Manufacturing System Technology - II
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Lecture – 23

Hello and welcome to this Manufacturing Systems Technology - part two - module 23. We have been discussing about the ways to represent order and unordered samples with respect combinations and permutations. Today, we are going to start off continue that and see if we can mention or we can arrive at a distribution which can actually helpful in a estimating probability. The estimation of the probability was done in respect of acceptance sampling.

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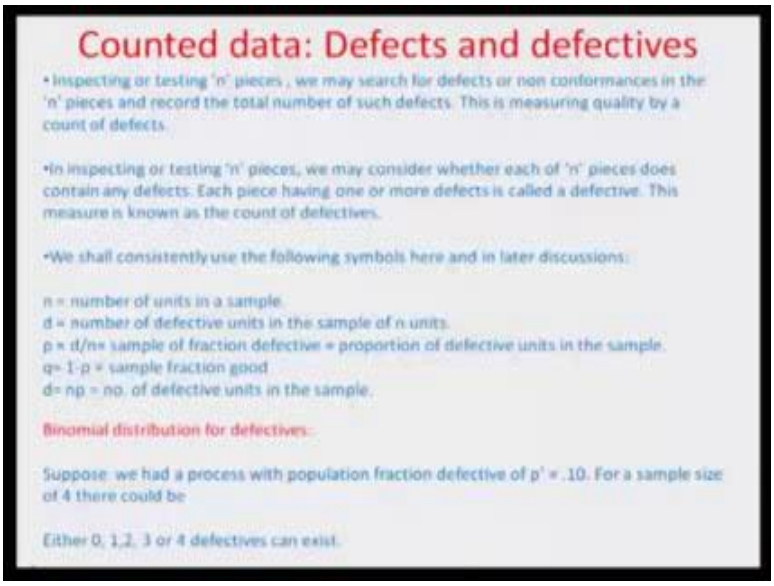

So, let say in a particular case, we are inspecting about n pieces, and we may search for defects or non conformance in the n pieces and record the total number of such defects. This is measuring quality by a count of defects. In inspecting or testing n pieces, we may consider whether each of n pieces does contain any defects. Each piece having one or more defects is called a defective. This measure is known as the count of defectives. So, we will consistently use the followings symbols, n is the number of units in a sample, d let say the number of defective units in sample units n. And therefore, the probability of having the defective which is the also the fraction defective is d by n. So, it is basically sort of indicative of a proportion of the defective units in the sample. And obviously, the numbers of good units as a proportion are 1 minus p or 1 minus d by n, so therefore exactly there are n minus d units, which are good as a respect to the total sample size which is n, so that is the fraction of good pieces one minus p. And then obviously, the number of defective units in a sample can be represented as n times of this fraction probability. So, obviously d has been defined in a manner so that d equal to n p.

So, suppose we had a process, we will going to start off see and evaluate this, so called binomial distribution, which I am just going to bringing in the next slide is somehow useful in the process of the estimation of all this probability given this particular situation of selecting d defective from n. Supposing, we had a process with population fraction defective of 0.1, that means, about 10 out of 100 sample are acceptable. And for sample size of 4, there could be you know if we look at let say a draw size of 4 from this overall n population size, there can be either case where all the samples are good samples. So, you have zero defective. Or you have at least one of the samples out of the four draws, which is defective two of the samples again which are defective, three in the defective and four all four are defective. So, these are some of the possibilities which will happen, when we are drawings samples from otherwise large population of n total samples.

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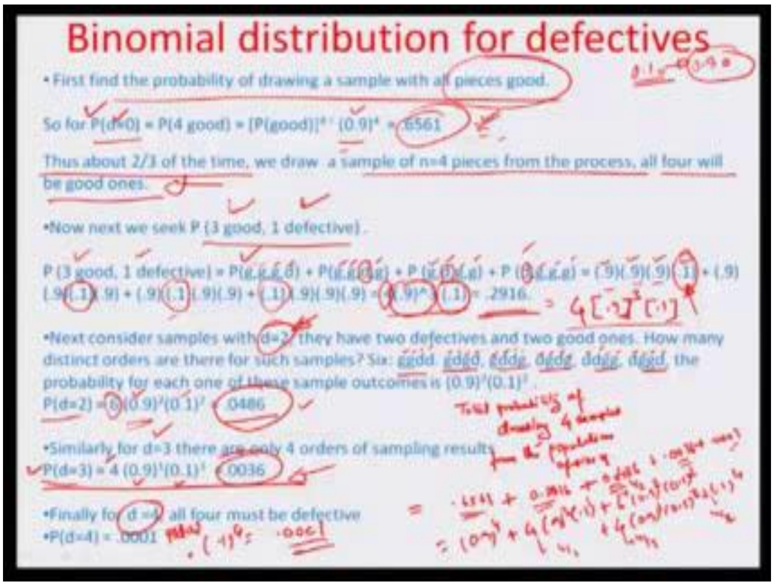

So, how do we write the probability of drawing all good pieces, so obviously, this probability can defined as just you know because of fraction defective of is 0.10, therefore the amount of good pieces or fraction of good pieces are actually 90 percent. So, they are about 90 items in the 100 lots, which are actually good samples. So the probability of d being zero, the d being you know the number of defectives being zero is basically that all the four items that are drawn are good items. So, therefore we can just simply because they are independent of each other these draws are not depended on each other, so we can say that 0.9 to the power of 4 is what the total probability of zero defective would be which is 0.6561. So, thus about two-third of the times about 66 percent time, we draw of a sample of n equal to four pieces from the process where all four will be good samples probability provided there are about ten percent defective in particular samples.

