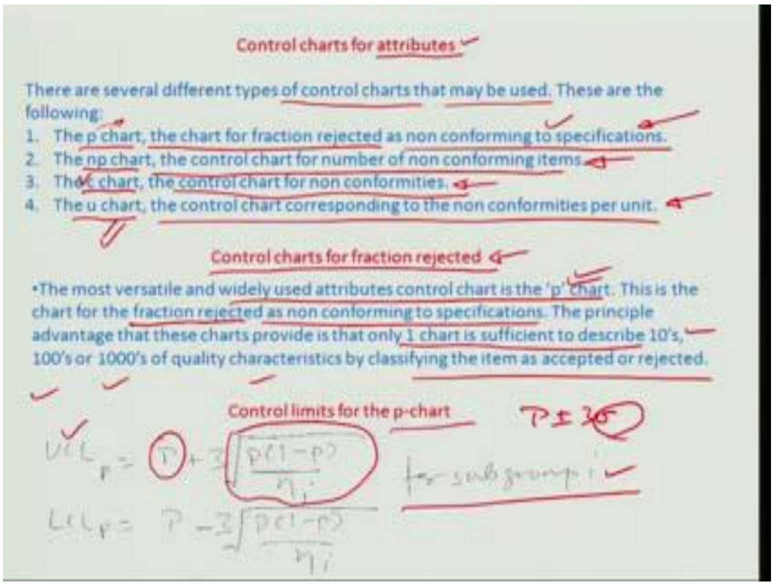
Manufacturing System Technology - II

Prof. Shantanu Bhattacharya
Department of Mechanical Engineering or
Industrial and Production Engineering
Indian Institute of Technology, Kanpur

Lecture – 27

Hello and welcome to this manufacturing systems technology; part 2, module 8. In the last module, we were talking about the different distributions that are the (refer time: 00:21) distribution, normal distribution, binominal distribution and how you can relate that to acceptance the sampling. This module, we are going to actually, start plotting some of the control charts, based on acceptance and ping plans, particularly the p charts.

(Refer Slide Time: 00:38)

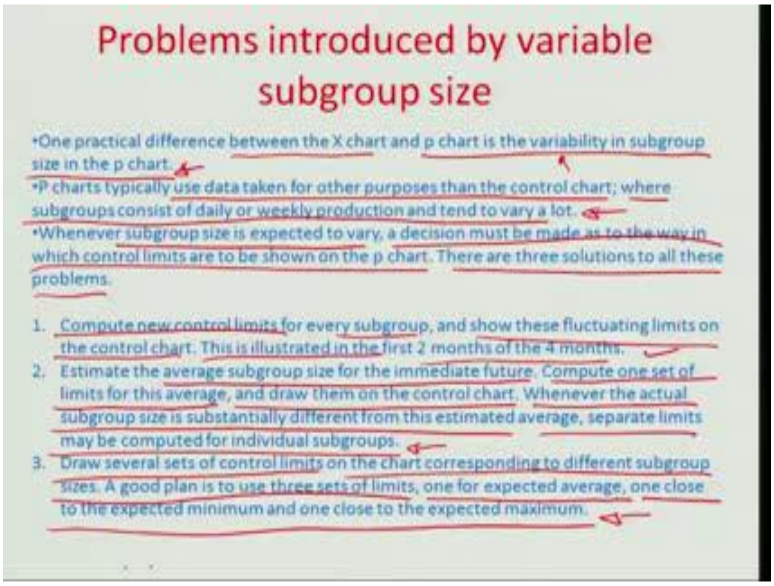


So, let us first look at what are the kinds of control charts, which are available for attributes or quality parameters, which can be judged without mentioning any appropriate quantity; only qualitative judgments can be passed. So, there are several different types of such control charts that may be deployed. One of them is the p chart. The p chart is basically, again the chart for fraction rejected as non confirming to specifications. You already found out with a particular p value in a certain situation, in a certain binominal distribution now, the mean and average, etc. can be calculated. Then there is the np chart, and this is the control chart for a number of non conforming items. So, one of them is non confirming to specifications. So, there may be some specifications and you know p chart basically, shows whether, at least one of the dimensions or one of the quality parameters, which are there, are away from the specifications. So, that makes the part to get rejected, and np chart is basically, in terms of the number of non confirming items; that is in a lot of, let us say, 100 of the piece, about 20 percent. So, the 200 of the 1000 are rejected, because of non confirming to specification and if you can plot the np somehow, value, that becomes the np chart. Obviously, there is another c chart, which you know basically, is the control chart for non conformities, and the c is typically, the number of occurrences of a certain non conformity and is being estimated though (refer time: 02:09) distribution as we have detailed earlier.

