Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)
Course Title
Basics of Finite Element Analysis

Lecture-01
Introduction to Finite Element Analysis (FEA)

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Hello, welcome to basics of FEA part I, this is a eight-week course, each course we will have six modules. One module on each day, Monday, Tuesday, Wednesday, Thursday and Friday. So overall we will have something like 48 lectures, each lecture will be something like 20 to 25 minutes. And then at the end of each lecture we will also have an assignment which you will be expected to do, most of these assignments will have multiple choice questions.

So please do those questions, even though they are multiple choice, to answer those questions you will have to have some good understanding of the theory of finite element analysis. So it is important that you listen to these lectures carefully and work accordingly. The book which I will be heavily relying on for this particular course is introduction to the finite element method, and the author of this book is J. N Reddy, it is a very popular book.

And the strength of this book is that it is extremely easy to understand, finite element analysis or finite element method is a technique to solve differential equations. And it is mathematically very intensive. So a lot of books in area finite element method you will see that they are extremely heavy on mathematics, and in that process sometimes the understanding part of the book through its language and through explanations is not that clear.

So the advantage of the book by J. N. Reddy is that it is easy to understand and yet it does not compromise on the mathematics of the course. So that is the book I will be referring to and then of course we will have examinations, mid semester examinations, and also in some exams. And that is how we are going to do. In this particular, so this is part 1 of the course these eight weeks, and then maybe in July onwards we will have a new set of mooks.

So then in that particular course I will do the second part of finite element analysis. What you will learn in this first part will be basically the theory of finite element method as applied to one-dimensional problems. And so, you will understand how complex geometries are broken, how they are discretized, how are assembly element level equations written, how are assembly is done, a lot of stuff about terminology, how do you apply boundary conditions, what are initial value conditions, what are boundary value conditions, what is -- what are the different mathematical approaches for this.

And I think if you have a good understanding of 1d type of problems then going from there to 2d of type of problems is relatively easy, and that is not a big jump. So most of the conceptual, you know understanding in context of finite element analysis it starts right away from 1d problems. So we do not have to think that 2d problems are 1d problems are simple, 2d problems are complex, and 3d problems are super-complex.

If you get a good understanding of 1d then going from 1d to 2d, the only additional step is how do you assemble elements into two dimensional systems. And then from two dimensions to three dimensions is actually that transition is fairly straightforward, because there is nothing new when you go to three dimensions. So that is -- so even if you are planning to do the part 2 of the course which will be after June or July, if the plan goes correctly then having a good grip on that particular course will very strongly depend on how we treat the subject at this level.

So that is the absolute, so once again I welcome all of you to this particular course, and I hope that you enjoy this course, and learn something worthwhile and useful, because finite element analysis is now very extensively used in almost all areas of science, and technology, and engineering. So you can be a mechanical engineer, or a civil engineer, or an aerospace engineer, you can be a material scientist, you can be a physicist, you can be a chemical engineer, and virtually all areas of engineering and science in some form or other FEA is used, because what FEA allows you to do is to solve complicated differential equations.

You know that suppose you have a function, you can have any function and differentiating any function is a pretty straightforward process, but integrating any function, any general function is not that straightforward, because the way we have defined integration or integral of a function is we actually consider it as an inverse process of a derivative.

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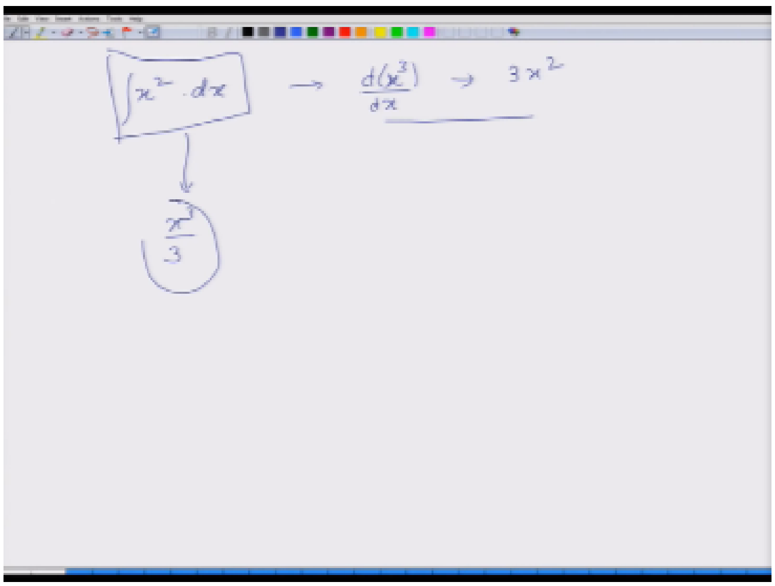

So suppose you have x^2 and I want to integrate it with respect to dx, then the way I do it is that I know that derivative of dx, derivative of x^3 is $3x^2$. So then what I do is, I know this information, so I know that the derivative of cubic of x is $3x^2$, so I say that okay, if that is the case then entire derivative of $3x^2$ is x^3. And from here I infer that integral of this will be x3 over 3 okay. So what this basic concept means is, that if I have to integrate a function I have to basically – I am trying to find its anti derivative which means that I have to figure out a function whose derivative is this thing.

And finding out that function is not a straightforward affair, but if I want to differentiate any function that is pretty straightforward. So the point what I am trying to make is, that when we have differential equations which actually capture several physical phenomena first instance, if there is a heat transfer happening across the body, then the way you capture the physics of that bodies through a differential equation.

You have a body, suppose this is a body and I am putting some force on it, and I want to calculate the stresses and strains in this body, then again I have to write equations of equilibrium in differential form. So again I have a partial set of partial differential equations which given that phenomena. And each time I have partial differential equations or ordinary differential equations I have to solve for unknown variables for instance, if I am trying to bend this pen the unknowns maybe displacements in the pen at different points or in space.