Now let us see the probability of the second case, which is three good and one defective; that means, in three draws, you have good samples and one draw you have defective sample. So, what are the basic combinations of when this defective can occurs. So, you have four different draws. So, the defective can occurs in the fourth draw or the third draw or the second draw or the first draw. So, you have a probability of a getting one order which is good in the first draw, good in the second draw, good in the third draw, and bad in the fourth draw represented by d. Similarly you can have a defective drawn in third draw, so the first two are good, and the last one is good again. The defective draw in the second draw whether first and the last two are good samples. And then one in the first draw and remaining three in the last three draws are good samples.

So, you can actually again represented this as you know the probability of getting good times probability of gain bad obviously, acceptance rate is 90 percent. So, 10 percent are defective. So, you can say the probability of getting three good samples in a succession followed by a defective is 0.9 into 0.9 into 0.9 times of 0.1. And similarly the all variation that is happening here in terms of where that is point one is place. So, you can see this is in the third place, second place, the first place and so on and so forth. So basically you can represent all this as four times of 0.9 to the power of 3 times of 0.1 which is actually represented as 0.2916. So, it is basically four times of 0.9 to the power of 3 times of 0.1.

So we considered next samples with where two of these four draws have defective samples. So, how many such combinations will exist, you will have two goods, two defectives. You can have one good, and one good defectives; again another good, another defectives. You can have one good, two defectives followed by a good. You can have a defective then a good, then a defective then a good or two defectives in the beginning and the two good at the end. Or otherwise you can also have two defectives sets you know, first in the fourth draw, and second and the third draw being the good sample. So, these are all what is needed for getting exactly two defectives in the success of four draws that you are making from the samples. So obviously, we can actually now put the outcomes together again as 6 times of 0.9 square that is the probability of having a good sample times of 0.1 square which is the probability having a defective samples, which is 0.0486.

We can do the same thing for d equal to three that means, the defectives numbers of defectives now are three out of the four draws. So, you can have you know four times of again the same you know as you had three good and one defective, the only thing here would change with basically is going to be if it is three defectives, similarly for the d equal to 3, which is basically number of defectives equal to three and number of good one, there are only four orders of sampling results. Where the p probability having three defectives, d equal to 3 is given by 4 times of one good that is 0.9 to the power of one times of 3 defective 0.1 to power of 3 in an identical manner before. So this comes out to be about 0.0036. Finally, for d equal to four, all four must be defective, so you have point one to the power of four defining 0.0001 as probability of having d equal to four defectives.

So, if we summarize all these data points together about the probability of having either zero defectives or three good and one defective and also two good and two defectives, or let say a probability having three defectives in the four draws or all four as the defectives. The summation which will come out to be also representing the total probability or probability of drawing four samples from the population of size n as all four being good which is probability of getting exactly d equal to 0 plus, three good and one defective, which is actually 0.2196 plus again the possibilities of getting two defectives which is 0.486 plus the possibilities of getting three defective which is 0.0036 plus the possibilities of getting all defectives or four defectives. So, there are actually five terms which are there.

And if I just correlate all these and write in terms of these powers this is 0.9 to the power of four. This is actually four times of as you can see here0.9 to the power of three times of 0.1 the power of 1 plus this actually represent six times of 0.9 to the power of 2 times 0.1 to the power of 2 plus 4 times of 0.9 to the power of 1 0.1 to the power of 3 plus again 0.1 to the power of 4. So, in general this can be recorded as case where this four is actually 4 c 1, this can be 4 c 2, this is 4 c 3, and this one can be 4 c 4, where c depicts the combinations as talked earlier.

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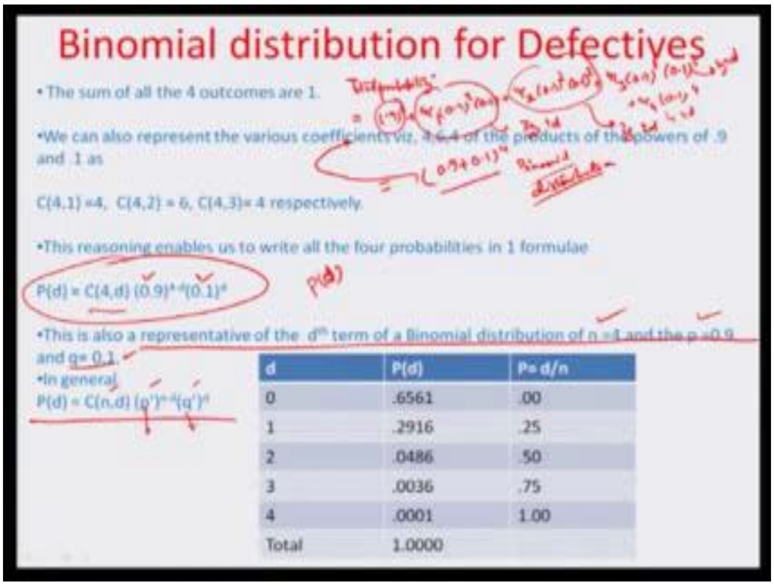

