Manufacturing System Technology - II

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Lecture – 24

Hello and welcome to this Manufacturing Systems Technology part two - module 24. This going to be a little more little bigger module time wise, because we wanting to complete some of the left over distributions like Poisson's distribution or normal distribution and how you can derive although those different distribution with respect to the acceptance sampling plan and binomial distribution that you have just done in the last few modules. So, let us first starts with the task left over of a representing or calculating the expectation or the mean value of the binomial distribution. And because as we said that it is a discretized distribution n x, so basically sigma x varying between one n x px where x is basically the number of defectives and p x is exactly the probability of those number defectives in a certain amount of draw sizes or sample size as you saw in the last example of good versus defective. So, in the same manner, we are going to calculate the mathematical expectation or the mean and the standard deviation of such a distribution.

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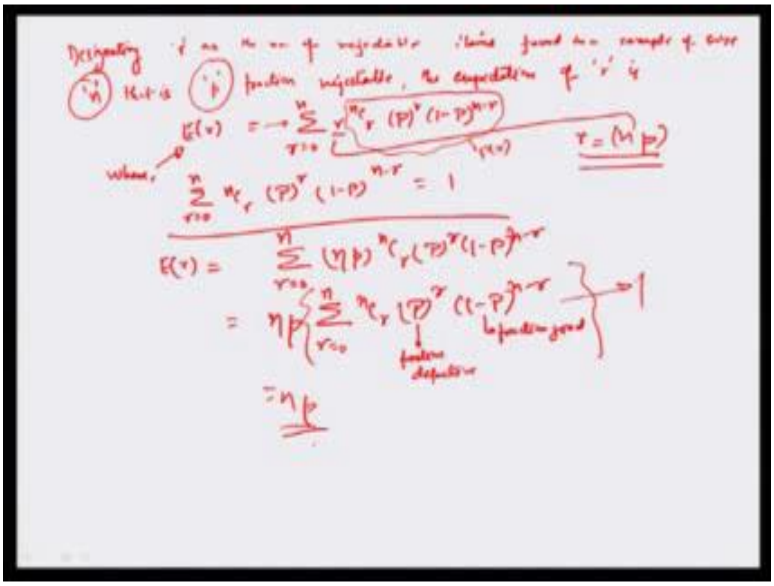

So, let say we designate r as the number of rejectable items found in a sample of size n, that is p fraction rejectable. So, this is exactly the percentage defective which is there in the sample. So, we call it fraction rejectable. The expectation of r is given as E r sigma r varying between zero and n r times of n c r p to the power of r 1 minus p to the power of n minus r, where sigma r varying between zero and n n c r p to the power of r 1 minus p to the power of n minus r is actually equal to one. So, we want to find out the expectation of this particular distribution. So, we are basically multiplying it by exactly the probability of r rejectables in a sample size of n and a multiplying that by r just because it is discretize x px. So, this is the p of r and we are multiplying that by r, and r varies between zero one in n.

So, let us calculate what is the expectation of r, if we expand this particular formulation. So, basically the term here can be, so in this particular case, if r is the number of rejectables in the sample of size n, and p is the fraction rejectable you can right r equal to n p in the same way as we did d equal to n times of p dash, where p dash was the fraction rejectable. So, if I substitute this value of n p in this particular expression here, the expectation of r is written down as r varying between zero and n n p, and these are constant n c r p to the power of r 1 minus p to the power of 1 minus r n p is taken as constant. And we have sigma r varying between zero and n n c r p to the power of r one minus p to the power of n minus r this is the fraction good, this is the fraction defective. And so obviously, this whole expression here comes out to be one and you can say this e r as n p.