Then obviously, the u chart where the control chart corresponds to the number of non conformities per unit. So, c chart is sort of an occurrence of a non conformity, but in a non conformity, you can have many you know, specifications which run out, because of which more than one specifications sometimes, creates a problem that you have two or more defects within the same non conforming pce or non acceptable pce, which you are doing in acceptance sampling. So, the u chart is a plot for actually keeping a track of the number of non conformities per unit, wiz a wiz, c chart which actually, talks about the number of non conformances in a production line. So, having said that, let us first think about the control chart for fraction rejected; the most versatile and widely used attributes control chart is obviously, the p chart; this is what we are going to stick to at this time.

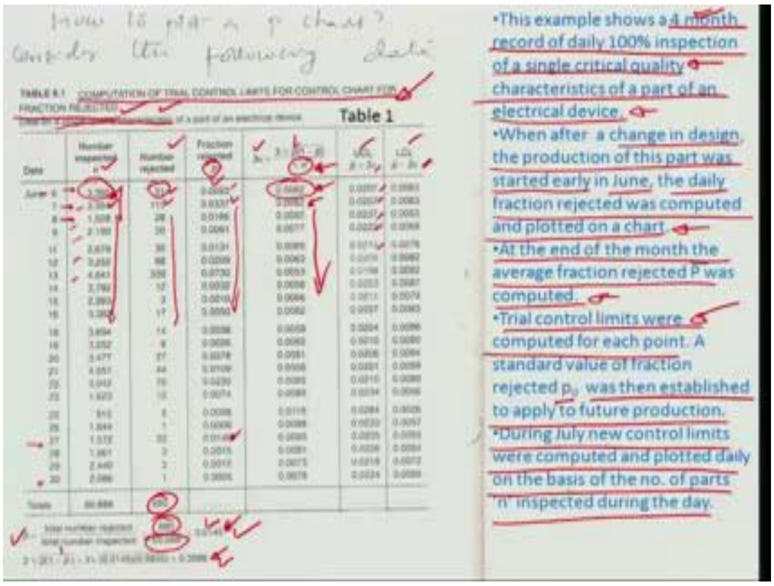
This is the chart for fraction rejected; obviously, you know what fraction rejected means, and this means that if refection has happened, because of the non conformance to some specifications. The principle advantages that these charts provide is that only one chart is sufficient to describe 10s, 100s, 1000s of quality characteristics by classifying the item as accepted or rejected. So, even if you have many dimensions, which are nonconforming on a single pce still, you can classify the pce to be either accepted or rejected, based on, whether one or more of the specifications are falling out of range. So, you basically, trying to increase the sample size by just comparing one characteristic, whether the overall sample is being rejected or the overall sample is being accepted. So, in a p chart, the control limit basically, comes out to be again, you know, this is the standard deviation for the p; this the mean for the p chart, as I think we had illustrated earlier for sub group size of I and so, you basically, trying to plot the you know, chart within p plus minus 3 times, standard deviation of p, which is p into p minus p into 1 minus p by ni or with the plus minus 3 to it.