And so those are the unknowns and the way I solve for those unknown is I have to integrate the partial differential equation which governs the bending phenomena of a pen. And as I explained in general integration is a much more difficult and complex, you know a non straightforward process than differentiation. So finding integrals of partial differential equations or ordinary differential equations, in some cases it is easy and straightforward, in most of the cases it is not easy and not straightforward.

So finding exact integrals of complex functions is not straightforward. So then what we do is that we try to solve these equations not in an exact sense, but in an approximate sense okay. So we will say okay, we are okay if my solution is not hundred percent accurate, but maybe ninety nine percent accurate. So we use some approximate methods to solve our to integrate these equations and that is their techniques like finite element method come into picture okay.

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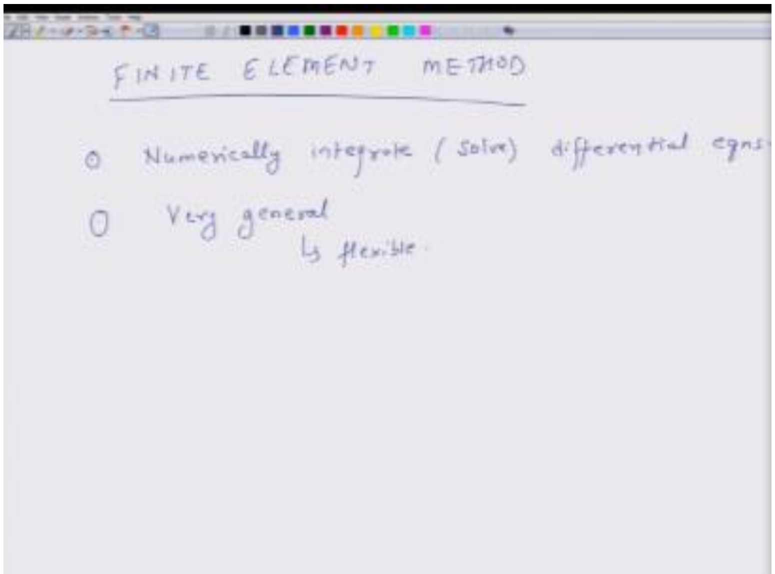

So I will give you some couple of comments on finite element method. So the first thing is that this method helps us numerically integrate, and we will explain this term numerically as we walk through this course integrate, or I can also call it solve, differential equations. Now these equations could be PD's partial differential equations or they could be ordinary differential equation it does not matter, these equations could also be linear or non-linear that again does not matter.

A very big advantage of finite element analysis that it is very general, and because it is general it is also very flexible, what do I mean by that. So if you have a particular ordinary differential equation or a PD you can integrate it under some special conditions there are several ways right, you have this variable separable method and so on and so forth. So there are several methods, but each method of integrating a partial differential equation is applicable to only a limited set of conditions okay.

It is only applicable to a limited set of conditions, but when we start using techniques like finite element method, this method is more or less insensitive to the type or farm of the partial differential equation, because the philosophy it uses and you will see it as we walk through in detail. The philosophy on which it rests on is very generic so it does not matter whether the solution is of a equation is of this type or some other type whether it is homogeneous or non-homogeneous.

If we understand the overall theory of finite element method reasonably well then we should be able to solve or integrate these equations which define or capture the essence of several physical processes fairly verify the amount of accuracy and these [indiscernible][00:11:59].

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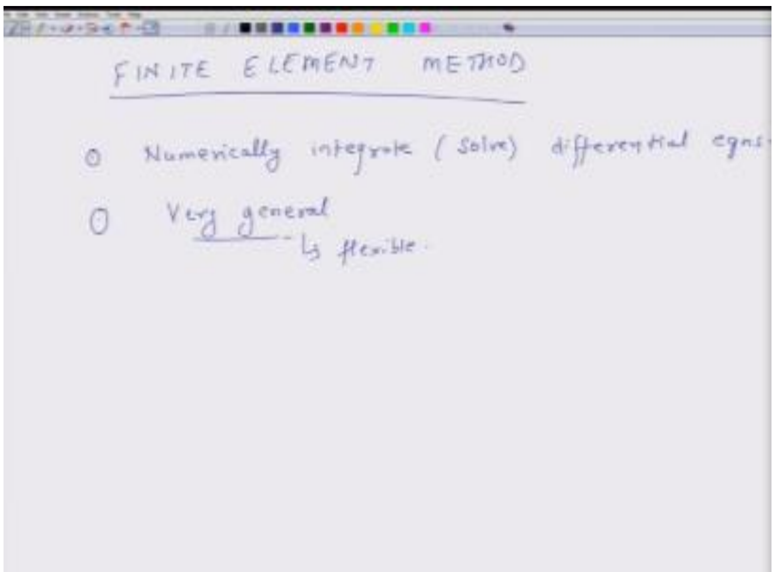

So this is very general, and because it is general I can use the same method to solve a heat conduction problem I can use the same method to solve a thermal diffusion problem, I can use the same method a solve problem in computational fluid dynamics or do stress analysis, or do dynamics, or understand vibrations, or understand the physics of the universe and so on and so forth.

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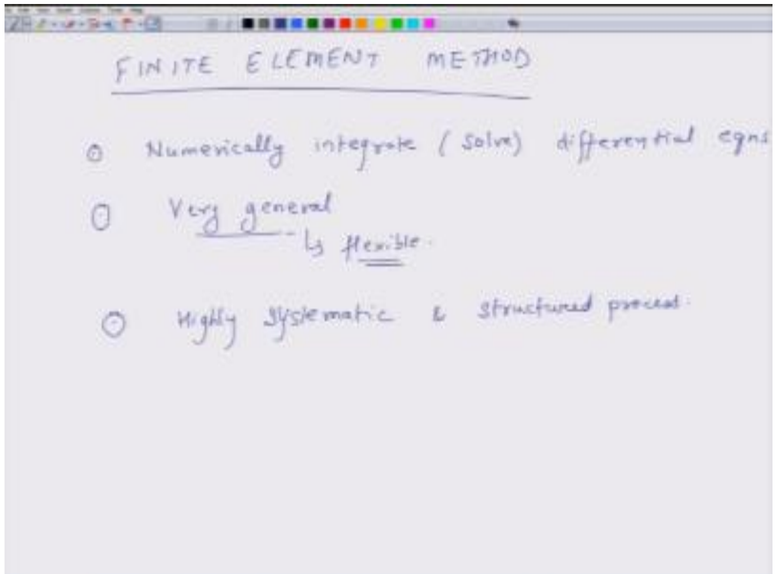

So because it is general it is also very flexible and another advantage is that it is highly systematic, highly systematic and structured process, what does that mean? So a lot of times when we try to solve differential equations we make guesses right we should guess a solution, and then we try that solution by plugging it back into the equation and see whether the equation is satisfied or not.

