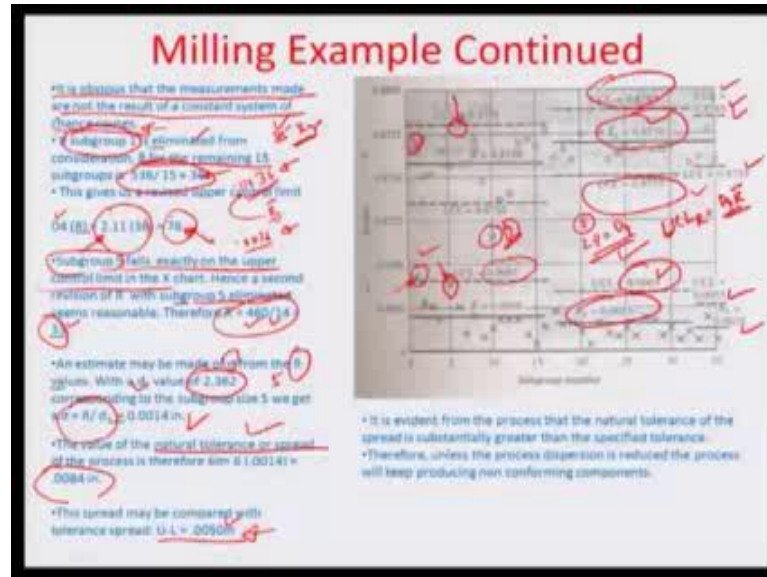


Manufacturing System Technology - II
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Module – 21

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Hello and welcome to this Manufacturing Systems Technology part two - module 21. We would like to now see given this control chart and given some of the out of control points can we really replot, so that we processes can show better control. So obviously, the first thing that we have to understand from our discussion in the last module was that it is not it is obvious that the measurements made are not the result of a constant system of chance causes. If subgroup one for example, which corresponds to R bar falling out of ranges eliminated from consideration, the R for the remaining becomes equal to 536 by 15 that is 36, so we are removing the 85 range which is out of the upper control limit of the ranges in order to replot. So, this actually gives up about 0.0036 has the new R bar which can again be because the subgroups size is not changing used to calculate the upper control limit. So, in this particular cases the upper control limit becomes 76 if you have eliminated the out of range point data point which is out of range corresponding to the subgroup number one.

So obviously, if we re plot the system now, the upper part again change because of that; and instead of this 0.0082, it would be 0.0076. So, let us actually look into the chart and

see what are the other out layers and then we get rid of those out layers and again tried to replot. So, as you know that in this particular cases we had seen the subgroup five falls exactly on the upper control limit of the x chart. So, lets say this subgroup here right here I am sorry this is not the one. So, the subgroup five is here falls exactly on the upper control limit and it can be rejected because this is actually very shady situation which can just about go a cross the limit for a little bit of processes the control. So, we eliminate this subgroup five. So, a subgroup five was eliminated the new range that can be computed is 460 by 14 now and this becomes is equals to 33.

So, an estimate can be made of sigma from R obviously, from control table with the corresponding five subgroups size the you know sigma and R can be related by $R = d_2 \sigma$; where d_2 value is about 2.362 from the control table, so that is 0.0014 inch that is the sigma. So, if I consider this sigma so obviously the six sigma is about 0.0084 inches and that the natural tolerance or spread of the processes and the spread may be actually compared with respect to the tolerance spread which is the upper and lower control limits sort of taken of from each other which happens to be 0.0041 inches in this particular cases. So, what are would like to do now is that you know just because the new range happens to be 33 while eliminating the subgroup number five, and subgroup number one because of their out layer properties and also you can actually eliminate this subgroup right here which goes out of the x chart does well.

So, the new limits by the eliminating these three control points that is subgroup one, subgroup five and then subgroup nine, which actually is completely out domain after the controlled chart we get fresh control limits. So, in our case the mean of means in this case if we remove the x bars from all these three subgroups are 0.8770, so obviously, the UCL and LCL are getting modified in the case of x chart. Similarly the range as you can see here from the range chart because you know we have completely eliminated the out layer which was there. So, this particular thing this particular range value which was on the sort of edge of the upper control limit, and this particular range value which is above the upper control limit have been eliminated, so that corresponds to a new range bar of 0.0033 and obviously, and to multiply with 2.11 to have 0.0069. So, 2.11 is corresponding to subgroup size five is the sort of d_4 value which is needed for estimating the upper control limit - UCL of the range chart, so which is $d_4 \bar{R}$ you know, so that how ((Refer Time: 05:15)) postulated the a plotting of the limits. So, here this is the new range value.

And then obviously, if you wanted to again neglect the point which is the ninth subgroup again the new range comes out to be 0.0026, and the upper control limit comes out to be 0.0055. Similarly, this particular case you can see that the new UCL and the LCL have changed quite a bit from 0.8755 and 0.8785, so that is about close to of 0.0030 inches spread in the upper and lower control limit of the \bar{x} bar. So, that is how you keep on modifying and changing the control chart it relatively as the situation may be. And finally, you are high vat by removing all the out layers, the exact control limits and then assign the out layers kinds comes you have to sort of a revised the control chart, so that the control chart can go for the exact processes held and does not really get offset because of the processes reject, which are never a part of the system. Obviously, if you have controlled, if you have eliminated something from a system that should not be used as a production data, so that part of that subgroup is completely neglected from being question.

So, it is a evident from the processes that the natural tolerance of the spread is substantially greater natural tolerance is actually six sigma which is 0.0084, and the specified tolerance limits here as you can see is point only 0.0030, you know these are the modified tolerance limits as you can see here at the end when you eliminate All the three subgroups you know this particular tolerance limit. So obviously, there is going to be quite a bit rejection in this case because six sigma is quite out site 0.0030 limits which is actually the processes spread.

