

Manufacturing System Technology - II
Prof. Shantanu Bhattacharya
Department of Mechanical Engineering or
Industrial and Production Engineering
Indian Institute of Technology, Kanpur

Lecture – 22

Hello and welcome to this manufacturing systems technology part 2 module 22. The last module we will be taking about some probability theory, and we would be trying to discuss how acceptance sampling can be linked up to the theory of probability, extra. And we are looking to the aspect of supposing there are 2 pieces and we are expecting them. What are the chances that one of them will be defective provided the overall percentage defectives about 8 percent. So, we try to investigate all that.

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Probability Laws

- If 2 events A and B might occur on a trial or experiment, but the occurrence of either one prevents the occurrence of the other, then events A and B are called mutually exclusive.
- For two such events: $P(A) + P(B) = P(A \text{ or } B, \text{ mutually exclusive events})$
- We have seen this in the earlier example in the 1 good case $P(1 \text{ Good}) = 0.736 + 0.0736$
- If one of the two events A and \bar{A} is certain to occur on a trial, but both cannot simultaneously occur, then A and \bar{A} are called complementary events.
- For any such pair of events
$$P(A) + P(\bar{A}) = 1$$
- We have seen this in the example of a single draw where p' was given as 0.08.
$$P(1 \text{ defective in 1 draw}) = 0.08 = p'$$
$$P(1 \text{ good in 1 draw}) = 1 - P(1 \text{ defective in 1 draw})$$
$$= 1 - 0.08 = 0.92$$

The 2 events were complementary

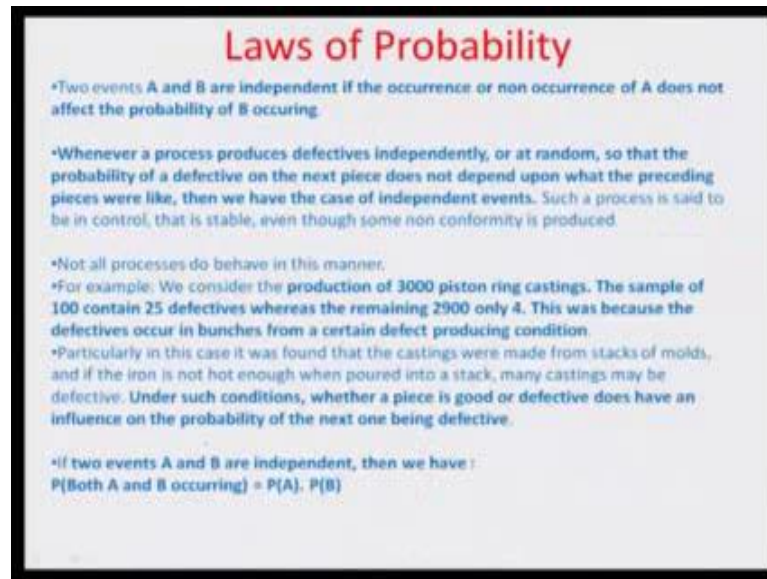
Today, we will be taking that little differently we will actually talk about mutually exclusive events. So, if 2 events A and B occur on a particular experiment or a trial, and further we occurrence of either one prevents the occurrence of the other then events A and B are called mutually exclusive. For example, let us say we would be talking about the percentage defectives and we would be talking about the defective pieces, and 2 draw either there can be 0 defective 1 or 2, let us consider the one defective case. So, if there is the one defective case which is there in such situation, then obviously out of the 2 draws that you are making on this whole laws; one is the defective to other has to be good. So, therefore they are mutually exclusive sets, and one of them would be differently

defective and other would be good depending on the first choice good. So, other one should be defective and vice versa. So, these are all. So, called mutually exclusive in nature, and sub if there is the defective future occur; obviously, your left with only a good occurrence, you cannot have a defective occurrence, because that will have the number of defective to be 2. So far 2 such events B probability of A plus probability of B is actually the total probability A or B mutually exclusive events, we have seen this in the earlier example.

For example, if 0.08 forefather 8 present is really the amount of the parentage defective of the population. So, in one draw what we would doing is either one defective which would be 0.08 times 0.81 good which is 0.9 be 2 that is 0.736, the other one to be first one is the good, the other one should be defective usually exclusive that how you update this all. So, this is the very, very good example mutually exclusive events for their 2 events, and a complimentary is certain to occur on a trial, but cannot simultaneously occur, and a complimentary we have know as by complimentary events for any such pair of the events. Supposing if there is the trailing which that see the events a has occur and you already know that the events that a should not occur that means the other events which are apart from, should occur by this a dash events.