It is satisfied, it gets satisfied or not. This type of a process where you have to in guess or you have to intuitively figure out the solution of a differential equation this type of a process cannot be easily implemented in computers, because a computer cannot guess solutions okay. In context of FEA we do not rely on this intuition, but the method is very systematic, very structured and there is a very well-defined algorithm which drives the whole approach.

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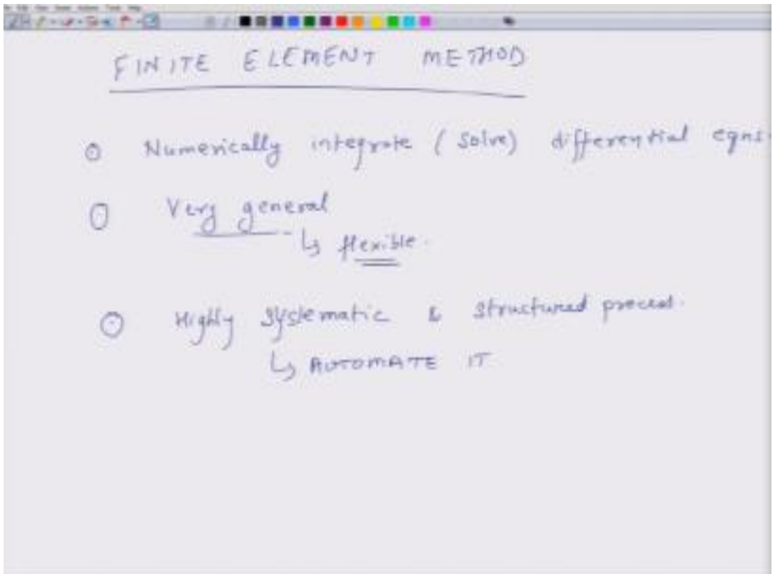

So if you have a very systematic and structured process then you can easily automate it. So it is easy to automate it and as a consequence you have a problem, and you have a very generic code that generic code for a stress analysis can solve how append bends, or how if I am stretching this strap of this watch, how does it deform, or if I am bending this plate and it is deforming. So the same code, the same software is able to solve a very large number of problems.

And the variety of problems that can address is not limited by size, or shape, or material properties, or things like that. So it is easy to automate, because it is highly systematic and it is a very structured process.

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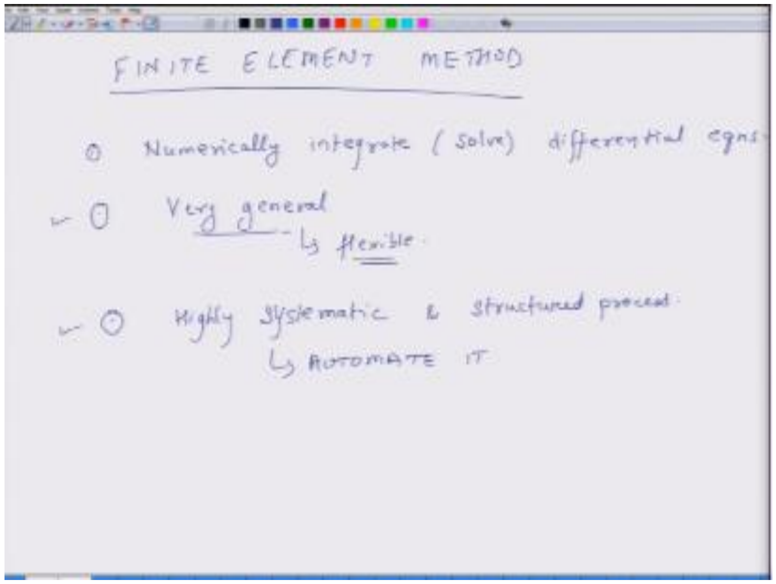

And as a consequence of these things that it is very general and also, because it is highly systematic and structured process it is used almost in all areas of science and technology okay, so that is there. So now what we will talk maybe your next 5-7 minutes is give you a very basic overview, again this will not make you experts in finite element method, but it will give you a very basic overview of what is the overall philosophy of finite element method.

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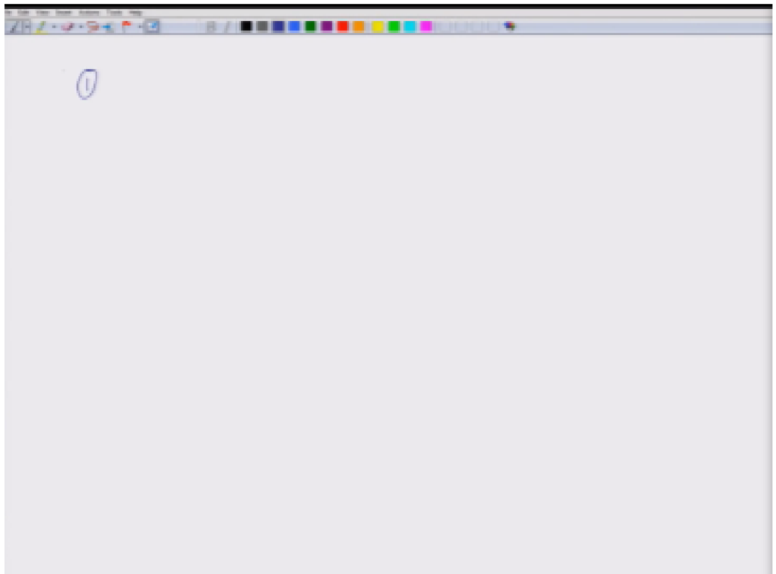

So the first step when you are trying to use finite element method, so let us say that I have a – let us say this is a beam and I am trying to bend it okay, I am trying to bend it and I have to understand how this -- when I apply a force F on this beam and let us say the beam is rigidly fixed here, and I am applying a force here, this beam is trying to bend and when it is trying to bend I have to figure out that, at each point along the beam what is the displacement in X direction, in Y direction and in the Z direction whatever. So and I want to – so let us say solve it using the finite element method.