So, I am go to sort of represent all this together as a distribution where I can say that the total probability in this case of the whole draw to happen is basically 0.9 to the power 4 plus 4 c 1 0.9 to the power of 3 0.1 to the power of 1 plus 4 c 2 0.9 to the power of 2 0.1 to the power of 2 plus 4 c 3 0.9 to the power of 1 0.1 to the power of 3 plus 4 c 4 0.1 to the power of 4. In other words, this can be represented as a probability distribution of the number of good samples plus the number of bad samples to the power of 4. So, this is actually nothing but the binomial distribution. When each term of the distribution is representing something for example, this terms represents the probability of having three goods and one defective; this represents two goods and two defectives; this represents one good and three defective, and similarly this represents all the four defectives that is have represented the total probability in this binomial distribution.

And generally you can write the generic term of representing d defectives is basically p d given as c 4 d 0.9 to the power of 4 minus d 0.1 to the power of d, this is also have represented the d th term of binomial distribution for n equal to 4, and the p equal to 0.9, and q equal to 0.1. In general, the p d becomes equal to c and d p dash to the power of n minus d q dash to the power of d where p dash is the probability of having a defectives sample d by n for example, in this particular case. And q is 1 minus p dash which is probability of having a good sample that is how you represent through the binomial distribution the whole acceptance sampling mode and also can estimate the number of defects in a particular sample in a subsequent draw which comes on. So, this is one important case which has been described here.

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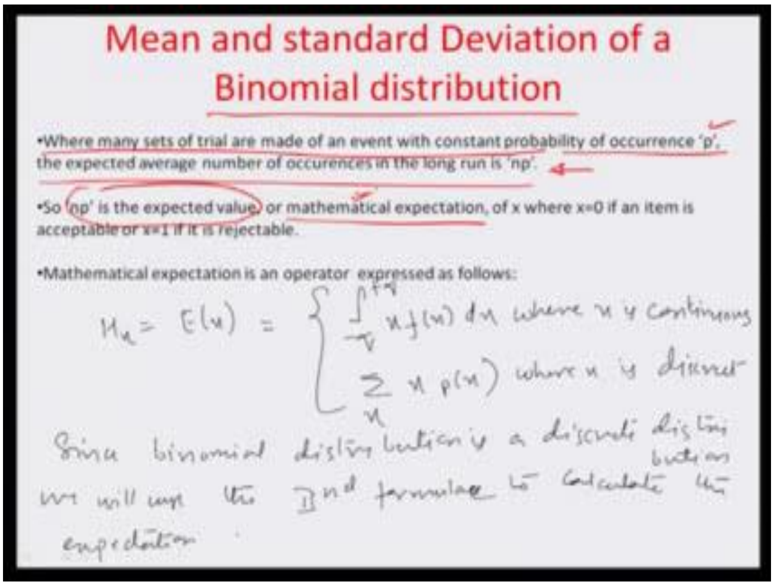

So, just because we are taking about binomial distribution I think it is very important and proved and for us to know about what really is a binomial distribution and what characteristic properties of the distribution. So, particularly where many sets of trial are made of an event with a constant probability of occurrence p, the expected average number of occurrence in the long run is obviously n times of p. And so n p is the expected value or mathematical expectation of where this mathematical expectation is concept probability taught in earlier you know first and second year level courses. And mathematical expectation of x, where x equal to 0, if an item is acceptable; or x equal to 1, if it is reject able. So, we distribute it in a manner, so the mathematical expectation is an operator which expressed as followed.

So, the mean of x, where x is basically again this zero, if an item is acceptable; or one, if it is rejectable that is how you have placed the function. So, mu of x is basically defined if x is continuous as integral x f (x) d x, x varying between minus infinity to plus infinity and in a case of discrete distribution, in fact this binomial distribution that you are taking about the discrete distribution, you can define it as x p x sigma, x varying between one and n. And so basically we will apply the second formulation to sort of calculate the mathematical expectation. So the interest of time, we will close this particular module, but in the next module, we are going to bring up two different formulation strategies where in one we will calculate what is the mean of a binomial distribution using this definition here of the you know in case of the discretized x of mathematical expectation or mean of the distribution. And then also the calculate or try to calculate this standard deviation of such a discretized distribution, so that we are enable now to a may be based on that estimate things like number of defectives or even number of units which are actually defectives and so on and so forth in a particular acceptance sampling plan.

So, thank you so much for this particular module.