So, the probability of A and probability of a dash combine to be other is more certain that either should occur or other events which are they are apartment from a they should occur the total probability is one that case. So, therefore the way that a should occur really one minus the probability of the compliment a, that is a should not occur in that particular case. So, we have seen this in the example of the single draw where p dash given to be 0.08. So, if it is the one defective and one draw case is 0.08, if it is one good in one draw case then should be 1 minus 1 defective in one draw case that is 0.92. So, the good chance of seventy to percent or excellently complimentary to be defective chance of a 8 percent in the particular difference.

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Laws of Probability

- Two events A and B are independent if the occurrence or non occurrence of A does not affect the probability of B occurring
- Whenever a process produces defectives independently, or at random, so that the probability of a defective on the next piece does not depend upon what the preceding pieces were like, then we have the case of independent events. Such a process is said to be in control, that is stable, even though some non conformity is produced.
- Not all processes do behave in this manner.
- For example: We consider the production of 3000 piston ring castings. The sample of 100 contain 25 defectives whereas the remaining 2900 only 4. This was because the defectives occur in bunches from a certain defect producing condition
- Particularly in this case it was found that the castings were made from stacks of molds, and if the iron is not hot enough when poured into a stack, many castings may be defective. Under such conditions, whether a piece is good or defective does have an influence on the probability of the next one being defective.
- If two events A and B are independent, then we have :
 $P(\text{Both A and B occurring}) = P(A) \cdot P(B)$

So, that is all classify some more laws of probability the 2 events A and B are independent 2, if the occurrence or non occurrence of a does not affect the probability of occurring. So, in that events again, you know let see if you talk about some example whenever a process produces defectives independently or at random. So, that the probability of a defective on the next piece a does not depend upon what the preceding pieces were like, then we have the case of independent events such a process said to being control, that is stable even though some non conformity is produced, let see look at. So, such a process really is said to be in control unfortunately all process not at control, for example let us talk about different situation where, let see we are producing some piston range, you know on the assembly line the about to go to 3000's part of piston range which as be produce. So, it made happened it may, so happened because of the certain fault which is they are in the system the fault to related to stack of molds at situation, when the iron is the not hot, and the etcetera, and this was realized much later down the production in the production of already started.

So, then will be bumping of the effete of the first 100 maybe defectives to and different person that let see around 25 defectives came in the first 100, and then after words when the process go on the ramie in 2900 only 4 defectives. So, there for there is some kind of and independents of this events between first lot really chance called that certain point of kind to be defectives in the taken, because of the defectives. So, such kind of events then can we will have to the independently or is taken publicly, and then our there is the start if A and B are independent of each other this coming of 25 defective lot of 100 or let see

4 out of 29 of 100 are compliantly independence of and the reason then there in the complicit different of each other. So, there for both A and B occurring we should actually the product of the probability of A, probability of B.

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Example of Dependence and Equal Likelihood

- As a second example of probability, let us consider drawing without replacement from a lot of $N=6$ speedometers, of which 1 is defective
- Let N = no. of pieces in one lot and D = no. of defective pieces in a lot
- Now consider the very simple case in which we just draw a random sample of 1 from a lot of 6. Random means that each of the six meters is equally likely to be chosen for the sample. Probability of each is $1/6$.
- There are only two kind of meters "good " and "defective" with $P(\text{good}) = 5/6$ and $P(\text{defective}) = 1/6$.
- Now next consider drawing a sample of $n= 2$ from the lot having $N =6$, of which $D=1$ is defective. This may contain no. of defectives either $d=0$ or 1.
- This is a case of two consecutive drawings which are not independent. Take first the case of the sample yielding no defectives, that is, two good meters. We need $P(2 \text{ good}) = P(\text{good, good}) = P(\text{good on first draw}) \cdot P(\text{good on 2}^{\text{nd}} \text{ draw given good on 1}^{\text{st}} \text{ draw}) = 5/6 \cdot 4/5 = 2/3$

Let now talk about second example of probability, then we talk about the dependence and equal likelihood, let us consider drawing without replacement from a lot of n equal to 6 speedometers lot of, which one is defective n is the number of pieces of one lot. And this case we lot defective size of n equal to 6 and the number of defectives lot is number is one this case one should be re probability right to be consider very simple case, which we just draw a random sample of one from a lot of 6 random means that each of the 6 meters is quality likely to be chosen for the sample probability of each is the 16.