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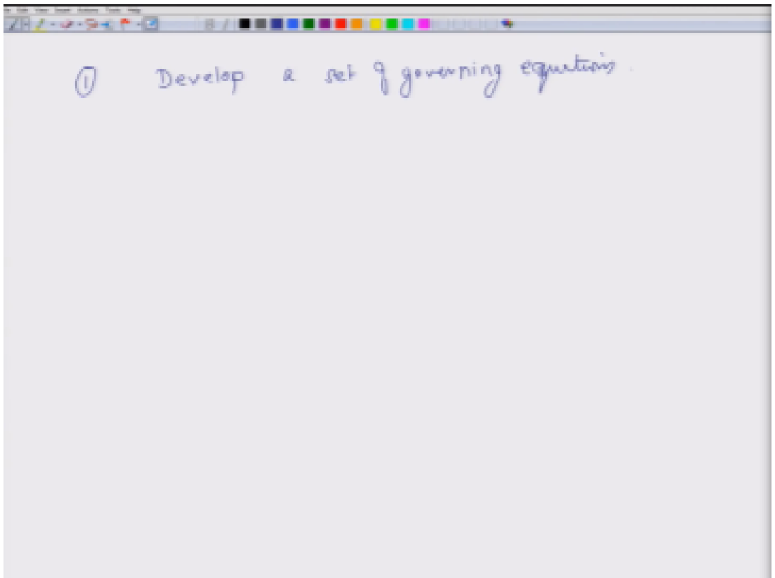

So the first process, the first step in this entire process is that we develop a set of governing equations. This step is invariant in the sense that whether you are trying to solve the partial differential equation in an exact sense by directly integrating it, or you are trying to use finite element method to solve the thing. This step has to be there, so you have to develop a set of governing equations.

For instance, in this case of the beam what is it that we are trying to do, we are applying a force and as a consequence of this force this structure is bending. So what I am trying to develop is a set of equilibrium equations, because there is mass here, there is a stiffness, there are forces, and the laws of Newton tell us that this thing will bend and move only if external forces on each small element of this material is not zero.

So I take a small piece of material, I see what all of the forces on it, and then I develop an equilibrium equation for this. So that is one thing, then using laws of geometry I try to figure out what is the relationship between the curvature of the beam, and the displacements in the beam. So those –

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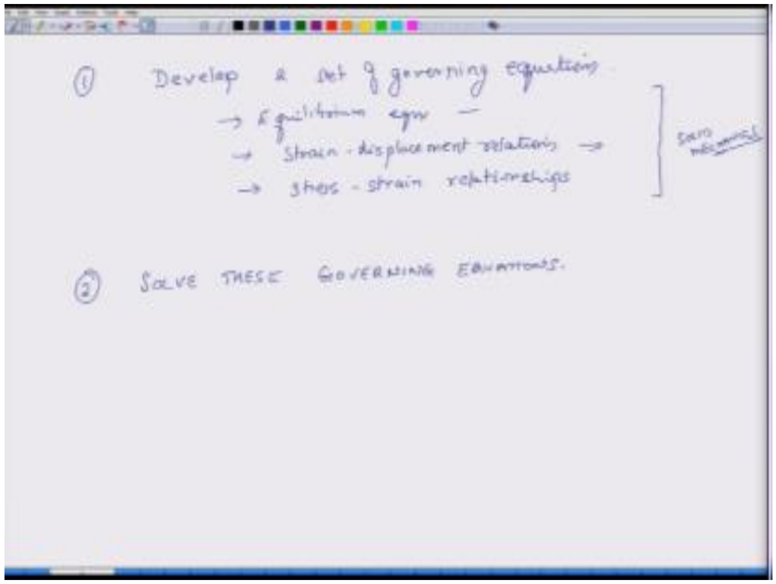

So first and at least in context of this beam, first set of equations would be equilibrium equations, second set of equations would be strain displacement relations okay. So equilibrium equations are basically statements of Newton's laws of motion. The strain displacement relations they are basically driven by geometry okay. And the third set of equations will be how stresses and the strains are related, stress strain relationships.

Now these set of governing equations are applicable to let us say solid mechanics okay. But if our problem is different say, it involves heat transfer then these set of equations will differ and we have to develop a different set of equations which are applicable to the heat transfer process. But regardless we have to -- we always start by developing a set of governing equations. Once we have done this, then we -- what do we so, we solve these governing equations and so we solve these.

And before I go deeper into how we solve these equations using finite element method we will see what are we will have a very quick overview in terms of what are different types of methods to solve these equations.