(Refer Slide Time: 04:09)



So, that is how you can plot the control limits. One practical difference between the x chart that you had plotted earlier and the p chart is the variability in the subgroup size in the p chart. It can have a subgroup, list of subgroup sizes, which are completely different; will show this in the example problem later on. P charts typically, use data taken for other purposes than the control chart, where subgroups consist of daily or weekly production or tend to vary a lot. Whenever a subgroup size is expected to vary, decision must be made as to the way in which, the control limits are to be shown on the p chart. There are three solutions to these problems; one is compute new control limits for every sub group; show that these fluctuating items limits on the control chart. This is illustrated in the first two months of the four months. I will show you one problem example, where I will actually do this. So, estimate the average subgroup size for the immediate future. Compute one set of limits for this average and draw them on the control chart, whenever the actual subgroup size is substantially, different from this estimated average. Separate limits may be computed for the individual subgroups, and the third solution is that draw several set of control limits on the chart, corresponding to different subgroup sizes. A good plan is to use three set of limits; one for example, one for expected average; one close to the expected minimum and one close to the expected maximum. So, I am going to plot and control all these things in a p chart.

(Refer Slide Time: 05:33)



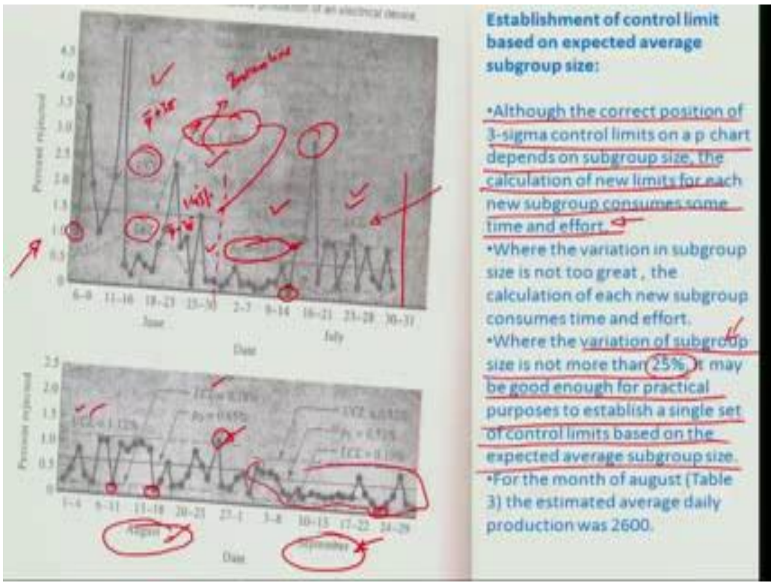
Let us look at the data base here. So, you are doing the computation of trail control limits for control chart for fraction rejected item, and this data is based on a single quality characteristics. We are not adding the complexity of many nonconformities in one sample. It is part of an electrical devise and you know, date wise, there has been recording as to how many have been expected and how many have been rejected. So, you can see that on 6th June, about 3350 items were inspected; out of which 31 was rejected. So, the fraction rejected came out to be about 0.93 percent. Similarly, you know on June 7th, it was 3354 number; rejected is about 113; fraction rejected is 3.37 percent so on and so forth.

This shows daily basis what is the number of items inspected and you can see that the subgroup size here of the inspected numbers are quite variant. So, that is one good issue about computing this particular control chart and so, the number rejected is also accordingly varies. What is interesting to see is the percentage rejects also, is varying quite a lot, because; obviously, the fraction rejected when the subgroup size or the sample size is too large, would be tending to be the average sample; average of the whole distribution. So, the three sigma limits are being computed according to the formulation; 3 route of p dash on minus p dash by route of n, where this p dash is basically, the average fraction rejected. So, let us look at the average here, right about here. So, it is basically, 880 total number of rejects out of total number of inspections 60688. So, the average number of fraction rejects in the whole distribution is 0.10145; so, about 1.45 percent. You can see here that you know, let us say the 27th day of June, more or less, whatever is the percentage reject, is very close to the distribution's overall average number of rejects. So, that is p dash value.

So, you put the p dash here and what you need to do is sort of keep varying n, because as you know the sample size every time, subgroup size every time is varying in this case. So, you will have this particular entry for calculating this 3 sigma. You only need to vary now, the subgroup size, which is actually quite variant as you see from between all these different days; June 6th all the way to June 30. The subgroup size is varying. Every time, the number of inspections on particular day is quite different than the others. So, the three sigma limit has been computed here, as three times of root, p dash times of 1 minus p dash and that comes out to be in this case, 0.3586. So, this value has been obtained by dividing this three times of route, p dash 1 minus p dash 0.3586, divided by root of 3350.