So, there are two kind of seed of meters; one is the good, one is the defective one and you known the such a lots where there us 6 pieces are one is defective ob one 6 is the pres defective and 5 6 is the pra to good speed of miters. So, now we consider drawing a sample of 2 from the lot of 6. So, you want to draw 2, and you known the oral lot size to 6, you have only one fiftieth or 1 6 of the lot complicity defective 5 6 is actually completely. So, what would be the probability of defective. Let see or good lets trying the out of the to draw making our the 6 lows, one of the that could be defective. Similarly one of them good or both of them would defective both of them be good both of them to be defective is not a case of, because there is only one such a defective when the lows. So, you have to eliminate that and you have let with only either both good or one defective one good. So, this is a case of 2 consecutive drawings which are not

independent.

So, take the first case of the sample yielding no defectives that is 2 good meters. So obviously, probability of 2 good is basically to good on first draw good on the second draw as you need there are 5 such good pieces in the sample the first draw exact probability 5 pieces, the second one is 4 fifth because already draw on then. So, the number could reduce 4 and out of which your drawing again we should get one of them. So, have to 5 6 this is actually 2 thread that the probability of to good and will you wanted to do it for, let at the defective we should either as one 6 in to 5, fifth one, because exited. If you had bad or defective draw in the sample also have 5 6 kind of one fifth, you know both of them optically this is good had the good on the first draw bed had the second draw.

So, we would all the result is 1 6 to be same as have in a probability of one defective. So, that is how we basically do the you can see dependent events given one conditions doing the second given them that we would drawing the good your doing, other one trying to draw, so there as to do between earlier and later. So, these are completely different from the independent in the layer side.

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Counting samples (Permutations and combinations)

- In combinations we consider for example we have "n" objects which we can distinguish between.
- Now how many distinct samples, each of one, can we draw from a lot of $N=10$? Obviously the answer is 10. So, we call this a combination of N objects taken 1 at a time, or in symbols:
 $C(N,1) = N$
- Next consider samples of two, from say four good pieces (g_1, g_2, g_3 and g_4)
 $C(N,2) = \frac{N!}{2!(N-2)!} = \frac{4!}{2!2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 6$
- Then the number of distinct unordered samples may be found from the number of distinct ordered samples. For example, for ordered samples:
 $g_1g_2, g_2g_1, g_1g_3, g_3g_1, g_1g_4, g_4g_1, g_2g_3, g_3g_2, g_2g_4, g_4g_2, g_3g_4, g_4g_3$
- The no. of unordered samples are only half as much, that is, six, because, for example, the one unordered pair g_2g_4 corresponds to two ordered pairs g_2g_4 and g_4g_2 .
- Now let us consider lots of 10 distinct pieces. The number of possible ordered samples, each of two is 10*9, because there are 10 choices for the first piece and having made a choice their remain 9 choices for the second piece. So we have 90 ordered samples and exactly 1/2 of this unordered. (45)

So, you can also founds sample using the permutations of the combinations this is the revere of what probability you have knowledge about, but we want to see in case of sample which will just about come in 5 slider. So, in combinations we considered for example, that we have about n object, which we can distinguish between clearly and you

know how many distinct samples, each of one can we draw from a lot of n a big size n equal to capital n equal to 10.

Obviously, the answer is 10, because you have drawing 10 different sample, there all distinct they find from each other. So, you can draw 10 different sample and obviously. So, we call this a combinations of n objects taken one at the time symbolism, we write it at C_n^1 and the C_n^1 typically mean and pictorial by 1P_n minus one pictorial and this result and n basically n here was 10. For example, for you said exactly 10 draw the possible, because there mutually excuse even they are you will there completely, you know insistence object there complete completely different of object we the respect each other. So, next considered sample of 2 from say 4 good pieces g_1, g_2, g_3 .