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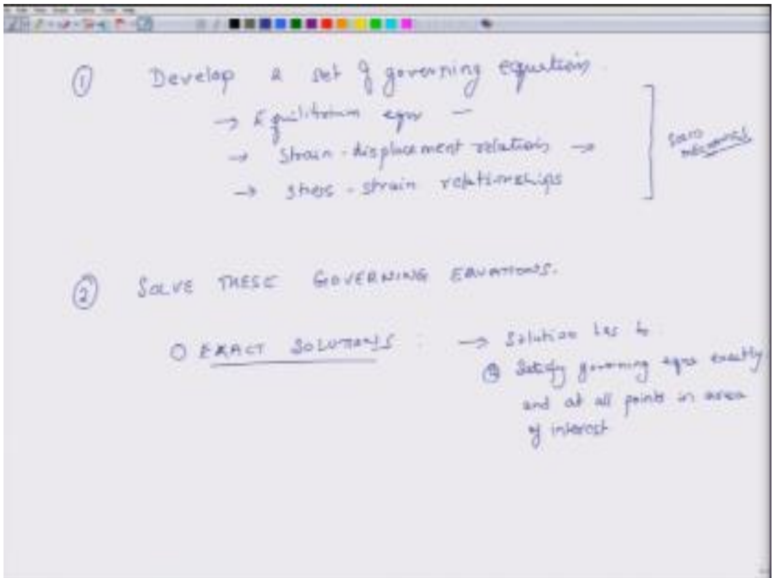

So the first way to solve them is exact solutions, okay. So one way we can solve is we get an exact solution of these differential equations. So what is the meaning of exact solution, it means that my solution has to do two things it has to a, satisfy governing equations exactly, it has to satisfy an at all points in area of interest okay. So that is the first thing that whatever solution I have, suppose I get a solution that displacement equals $ax+bx^2+cx^3+dx^4$ and so on and so forth.

This solution if I plug it back into my original differential equation then the left side of the equation and the right side of the equation they have to match exactly for all values of x, which are -- which lie in my area of interest okay. So that is the first, condition that is the first condition.

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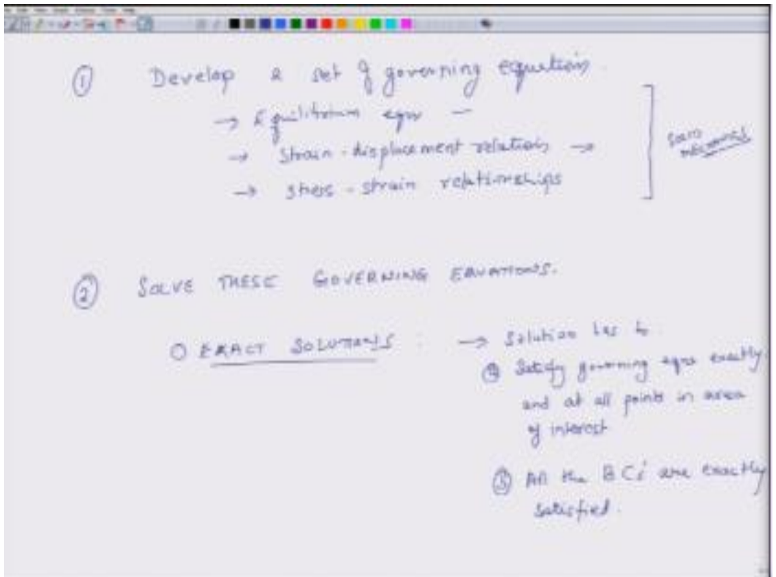

The second condition for an exact solution is that all the boundary conditions are exactly satisfied okay. So there could be a situation that the differential equation is exactly satisfied, but the boundary conditions are not. In that case the solution may not be exact there could be another situation where some boundary conditions are satisfied, and some -- at some points the differential equation is satisfied, but all boundary conditions are not satisfied or whatever.

So all the boundary conditions have to be satisfied and the second one is that the governing differential equations which we are interested in, they have to be satisfied at all the points, suppose I say that my displacement, suppose I guess that the displacement in this vertical direction if I apply a force is, you know $c1x + c\ 2\ x^2 + c3x^3 + c4x$ then at all the points in the beam, at all the points in the beam that equation has to satisfy the governing differential equation which you know supervisors or overseas the behavior of the beam at all the points so that is there.

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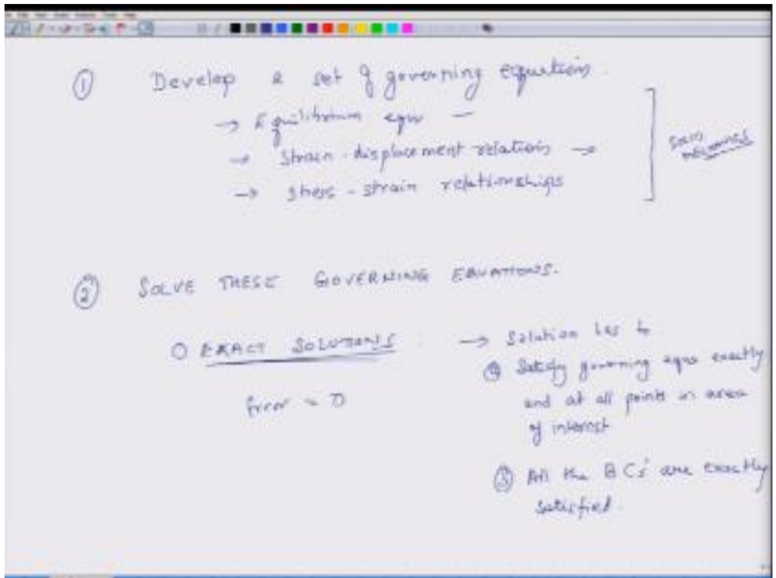

Now so if the solution is exact then our error will be zero okay. The other thing, so that is the first set of solutions, but then in most of the cases unless our problem is very simple these exact solutions are not possible.