Similarly, this value has been calculated by this 0.3586 number, divided by root of 3354 and so on and so forth. You know that is how you calculate this three sigma limit. So, obviously, UCL and LCL are p plus and minus 3 sigma. So, p plus has already been initiated at 0.0145. So, you are basically, changing the values, based on whatever is their numbers from case to case by changing a variation or by looking at a variation in the sigma, and you are doing p dash plus 3 sigma and p dash minus 3 sigma for UCL, LCL. So, obviously, this is the data out of larger, much larger data actually. It is actually a four months record of daily 100 percent inspection of a single critical quality characteristic, which is a part of an electrical device, and we will show you the later on data for the next three months, all the way up to September. Only thing in this case is we will need to understand thereafter, the first month how you can vary the control limits so that, you can keep plotting the p chart in accordance. When after change in design, the production of this new part was started early in June, the daily fraction rejected was computed and plotted on a chart and at the end of the month, the average fraction rejected p dash was computed every time. So, at the end of June for example, the average fraction rejected is about 1.45 percent. So, now, the trial control limits have to be plotted for each point. A standard value of fraction rejected p 0, was then established to apply to the future production and during July, new control limits were computed and plotted daily on the basis of the number of parts and inspected during the day.

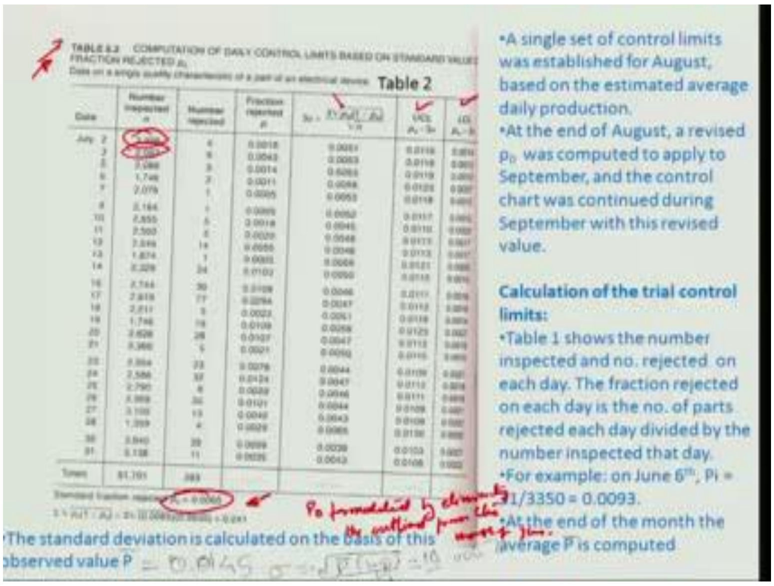
(Refer Slide Time: 10:03)



So, let us look at how this control limit is established. So, here for example, you know if you look at the 3 sigma control limits on a p chart, depends on the subgroup size, this is the control chart plot for the p chart. For the first month, let us say, we are talking about up to June 30^{th} and as you can see here, we are plotting the varying UCL and LCL based on p dash plus 3 sigma and p dash minus 3 sigma and these are typically, the assembly of points, which are being computed here. Let us go back to the table. So, you have a UCL and LCL and you have actually, different values of the UCL and LCL on a daily basis, which are actually plotting as the upper control limit and the lower control limit, through this broken line. So, this is broken line. Obviously, the average value of the fraction rejects is about 1.45 percent in this particular case.

So, as it has come out from the p dash value, if you remember as calculated in the table, that is how you can plot the upper and lower control limits. Once the limits are defined now, in the p chart, we now start looking at the individual points and see what are the outliers and here, in this particular case, I can say that there are a many points. So, these are the individual points; for example, this is corresponding to the case 7th of June, where there is an outlier 0.037, and it goes above the upper control limit. Similarly, there may be many such cases, which are there from time to time; for example, this again is an outlier, June 12th is an outlier, because 0.02909 is greater than the upper control limit in this particular case. So, similarly, there may be cases, where it goes below the lower control limits.