Then the number of distinct unordered samples may be found from the number of distinct ordered sample for example, for ordered for ordered sample you can either have g_1, g_2 or g_2, g_1 ; these are the essentially same on the thing is the first draw one case you are having g_1 and other case g_2 similarly $g_1, g_3, g_3, g_1, g_4, g_4, g_1, g_2, g_3, g_3, g_2$, similarly $g_2, g_4, g_4, g_2, g_3, g_4, g_4, g_3$.

So, we do not have a president of what comes first in this particular case, and your basically arranging all the sample you certain ordered and certain arrangement, so but then essentially when you are drawing. And you talk about another samples you essentially meaning the same when your drawing g_2, g_4 and g_4, g_2 that use in one case you drawing g_4 , but it is the head of g_2 in other case g_4 completer. So, in that ordered dish matter to us in the case of selection, then we represent it little bite different manner. So, you can see hear that if you talking about just we the represent of all this order the sample there is a essentially 12 such sample which are there, but if you are talking about the another sample there only 6 such sample at all here, because your just by changing the sequence what come first and what come next.

So, in one case that is the order sample and you are drawing to out of a lot size of 4. So, C_4^2 would typically correspond to factorial 4 by to factorial 2 factorial and that. So, this is $4 \times 3 \times 2$ which is actually equal to 6. So, this C for 2 then there for represent unordered sample the order is not important in that sample and the similar kind of the thing. So, if you talk about out of 4 out of 2, you know we talking about the order sample were we talk about just 12, which is the actually this many 6 which all being clear in the process of the draw. So, now let us consider lot of 10 distinct pieces number of possible

ordered sample, each of 2 is 10 into 9; that is ninety because there are 10 choices for the first pieces and having made a choice their remain 9 choice for the second pieces; obviously, 10 into 9.

So, 90 ordered sample and exactly half is number is when her having unordered that mean the sequence is not important, and which your selecting that time you are having exactly 45 in this and this you can represent 1 2 computation. So, the ordered sample always expressive to combinations and the unordered sample are always extraneous combinations, you already probability ignore the form here basic class knowledge about how you know a partition can be n pictorial by n pictorial minus 1, and the combinations form the n pictorial divided by i 1 pictorial minus 1 pectoral.

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Counting samples (Permutations and combinations)

- Now let us go to a sample of 3 from a lot of 10. The no. of distinct ordered samples is $10 \cdot 9 \cdot 8$: 10 choices for the first, 9 choices for the second and 8 choices for the third. But now six of this ordered samples correspond to just one unordered sample. For example: g1g4g6, g1g6g4, g4g1g6, g4g6g1, g6g1g4, g6g4g1 all correspond to g1g4g6 unordered sample.
- Hence the number of distinct or unordered samples or combinations is $10 \cdot 9 \cdot 8 / 6 = 120$
- Ordered samples are also called permutations and they are calculated by the general formulae $P(n,r) = \frac{n!}{(n-r)!}$ In the earlier case $P(10,3) = 10! / 7! = 10 \cdot 9 \cdot 8 = 720$
- We call the number of distinct unordered samples a combination and is given by the general formulae $C(n,r) = \frac{n!}{r! (n-r)!}$ In earlier case this would be $= 10! / 3! \cdot 7! = 10 \cdot 9 \cdot 8 / 3 \cdot 2 \cdot 1 = 120$.

So, having said that this is a step to what is a counting of sample particularly, when you walk about exalting sampling and already thing describer this p and r and pictorial by n minus r pictorial, you can the present either and this manner or the manner showing before. An; obviously, would be then n pictorial by r pictorial by n minus r pictorial. So, formulas is very clear then how you calculate the different ordered, and unordered samples as I told in the last slide discuss in the last slide.

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Counted data: Defects and defectives

- Inspecting or testing 'n' pieces, we may search for defects or non conformances in the 'n' pieces and record the total number of such defects. This is measuring quality by a count of defects.
- In inspecting or testing 'n' pieces, we may consider whether each of 'n' pieces does contain any defects. Each piece having one or more defects is called a defective. This measure is known as the count of defectives.
- We shall consistently use the following symbols here and in later discussions:
 - n = number of units in a sample
 - d = number of defective units in the sample of n units
 - $p = d/n$ = sample of fraction defective = proportion of defective units in the sample.
 - $q = 1 - p$ = sample fraction good
 - $d = np$ = no. of defective units in the sample

Binomial distribution for defectives:

Suppose we had a process with population fraction defective of $p = .10$. For a sample size of 4 there could be

Either 0, 1, 2, 3 or 4 defectives can exist.