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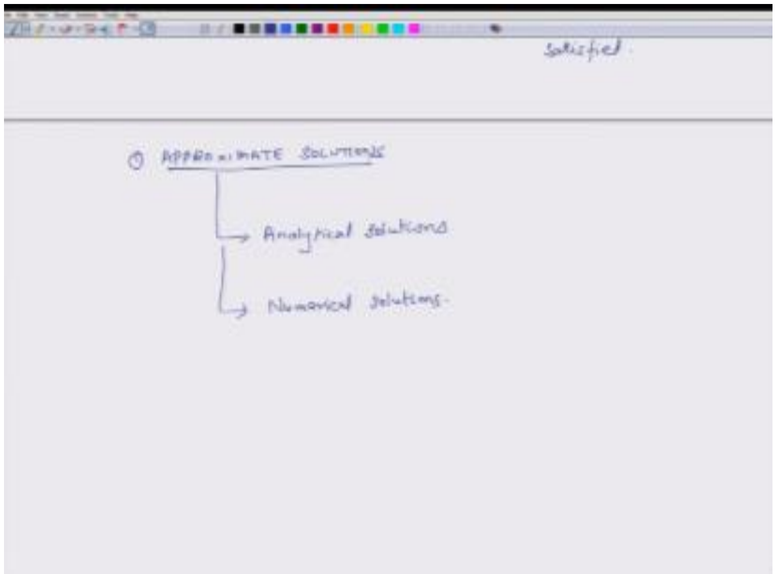

So then we go for approximate solutions okay. And there are broadly speaking two categories of approximate solutions, first one is analytical solutions and then other one is numerical solutions. Now in approximates in exact solution the error and what is the error, the difference is you have a governing differential equation the difference between right-hand side and the left hand side, that difference will be always zero.

If the difference is not zero then that is regarded as the error okay, that is defined as the error of the solution. In exact solution for all points the error is zero, in approximate points the error is not zero at most of the points.

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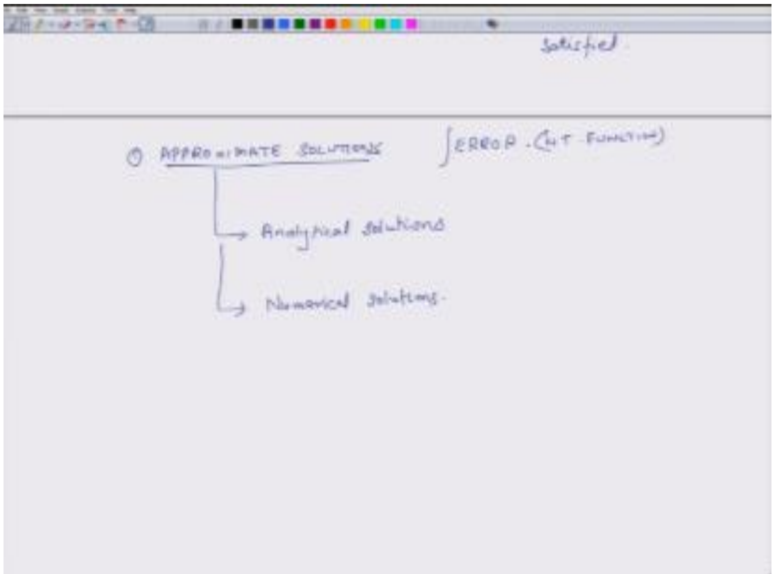

But what we say is that in this case, one way what we try to do is, that will say okay if I have an error okay, and I multiply it by some weighting function by some function at this point of time we will just call it some function, some weighting function we assign it some weight, and we integrate it. And suppose the beam we are -- if it is a beam then we integrates it over the length of the beam, then the weighted integral of the error is zero.

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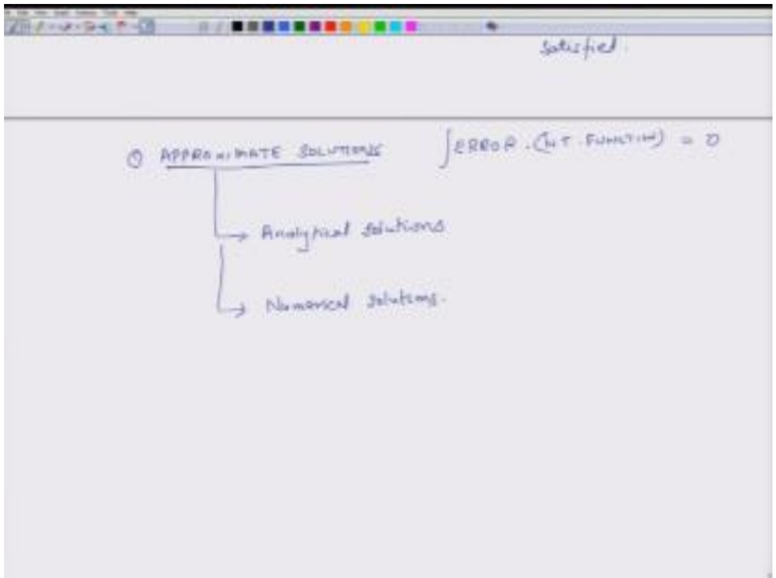

This is one strategy to achieve an approximate solution. So point-by-point the error may not be zero, but error maybe a little bit high at one point, a little bit low at one point, and if you add up all these errors after waiting.

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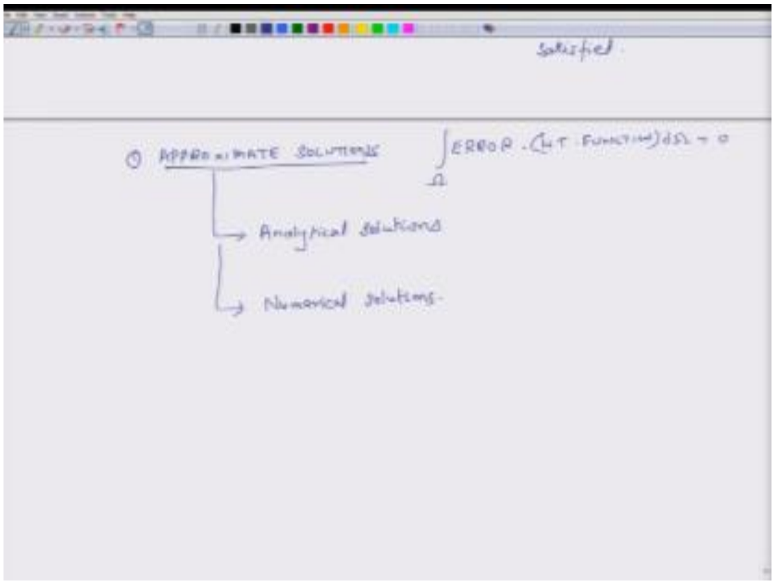

You know then that is the integral right that is the integral. So this is Ω it represents or domain, or domain means the area of interest, the region of interest. So if I integrate this error in a weighted sort of way and I will use this stuff weighted again and again and slowly it will become more and more clear to you what it means, in a weighted sort of way, if I integrate this error over the domain which means it is the region of interest.