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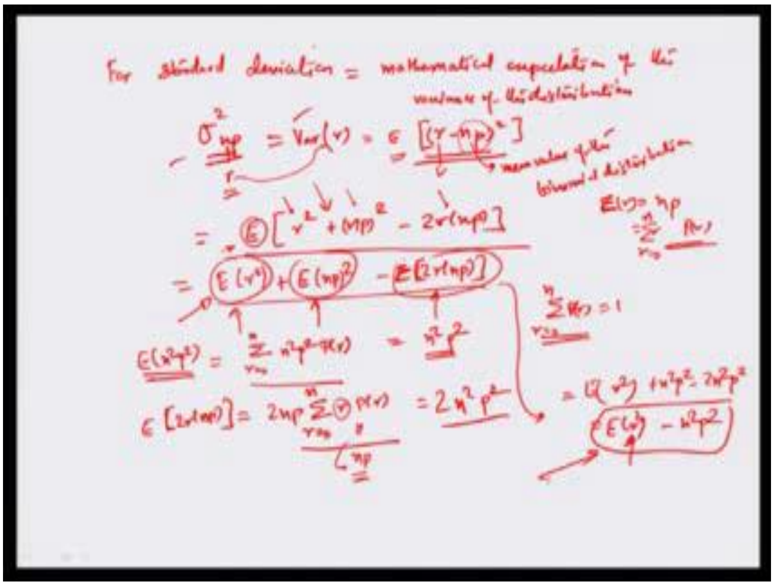

For the purposes of further mathematical development, the expression for standard deviation also needs to be computed because that is going to be sort of added onto studying this distribution further. So for standard deviation on the binomial distribution, we can really calculate this by developing the mathematical expectation of the variance of the distribution. So, when we are talking about variance is basically the square of the standard deviation, and we are talking about the variation or variance of a case where we have a exactly r rejectable in a sample of size n would the fraction defect is p, the sample, the drawing condition, the overall lot size does not really change in this particular case. So, this can be written down as the variance of r, r being n p equal to the expectation of r minus n p square. So, n p being the mean r or the mean value of the binomial distribution as you had just calculated in the last step and r is the rejectables.

So, this r can vary it can be more than the mean or less than the mean; r equal to n p is really one value of the particular distribution because in questions. So, the expectation the average of r minus n p would be the variance of r. So, the variance of r r minus n p square I am sorry is a variance of r. So, mean of this squares. So, if I just expand this we get e of r square plus np square minus twice r time is of n p. And so this expectation can be actually further split a part because this expectation many event of this sigma and you know that how we calculated that the expectation of r rejectable exactly is equal to n p, so it basically r p r, r varying between zero and n, so discrete distribution.

So in the similar manner, you can actually split of this function as a summation of individual variables which are recorded in this expression within the bracket and you can write this as expectation of r square plus expectation of n p square minus expectation of twice r n p and so on and so forth. And because there all in any events summations, so this is r square p r this n square p square p r summation, this is r n p p r summations and so on and so forth. So, let us first of all start calculating what these individual expectations are in order to find out what is the variance in subsequently what is the standard deviation in this particular case. So, when we talk about the expectation of n square p square, so obviously this means r varying between zero and n n square p square times of the probability of having exactly r rejectables.

So, this comes out to be simply n square p square because obviously the overall summation of having zero or n rejectables in a lot sample of n is exactly summations of p r that is probability of zero rejectable one rejectable, two rejectable and so on up to n rejectable, so that is defiantly one. In a similar manner, if we try to calculate the expectation of twice r n p, we can represent this as twice n p sigma r varying between zero and n r p r. So, this we already know is n p from our past derivation about the mean, which is actually again you know assuming that there are about p percentage rejects in a sample of n, where n p is equal to r as we have done exactly in a similar manner for the mean of the probability distribution in the earlier slide. So, this becomes twice n square p square. So, now, the task that is there is to really we able to calculate the E r square - the expectation r square so far whatever we have done results in a total variance so far, which is equal to expectation of r square plus n square p square minus twice n square p square which is actually expectation of r square minus n square p square, so that is how the final variance of r would be calculated.

