Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture – 11
Gradient and Divergence Theorems
Part-II

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Hello, again welcome to basics of finite element analysis. In last class we had introduced two different theorems, the gradient theorem and the divergence theorem. And today what we will do is, we will explore these theorems further, and see how we can apply them in context of weighted residuals. And how can -- we can use theorems to shift differentiability operator from one function to the other function.

And this technique becomes very useful in context of problems when they are two-dimensional or three-dimensional in nature.

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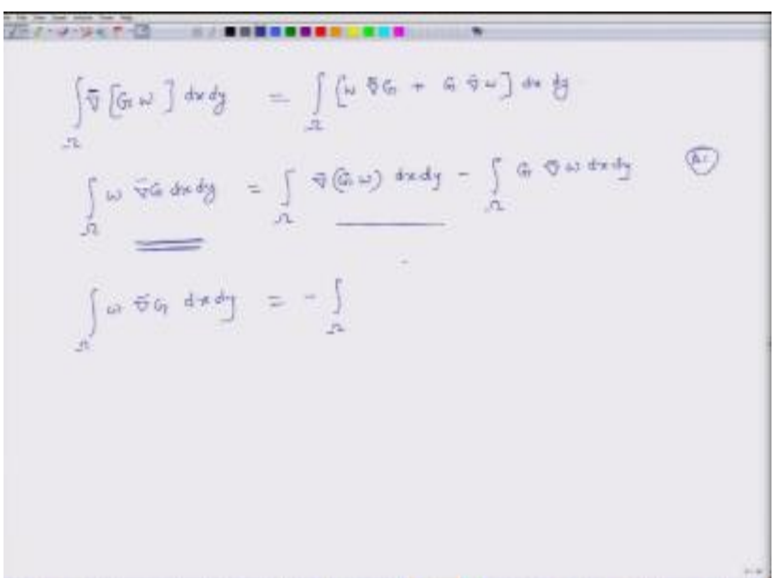

So let us consider a function G times w, so G is a scalar and w could be a weighting function okay. And then we take its gradient and we integrate it over the whole surface. So I can express it as integral and I will first -- I apply operator on G and then the second case I will operate it on w. So it is w times gradient, so this is a vector of g + g times gradient of w, dx, dy.

So I can write this as w times gradient of dx, dy integral equals integral of gradient of G times w minus integral of G gradient of w, dx, dy Ω. So now we continue this thing and we know that the left side can be written as integral of Ω, so this is equation a1, so it is actually not Ω it is w times gradient of G times dx, dy is equal to using the gradient theorem.

So this we keep it as is, and here we apply the gradient theorem. So now we have developed this equation.

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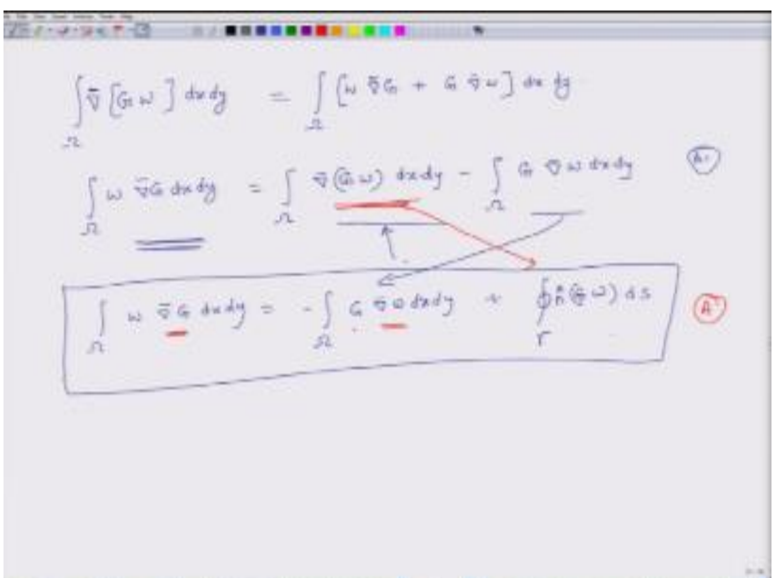

And then the next thing we do is that we apply the gradient theorem on this particular term. So what we get is left side does not change, w times gradient of G times dx, dy equals, and here I am going to apply the gradient theorem, but before that I will write this term here, so I am also reorganizing my terms g times grad of w dx, dy and for this term I will use the gradient theorem.

