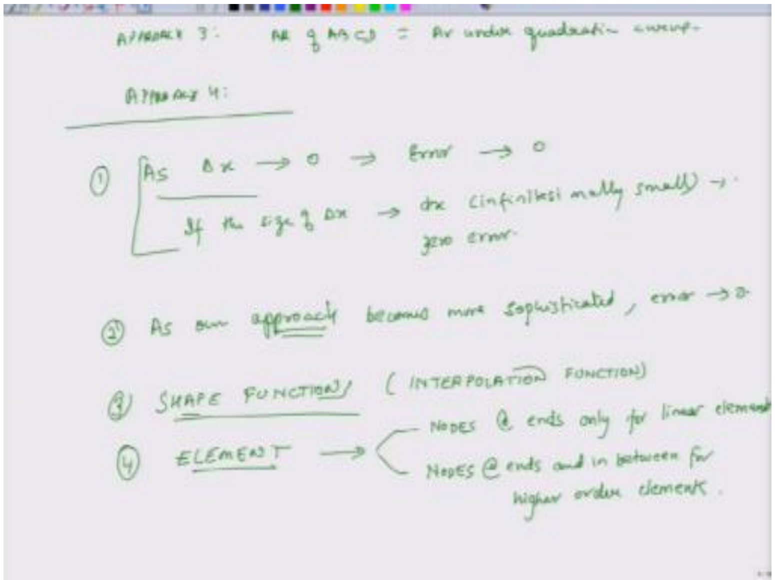

So in this class I will use shape function and interpolation function interchangeably. They are known as interpolation function because they help us interpolate the value of unknowns between two nodes, okay. If I know the function, using that function I can interpolate the value of the unknown between two nodes using interpolation methods.

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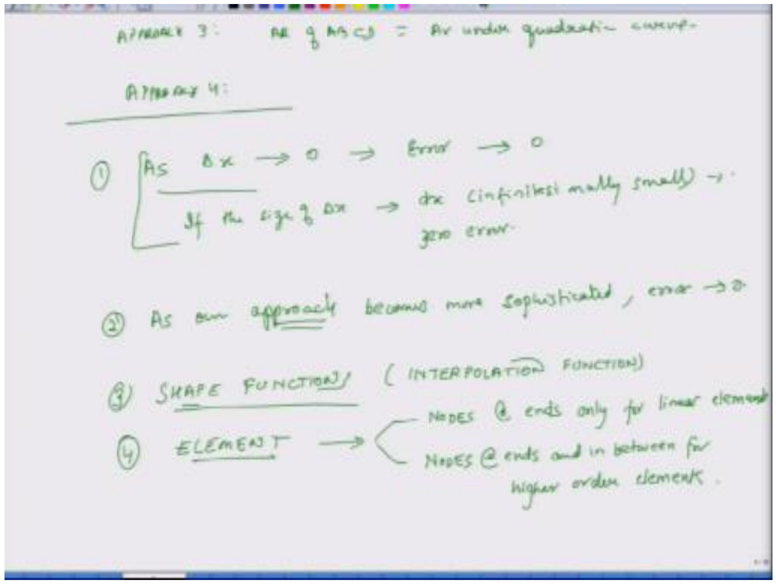

So they are known as shape functions or interpolation functions, some people also call them as basis functions, basis functions. So these are, so if we use higher order shape functions that also helps us increase the accuracy, if you reduce the size of the element we have seen that also helps us reduce the error and increase the accuracy. These are two ways to increase accuracy of at least in context of this example the area under a curve.

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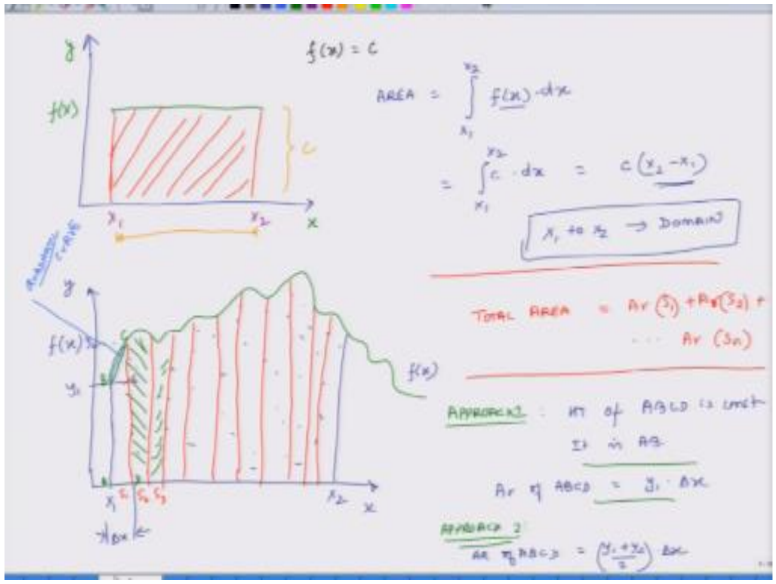

We will extend the same thing in several other, we will continue you know subsequent lectures to; we will use the same idea to when we are trying to solve differential equations, thank you.

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