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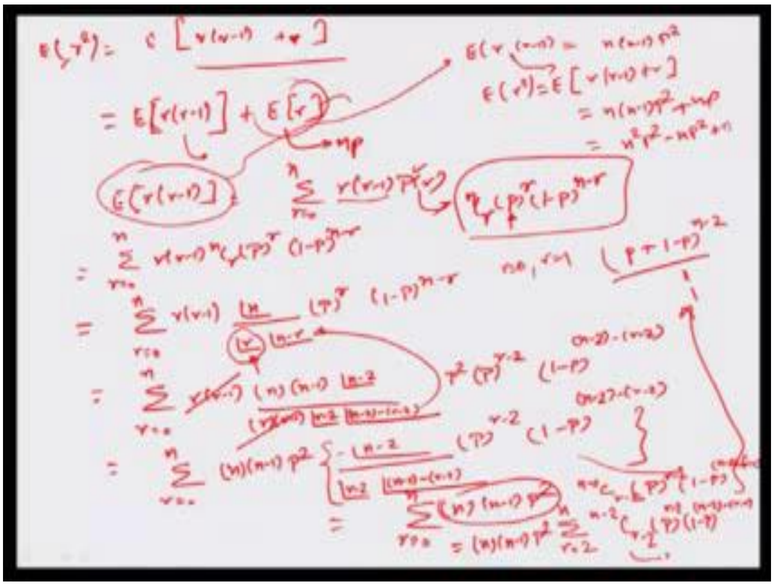

Let us now talk about how to find out the expectation of r square. So, I am going to write down r square in little different manner here. So, you can write it as expectation of r times r minus 1 plus r. So, essentially you are adding and subtracting r to the term r square here. And so therefore, in a similar manner or in a similar logic, you can write this as a summation of this expectation of r into r minus 1 plus the expectation of r, this already we know is n p from where earlier statement on means. We need to calculate what this is. And for doing that let us actually expand the formulation. So, the expectation r times r minus 1, it can be written down as sigma r varying between zero to n, r times r minus one times of probability or r. And if I may recall this probability of r is further recorded as n c r p to the power of r, where p is the percentage defective times 1 minus p to the power of n minus r.

Just putting out to the binomial distribution for r rejectable in a sample size of n which the probability of getting r rejectables exactly in a binomial distribution. So, we can write this down further or expand this further as sigma are varying between zero and n r times of r minus one times of n c r p to the power of r 1 minus p to the power of 1 minus r. So, this a form can be slightly manipulated here I can write r r minus 1, and then I can write this as factorial n by factorial r times of factorial n minus r, and further p to the power of r 1 minus p to the power n minus r. So, we had do a small trick here would basically try to remove r and r minus 1 terms from this r factorial.

So, let me just write it down in a appropriate manner as sigma r varying between zero and n r times r minus 1 times of let us write n times of n minus one times of n times of 2 factorial which we can very well write times of r times of r minus 1 times of r minus 2 factorial and this we can write down as n minus 2 minus r minus 2. It really does not matter because we have added and subtracted 2 in the process and this can be recorded n minus r. And this a further can be expanded as p square times of p r minus 2, and 1 minus p times of again n minus 2 minus r minus 2. So, I am just trying to write it down in algebraically correct manner, but what are in process we are doing is very interesting that these r r minus 1 get canceled in they have same in the numerator and the denominator.

And we are left with the situation where we can say this is equal to r equal to r varying between sigma r varying between zero and n, n times of n minus 1 times of square of p times of this old term here which is sigma n minus 2 divided by was factorial n minus to divided by factorial r minus 2 divided by factorial r minus 2 divided by again factorial n minus to minus r minus 2 times of p to the power of r minus 2 1 minus p to the power of n minus r minus 2 again. So, this comes out to be equal to sigma r varying between zero and n, n n minus 1 square of p; and we can write this whole expression here under this third bracket as the probability or the distribution whose generic term can be represented as a n minus 2 c r minus 2 p to the power of r minus 2 one minus of p to the power of n minus 2 minus r minus 2.

So, straight forward of this whole term n n minus one p square can come out the sigma. So, you can record this is n time n minus one time of square of p times of sigma r varying between now here is a very important issue that I would like to address corresponding to r equal to 0, and r equal to 1, this whole expectation of r into r minus one since to zero because there is a r into r minus 1 term here. So, I can say that r is varies from 2 and n, n minus 2 c r minus 2 p to the power of r minus 2 times of one minus p to the power n minus 2 minus r minus 2. So, this is nothing but one because this actually is a representation of p plus 1 minus p to the power of n minus 2. So, that is how you represent these distributions and obviously, this becomes equal to 1.