So this is our domain integral and I can express it in terms of boundary integral using the gradient theorem. So that term comes to plus boundary integral of G Ω not Ω, I am sorry w. And there should be a unit normal here times ds okay. So this is one equation okay, so what do you see in this equation that here the gradient was applied on the scalar G and that has gotten shifted to w variable.

So if w is the weighting function then it gets shifted to w, and then of course we have a boundary integral. So this is one important statement and we use this to transfer differentiability. So in one dimension we use integration by parts so this is somewhat of a similar process here also.

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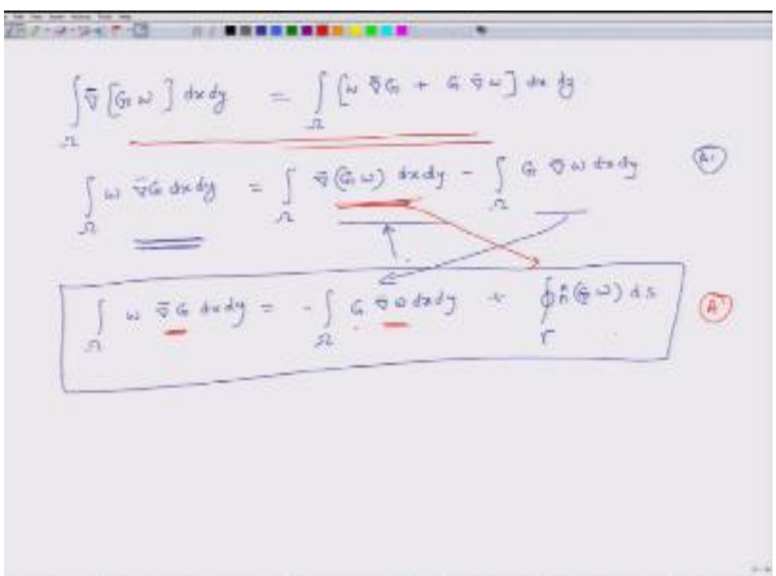

Because we start from here and then we shift differentiability operator are not necessarily differentiate with differentiability, but the gradient operator from one function to the other function.

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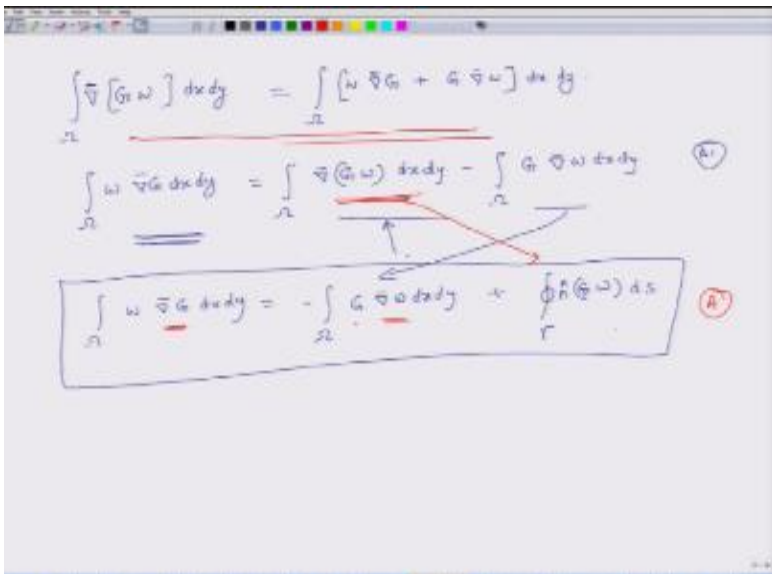

So this was one equation and now we will develop another equation.