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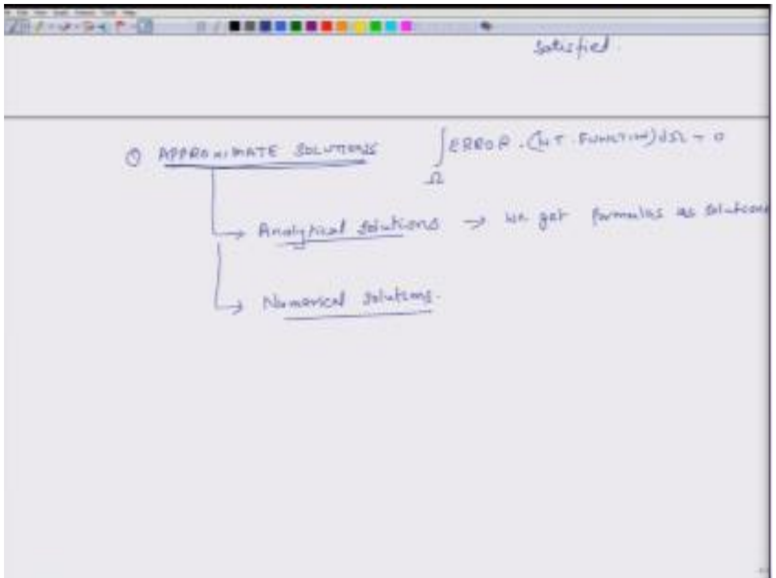

Then that error is going to be zero okay. So then these approximate solutions there lie in two categories numerical and analytical. And in analytical solutions we get in plain language we get some formulas as solutions okay. We get some sort of formulas, so maybe the deflection will be some sort of a series expansion or it may be a polynomial expansion series, or a sinus, you know this harmonic series or some sort -- or an exponential series.

So there will be some relationship between -- some relationships -- some formulaic relationship for the unknown variables in which we are interested in.

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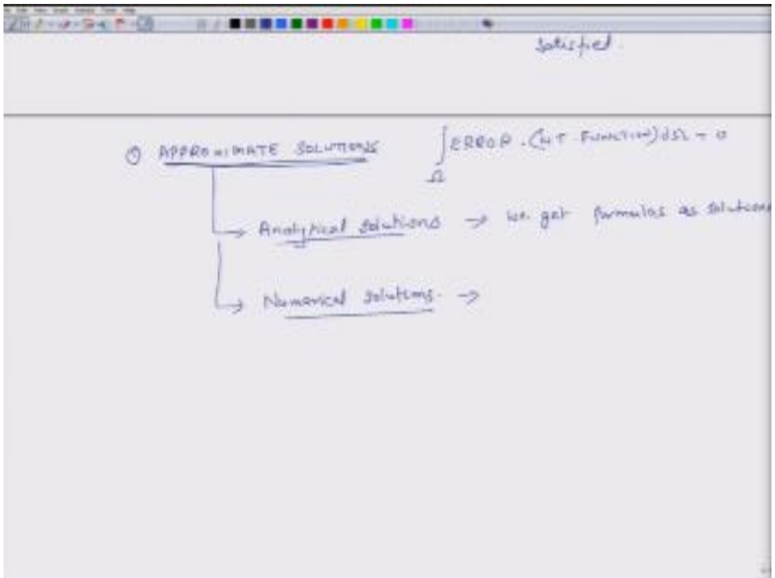

In numerical solutions we do not get formulas, but we get numerical values at different points in the region of interest that is the domain, in domain.

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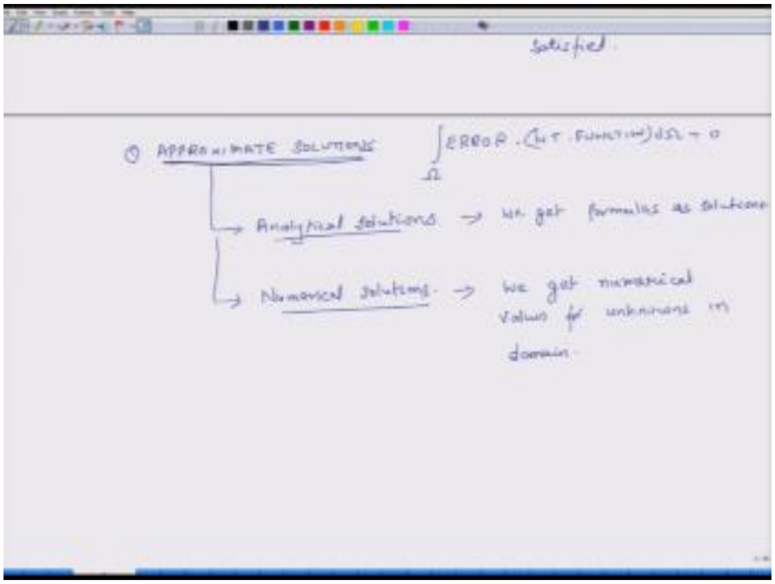

And once again what by domain I mean, the region of interest in which I am trying to find the solution okay. So suppose I have this piece and I am applying some force here, you know if I try to – if I apply a force here and if I am trying to develop an analytical solution I will get some sort of -- I will have to develop some sort of function which will explain or which will define how UVW suppose displacements is what I am interested, how UVW are changing with respect to XYZ in this cone.

But if I am using a numerical solution then I will not have this kind of a formula, but rather I will have maybe at this point the stress is 3.1, at this point displacement is 2.7, so I will get numerical values. So the finite element method is a numerical approach it is a numerical approach, another approach which is numerical in nature.