So, I can say that the expectation of r times of r minus 1, which you started deriving in this particular step happens to be n times of n minus 1 times of square of p and so. The expectation of r square which was really the expectation of summation you know r times r minus 1 as calculated above here plus r is basically n times of n minus 1 p square plus the expectation of r which is n p. So, if I further calculate this, this becomes n square p square minus n p square plus and that is what the expectation of r square is. So, finally, let me just recall goes one slide back and see what has been left over here. So, we have already computed n square p square expectation and twice r n p expectation, and these values come out to be n square p square and twice n square p square. And finally, we had the expectation of this whole term r minus n p whole square which is actually the variance of r was expectation of r square minus n square p square.

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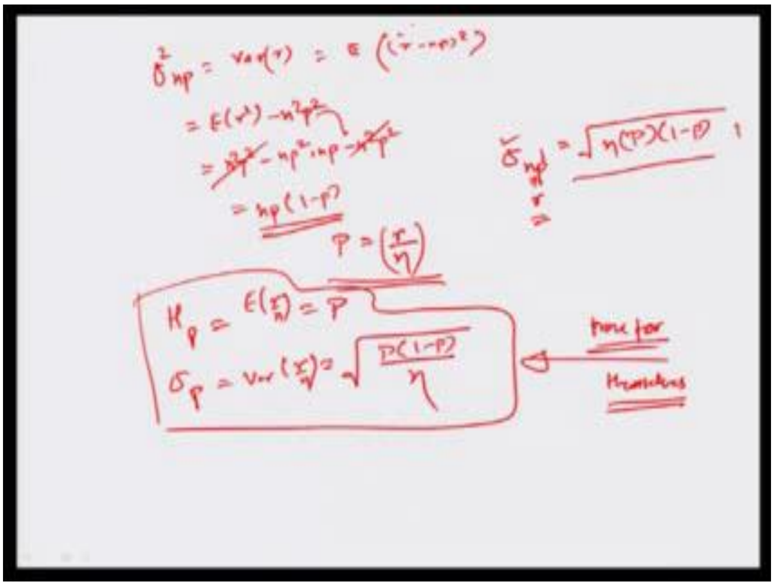

So, from this value write about here I would like to plug in and calculate what is the variance of r that is expectation of r minus n p square and we find out that this variance can be expectation of square of r plus or minus n square p square which can further be which is n square p square minus n p square plus n p minus of n square p square again as applied by this term. So, n square p square cancels each other and we are left with n p times of 1 minus p. So, the standard deviation of number of rejectable r which is n p, p being the fraction defect can be given as root of n times of the fraction defective times of fraction good that is how you calculate the standard deviation of the binomial distribution.

So, if you are talking about a case where let us say you had a variable which related to the term p itself, p being equals to the number of rejectable by n and you wanted to calculate the mean and the sigma that means, the mathematical expectation of mu or mu p mathematical expectation of p as well as the sigma of p, this would be written down as simply p that is 1 by n times of the expectation of r it is n p. And the variance in the same manner can be written down as p times of 1 minus p by n. And I would really like to leave it to the audience to prove for themselves using the similar algorithm or similar approach has been used in the case of this expectation of r or variance of r, so that is about it. So, it is about more or less about how a binomial distribution can be understood or determined and how you can predict the mathematical mean as well as the variance of such a distribution.

I would like to look at some of the distributions which naturally occur and let us also look at some cases where a data has been recorded in a manner which can be applied to some of these distributions. And obviously, binomial is only one case of it, there are many other distributions which can be fit into such data one of them is called Poisson's distribution, another is also known as the normal distributions. And we will try to see what are the certain differences in the data from which we can identify really whether this data would be more suite for a Poisson's distribution or a binomial distribution; obviously, the acceptance sampling and the fraction reject whenever these term come into picture what kind of envision yourself to be with the binomial distribution to force fit such a case as you have seen repeatedly in the example which have been done in the last about two or three modules.