(Refer Slide Time: 11:55)



So, in a nutshell, if you look at the days June 7, 12, 13, 22 are basically, the outliers. If I just look back in the data calculated in the June points, 7, 12, 13 and 22 are the outliers as you can see here. So, in case of 7th again, the upper control limit is exceeded. In case of 12th again, the upper control limit is exceeded. In case of 13th again, the upper control limit is exceeded to a large extent. In case of 22nd again, the upper control limit is exceeded to a large extent. The remaining values are all within the control limits and so, you can safely take them. So, what you have to do now is to take a decision for the next month to determine the UCL. So, you basically remove these values, which are outliers that is June 7th, June 12th, June 13th, and June 22nd from the overall calculation; that means, from the overall p average value that you have computed. So, you take out you know 70 items, 339 items, 68 items and 31 items from the number of rejected and similarly, you also take them out from the number inspected like 3350, 3354 item on 7th, 3252 item on 12th, 4641 items inspected on 13th.

So, you basically completely, eliminating these four points, which are outliers and finding out the fresh p dash value, corresponding to that and that comes out to be 0.0065. So, because of elimination of these outliers in this second month, when you will try plotting the control limits, you have to take an average value p 0, which is formulated by eliminating the outliers from the month of June. So, once that is done now, you can calculate the new limits, based on this p 0 value again, you know, considering the subgroup size, which is there on local basis and then, for the month of July subsequently, based on this new value 0.0065. You can see that the control chart now, has about 65 percent. So, the average changes from 1.45 percent to 65 percent, because of elimination of these outlying points and float us the UCL and LCL. So, the LCL and UCL in this case, are quite lower, because you have eliminated the major contributions, which are outliers in the first month.

So, that the second month is more smoothly plotted. It goes between 2 to 7, all the way to 30 to 31, and you can see here that now, you have again, a method of plotting the outliers. So, once the limits are determined, you are plotting the individual points, corresponding to 2nd of July, 3rd of July, 4th of July so on and so forth. Let us look at this particular table to find out if there are any outliers. So, you can see again, that you know, the UCL here is defined as p 0 plus 3 sigma, p 0 minus 3 sigma and we compare the daily fraction reject with respect to the LCL, UCL. So, we find out that there are some points like for example, this case of July 17th, when the fraction rejected is 2.94 percent and which is completely, an outlier again, in this particular case, or for example, July 24th when again it is 1.24 percent and then again, the outlier, because the upper control limit is 0.0112.

So, after plotting this, we try to find out what these outliers are. These outliers are represented here. So, this probably, corresponds to the first outlier. There is another one, which actually goes below the lower control limit. So, these outliers again, need to be eliminated in a sequential manner and then, the fresh limits are computed; the fresh UCL and LCL values are computed by eliminating this outlier so that, this can be the new governing LCL UCL for the month of August. So, let us look at how these data is calculated now. So, we calculate a fresh p 0 value. So, once you have found out that the number of outliers or lower from June to July, as you can again see back in this curve. So, in June the outliers are much more in comparison to July. So, you have a sort of standardized the way that we are having a control and fraction rejects. So, from this onwards, where you know, there are may be, very less outliers or the magnitude of outlying is also very low. You can actually assume now, the sample size or the group size or subgroup size, whatever we have considered to be varying so far, to be the average daily production. So, therefore, it will lead to (refer time: 16:36) of the upper and the lower control limits if you assume the average daily production as now, the basis for calculating the UCL and LCL. Let us look at this again, on this table 3 right here. So, here is the estimated daily average production is basically, out of these 31 days. The total production is 73523. So, it is about 2600 per day and based on that, we are now calculating fixed you know, UCL and LCL values by looking at the fraction reject.