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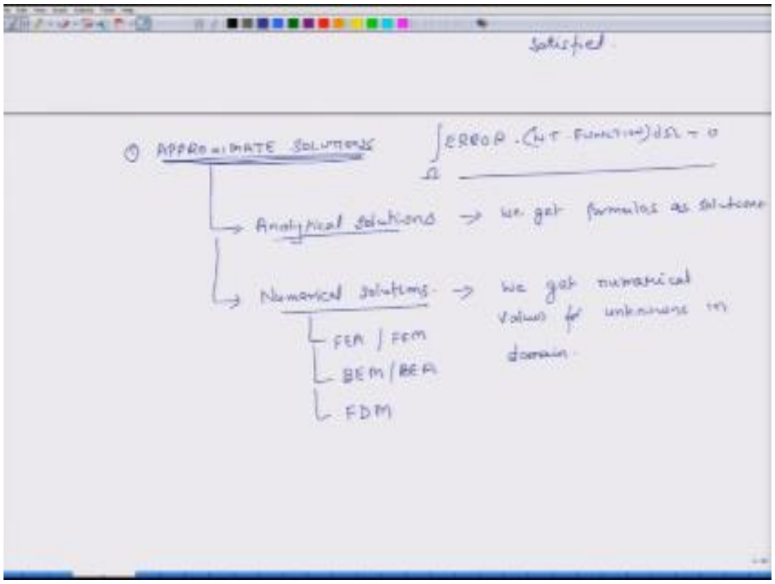

So two approaches are popular, one is FEA, other approach is boundary element method or FEM it is also called analysis, and the third one is finite difference method, these are three methods okay. One more thing, so we can do approximate solutions using this approach, so we said that approximate solutions, in approximate solution there are two categories analytical and numerical, and in both these approaches what we are trying to do is, we are trying to have a weighted integral of the -- sorry weighted the – we weight the error and we do a weighted integral of the error, we add that up over the domain and then we equated it to zero.

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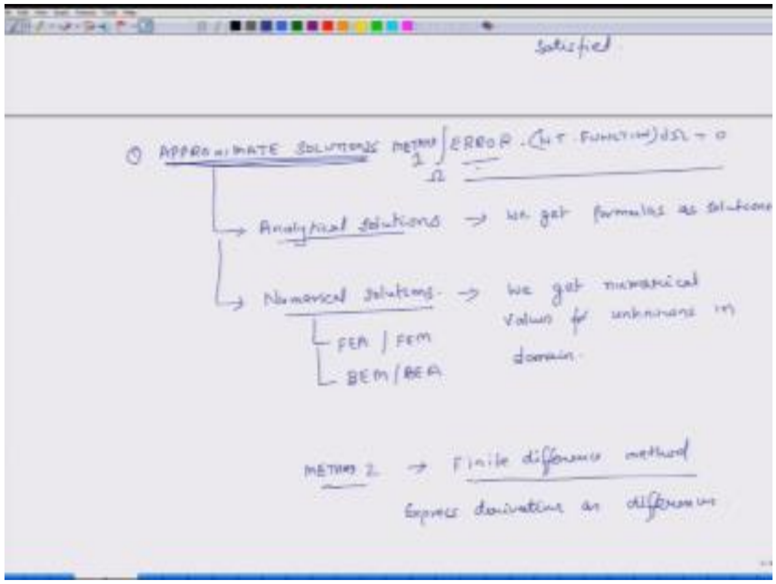

So that is what we do. We will learn what this weighted integral is later. So you have to be patient, but that is what we are doing, another way to if I do approximate solutions is to define, so this is method 1 this is method 1, the other method, method 2 is known as finite difference method. So actually I should have -- I should erase FDM from here, and here we do not do the weighted -- we do not calculate the weighted integral of the overall system rather what we do is that we express derivatives as differences okay.

So suppose we – I have to calculate del U over del X, then what we do is we calculate the value of u at point 1, at x = x1, and you calculate the value of U at another point where x = x2. And then we say okay, the partial derivative of U is u2 – u1/ x2 – x1. So once we do that we very rapidly convert these partial differential equations into algebraic equations and then we can solve those so that is another advantage.

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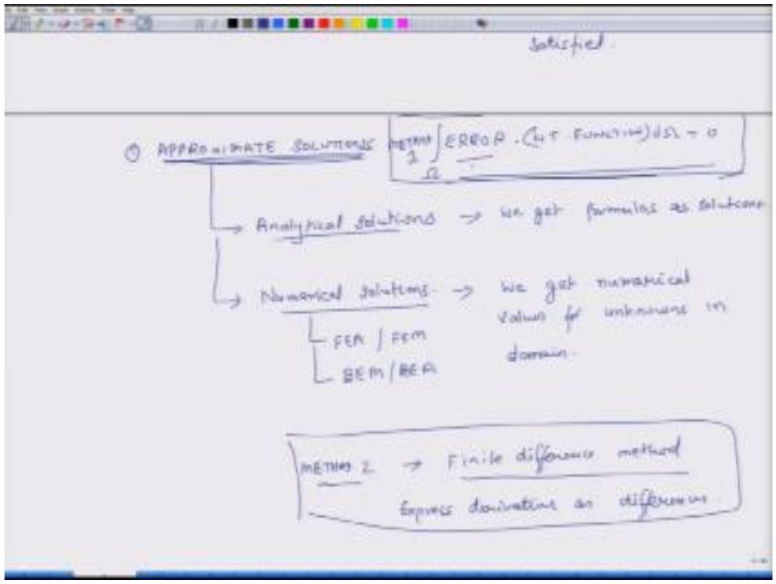

So we have an integral approach this is method 1, we have another one method 2 finite difference method. In method 1 we can have analytical solutions or numerical solutions, method 2 is finite difference method, and that is only a numerical method. So this completes our first module and we will again meet tomorrow for the next lecture, thanks.

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