So, when we talk about inspecting or testing n pieces are, and sample we may search for defects or non conformance in the n pieces, and record the total number of such defects this is measuring quality by a count of defects inspecting or testing of n pieces, we may consider whether each of n pieces which may not contain any defects. So, there all good pieces or maybe that pieces, which are actually defectives each pieces having one or more defects is called defectives this measure is known as the count of such pieces are known as the defectives. So, we will concerns consistently use the following symbols here probably and also in later discussions, then n is a number of units in a sample d is the number of defectives units in the sample of n units.

So, basically d by n is the fraction defective and then you can represent that by the term q , sorry p which is the sample of fraction defective or proportion of the defectives in the sample, and then you also represent q equals one minus p that is the sample fraction good. So, if there are exactly d defectives; obviously, n minus d are the good one. So, n minus d is the percentage of n is basically the q value, which is the sample fraction good and when you talk about the number of defective units you basically multiplying just n by n time of p to obtain the number of defectives units. So, basically of define the acceptance of percentage in a manner that if you have an idea of the initial sample size for which you are actually drawing a lots size, which you are drawing just multiplying the fraction defectives with respect to that lot size would give you the number of defectives is exactly, and similarly the number of goods also. So, suppose we had a processes with population which had a fraction defectives of 10 percent; that means, out

of hundred 10 are defectives and for a sample size of 4 there could be either you know 0 defective or one defective or 2 or 3 or even 4 defectives which can exist we can simple.

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Binomial distribution for defectives

- First find the probability of drawing a sample with all pieces good.

So for $P(d=0) = P(4 \text{ good}) = [P(\text{good})]^4 = (0.9)^4 = .6561$

Thus about 2/3 of the time, we draw a sample of $n=4$ pieces from the process, all four will be good ones.

- Now next we seek $P(3 \text{ good, 1 defective})$.

$P(3 \text{ good, 1 defective}) = P(g,g,g,d) + P(g,g,d,g) + P(g,d,g,g) + P(d,g,g,g) = (.9)(.9)(.9)(.1) + (.9)(.9)(.1)(.9) + (.9)(.1)(.9)(.9) + (.1)(.9)(.9)(.9) = 4(.9)^3(.1) = .2916$

- Next consider samples with $d=2$; they have two defectives and two good ones. How many distinct orders are there for such samples? Six: ggdd, gdgd, gddg, dgdg, dddg, dggd, the probability for each one of these sample outcomes is $(0.9)^2(0.1)^2$.

$P(d=2) = 6(.9)^2(.1)^2 = .0486$

- Similarly for $d=3$ there are only 4 orders of sampling results

$P(d=3) = 4(.9)(.1)^3 = .0036$

- Finally for $d=4$, all four must be defective

$P(d=4) = .0001$

There are present this processes by looking at the various probability let us say for 4 good pieces or 0 defective pieces, you have exactly 0.9. So, therefore, ninety percent of the goods are completely defect free. So, 0.9 to the bar of 4; that is 6 5 6 1 about 2 third of the time we draw a sample n equal to 4 pieces from the process all 4 will be good ones. Now next we seek p 3 good defective 3 good and one defective. So, this could either be the first one good second one good third one good and the 4th draw defective or the first one, second one, both good. And then the third draw defective and the 4th one good again first one good second one defective and third and 4 th; both defective good pieces, so also 4 th.

So, we can actually multiply the probabilities because they are all sort of independent of each other if the good are not consecutive in nature, it can happen any draw can happen at any time. So, it results in 0.2916 or 29.16 percent probability of having 3 goods and one defective, you could do the same for d equal to 2 and d equal to 3. And finally, d equal to 4 and d is probability is come out to be 4.86 percent, 0.3 percent, 0.36 percent and 0.01 percent respectively. So, we will kind of round of this particular module and the next module we will see how using combination, we could arrive at the same probability just in a way I have shown the earlier example, and after all these example we will try to look at the distribution that is being made and can be related the combinatorial with the distribution. So, that we can bring out an overall, you know way of representing the

acceptance sampling method in terms of fraction good and fraction defectives. So, that at one go you can do the calculation straight away and find out what is the overall probability in that particular case or situation. So, having said this will close this module and then in the next module will just wait on or will discuss the remaining portion.

Thank you.