Again, this fraction reject would be computed, based on you know, what you are eliminating. So, basically, here in first module, you have to eliminate June 7, 12, 13 and 22nd. In the second module, as I told you, had eliminated July 17th, and 24th and may be also, July 7th, which is an outlier again, on the lower side as you can see. By eliminating these three, we have calculated fresh p 0 value, which comes out to be here as 0.0065, but now; obviously, because you know, you want to straighten out the UCL and LCL, we are using instead of these varying subgroup sizes on a daily basis, the average production, which is 2600 on a daily basis, and that helps you to calculate the fixed set of UCL and LCL in this particular case, which is 0.0112 and 0.0018. So, this is a very critical decision making aspect for the management that when you want to straighten out your control limits.

Obviously, you have to look at what is the outlying capacity or what is let us say, I want what are the number of outliers or to extent, which the outlying capacity is there. If it gets minimized, because of one or two iterations, that should be time enough for the management to decide that we now, need to have straightened control limits and then, you can calculate everything on the basis of the average daily production. Similarly, in the case of September, you can see again, the same strength continues; obviously, there were some outliers in the case of August, which need to be further eliminated. So, there was one outlier particularly, here; there was another outlier region, which is below the LCL, again below the LCL. So, now, these values need to be taken off from the August. Let us look at the table corresponding to which outliers these are. So, one of them for example, is the August 27th, where the total production has gone above the UCL or the total fraction defects upon above the UCL, being 0.0112 and this has gone to 0.0116 and similarly, there are cases where for example, during let us say, August 21st, then again, the outlier; although the extent is very low, but it has gone below the lower control limit, which is 0.0018; this is 0.0017 and then, there is again, another August 16th, the same thing has happened here. It has gone to 0.08 percent, whereas the LCL was 0.18 percent. So, there are three such outliers. You remove these outliers; calculate the new value of p 0, which is p 0.0051. Again, you know, the average daily production is 2700 calculated, because once you have reasoned or you have made the UCL LCL straightened; there is no way to revert it back and then, the next straightened limits could be calculated by based on p 0; new p 0 value by eliminating from the last month and then, calculating 3 sigma based on the average daily production.

So, here the n value that you are taking is equal to the average demand. Just as you, took the n value as the average demand; subgroup size to be 2600, the average demand on the month of August and September. So, you have taken that critical decision making to straighten the limits from August onwards, based on the previous two months experience, June and July, because you see that the control limits are within; the points or the fraction rejects are within the control limit and then, the extent of the outliers also have reduced; obviously, the number of outliers have reduced quite a bit. So, it is good time to call on straight limits. So, this way, you can plot for the remaining two months; that is August and September, as you can see here, and in the month of September, in fact, the number of outliers have almost eliminated. There is only one particular value, which is outlying and this can continue from month to month.

So, this is how you basically, try to control your quality, based on the various aspects of the p control chart here, because it is attribute related; you need not quantify. You can just consider a pce to be a rejected or accepted, and it gives you a lot more flexibility. Then the x chart or r chart, where you can only mention one characteristic dimension, based on which, you plot the x and r chart. So, this is the Sheward's (refer time: 21:20) acceptance sampling based attribute control chart. You know although, the correct position of the 3 sigma control limits on a p chart depends on subgroup size, the calculation of new limits for each new subgroup, consume some time and effort. That is a problem at least, for the first two months, when you are having a unstraightened UCL and LCL; you will have to compute on the various subgroup sizes on a daily basis; although, if the variation of the subgroup is not more than 25 percent. It may be good enough for practical purposes to establish a single set of control limits, based on expected average subgroup size, and that decision can be taken based on looking at the outliers also and the extent of outliers also.

Once the straightened limits are established, then there is no reverting back, and by and large, the acceptance sampling plan should be based on operating the values, within plus and minus 25 percent of the actual average daily production, which would be taken in this particular case. So, this, in a nutshell, is what a p chart is or how it is plotted. So, you have already learnt about the various statistical distributions. So, you have learnt about how to plot the Sheward's(refer time: 22:23) control charts with x and r; that is the dimensions or for specific dimensions or quantitative control charts. You have also learnt about attribute control charts. So, we will try to round off this statistical quality control area here, and will start with the new topic in the next module, which would be more related to the robotics and automation in factories of the future.

Thank you.