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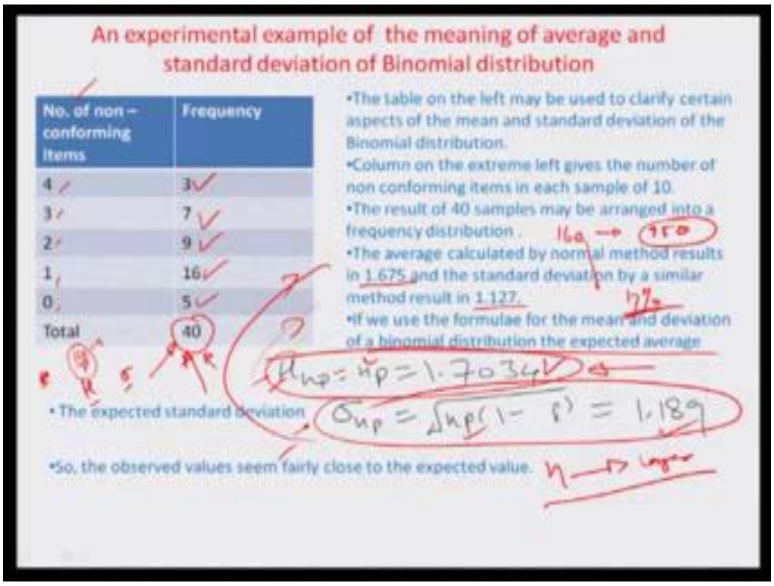

Now let us look at some more realistic data and see how this data can be utilized. But just before going to that I would just like to point one more example or one more case where I would show that if we really calculated the average the mean or the standard deviation in the normal manner, and did it through the binomial distribution would it really converge or is there some difference. So, let us look at this particular case here where we are talking about number of non-confirming items to the 4, 3, 2, and 1 and 0. And the frequency at which this occurs you know in a particular draw of 4, you could either have all four good; or three good, one defective, two good, two defective; one good, three defective; or zero good and all four defective.

So, the cases that have happened for a total 40 numbers of draws of this case are recorded. And the frequency table here shows that at least 3 draws have come out with all four goods or at least seven draws are come out in this 40 with all draws having three defectives and one good. And similarly 9 draws have come out with two defectives and defectives and two good or 16 draws have come out with only one defective and three goods. And similarly 5 draws have come out with zero defective and all good items. So, if I wanted to just calculate the binomial the mu as well as the sigma for this particular distribution, obviously, it is something like a frequency distribution and you can calculate it using normal to formulation as has been earlier taught.

The average comes out to be 1.675 and the standard deviation comes out to be 1, 1, 1.127 whereas we use the binomial distribution and the expected mean and expected variance the mu p here would record as 1.7034 and the sigma n p here would record as 1.189 by means that have been shown here n p, and n p 1 minus p here I have just described before. And what I would like to say if this n becomes larger that is the slow convergence of these two standard deviations as well as the mean calculated through the mathematical expectations method has been done before with the actual means and the standard deviations calculated with frequency analysis.

So, basically the sample size is very large then there would be a general convergence of the mean and the standard deviation of a binomial distribution with the actual average and actual standard deviation of the distribution. Although in this particular case, so the values seem very close, so 40 is really a good sample size for basically you are drawing every time you are drawing 4 pieces and there are 40 such trails you have made and basically you have drawn 160 items from a lot which is otherwise. So, in this particular case we are probably talking about 160 samples drawn from close to 950 overall population size with an acceptance rate of, so these 160 samples that you are drawing from 950 is the overall statistic its acceptance rate is about 17 percent. So, basically you 0.17 is p and 0.83 is the 1 minus p or percentage good. And so in this manner, you can see that the mu p and sigma n p are quite close to the actual averages and the standard deviations which are calculated by the other frequency method.

So, I think I will close this module in the interest of time, but in the next module, we will now try to goes bit of Poisson’s distribution and also a normal distribution to certain situations where we will first up all understand in the case study manner, what are those criteria in a particular data which you would needing to know to predict whether you are going to do we are going to use the Poisson’s distribution or you are going to use a normal distribution, so that we will do and cover in the next module.

Thank you so much.