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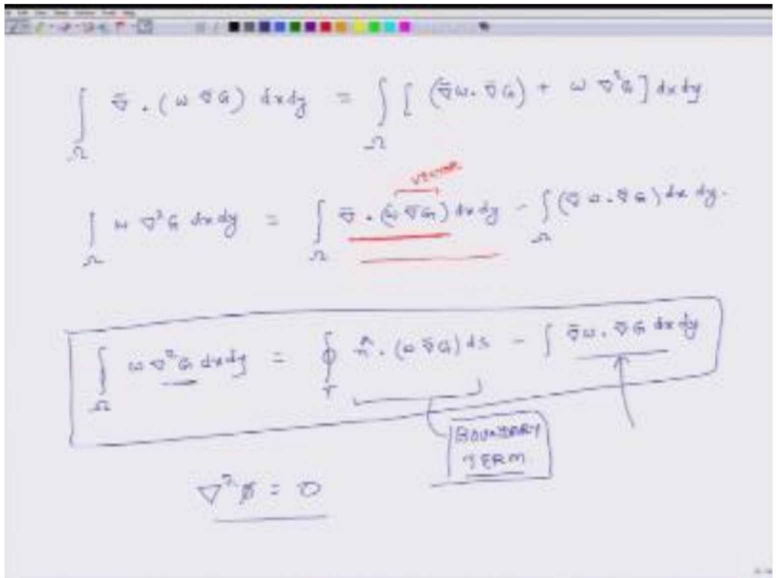

So consider this, so suppose there is some function w times gradient of G, and I take a divergence of this entire function. So it is a dot product, so this is equal to -- first I operate it on w, and then inner product with G. So this is 1, so gradient of w and gradient of G, the inner product of these two plus w times grad square of G dx, dy. So I am going to rearrange this w times grad square of g dx, dy Ω equals this is there, so the G is dy minus inner product of gradients of w and g dx, dy.

Now what I will do is, I will apply the divergence theorem on this guy. So this is the vector, because w times gradient of G is a vector. And when I operate this with the grad vector and take its inner product this is basically the divergence of that vector right, and it is the integral of the divergence of that integral of that vector. So that is nothing, but the boundary integral of this thing ds minus -- and the right side left side does not change.

So this is the second equation okay, so here also this is the boundary term, so several times you may have seen that there is a differential equation something like this grad square FE equals some constant r0, some physical phenomena is defined like this in heat transport equation, diffusion equation, several equations. And if you multiply that by the residue w, and then you do all this process then this is the right side you have the gradient operator being applied on w and on G.

And again you have a balanced system the differentiability requirement again become balanced. So here its second order differentiability requirement, here its first order differentiability requirement. And if its fourth order equation then you can get iterate it and you can get similar equations. And then of course each time you do this you get a boundary term.

So this is the analog of what we learnt two lectures earlier in context of 1d systems. Whereby using the integration by parts we were able to shift differentiability operator from one function to the other function.

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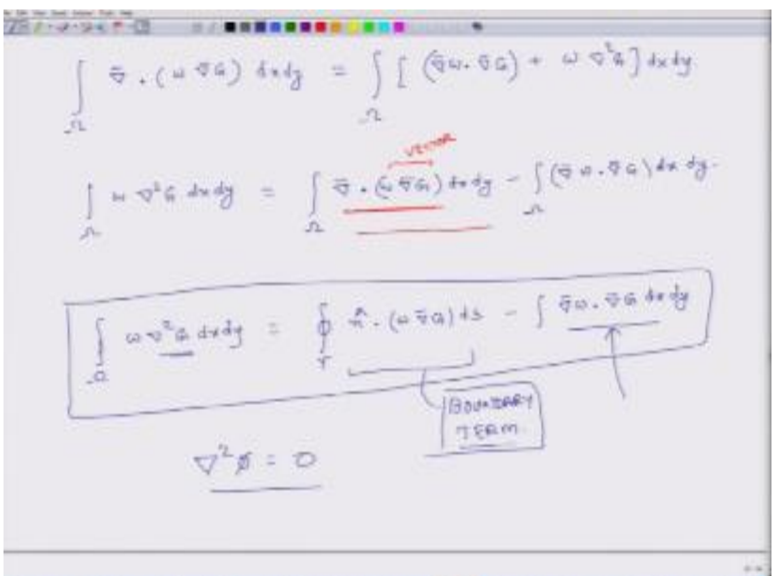

Here we are doing somewhat same thing using divergence and gradient theorems. So this gives you a mathematical foundation how you can shift the differentiability operator from one function to the other both in one-dimensional domains, and also in for two dimensional systems. And you can use similar mathematics for three dimensional systems as well. So this concludes our lecture for today we will again meet tomorrow and that will be the last lecture of this week, thank you bye.

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