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Revision of theory of Probability

Definition:

- Probability is concerned with the likelihood of an event occurring. The scale of probability of an event varies from 0 to 1.
- If an event cannot occur on a trial, then the probability of its occurrence is 0.
- If another event is certain to occur then its probability of occurrence is 1.

•As an example suppose that a trial is drawing a piece at random from some production line, and that the event in question is that the piece drawn is a defective or non-conforming one.

•Let us suppose that the probability of a defective is 0.05. This means that 5% of the time when we draw a random piece from the line, it is defective.

•The complementary event is that the piece drawn is a good one. It's probability is .95.

$P(\text{good or defective}) = 1$

Occurrence ratio

•Suppose that we have a production process for which the probability of a piece containing at least one "minor defect" is constantly 0.08.

•We then say that the probability is 0.08 for a "minor defective".

•Now what happens to the observed proportion of minor defectives as we continue to sample? This observed proportion of minor defectives is what we know as the occurrence ratio.

And I would say that the process dispersion as to be reduced in order to keep the

processes from producing non conforming components of parts, so that is what about you know that is all that I have about the \bar{x} and the \bar{R} chart which are one of the big reference in the industry for studying the processes control. We will actually now go to other aspects of control chart which is about the fraction defectives. So, there are sudden parameters of quality which cannot be given to by a particular unit or a value or a dimensions or a quantity. And in this case a for example, let say visually how a product looks you know if never quantify able with any measure. So, there can be some kind of a acceptance sampling in that case where we are talking about either a complete elimination by looking at the product which does not ((Refer Time: 08:15)) to a quality characteristics or accepting the product based on the perception of the customer. And there we need to understand acceptance sampling we also need to understand the probability of accepting a certain product rejecting.

And so we will do some basic revision of the theory of probability before beginning you know this kind of sampling. And in fact this sampling is also plotted as what is called as c chart which will come later once we are across all this theory a probability and estimates which would be needed for predicting the acceptance level of a product. So, what is probability, so definition probability is like the percentage chance of occurrence or may be lets put it as a likelihood of an event to occur. And as you know the scale of probability is a fraction, so it varies between zero and one. So, even cannot occur a definitely on a trial if the probability of occurrence is zero, but if it almost certain to occur you can say that occurrence probability of occurrence is one.

So, you can take an example for example, a trial is drawing a piece of piece at random from some production line and that the event in question is that the piece drawn is a defective or non conforming one. So, let us suppose that the probability of defective is of a five percent, so therefore, in 100 parts of 5 parts may not need the quality specification. So, it means that 5 percent of the time when we draw a random piece from the line, it will be defective; in 95 percent of the times, if we draw a random piece from the line it will be non defective or conforming to this specifications. So, the complementary event is that the piece draw is a good one, and the probability in that case is 1 minus 0.5 that is 0.95 or 95 percent, so that is straight away what probability would mean in terms of acceptance samplings.

So, what is occurrence ratio. So, suppose we have a production process for which the probability of a piece containing at least one minor defect is constantly 0.08 or 8 percent.

So, then say that the probability is 0.08 for a minor defective. So, now, what happens to be there to be observed proportion of minor defectives as we continue to sample, this is observed proportion of minor defectives is what we know as the occurrence ratio which I will slowly delve into what really it means in terms of big samples or small samples.

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Theory of probability

- Let p' = constant probability of a minor defective
- Where the letter 'p' means the probability and the prime means population probability.
- d = no. of defectives observed
- n = no. of pieces inspected or tested
- $p = d/n$ = sample proportion of defectives
- Let us see how $p = d/n$ behave as we sample more and more, that is, increase n .
- Would we not expect that the observed occurrence ratio $p = d/n$ would tend to approach $p' = 0.08$?
- We start with five samples of 10, then samples of 50.
- The first 2 columns are for current sample of 10 or of 50.
- The 3rd and the 4th column are for the total sample size and the cumulative total no. of defectives.
- The 5th column is based on the 3rd and 4th columns and gives the current overall proportion defectives and the occurrence ratio (total defective/ total inspected).

So, let p' be a constant probability of a minor defective, where the letter p actually means the probability and the prime means the population probability. So, p' is actually the constant probability that minor defectives as regards the sample. So, in terms of values what it means is the d says these the numbers of defectives observed, and n is the number of pieces inspected or tested. So, the p should really be equal to d by n this is sample proportion of defectives. Now let us see how p equal to d by n behave as we sample more and more data or let say if we increase this n value then does it change does it really remains same lets just observed that. So, the occurrence ratio p equals to d by n would tend to approach p' as we have n going to very large value. So, the p actually that is the probability of the sample would approach the probability of the population of you know having a minor defect which is 0.08 as this n size would increase or n would be large. So, we start with say five samples of 10 and then samples of 50, and we sort of see what is the percentage acceptance and mention this in a columnar manner.

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Sample	D	Σn	Σd	$p = d/n$
10	0	10	0	0.000
10	1	20	1	0.050
10	1	30	2	0.067
10	1	40	3	0.075
10	0	50	3	0.060
10	5	100	8	0.080
10	4	150	12	0.080
10	6	200	18	0.090
10	4	250	22	0.088
10	8	300	30	0.100
10	1	350	31	0.086
10	3	400	34	0.085
10	5	450	39	0.087
10	3	500	42	0.084
10	5	550	47	0.085
10	5	600	52	0.087
10	5	650	57	0.087
10	4	700	61	0.087
10	1	750	64	0.085
10	4	800	70	0.087
10	6	850	74	0.087
10	1	900	81	0.090
10	4	950	85	0.089
10	4	1000	89	0.089

The proportion defective only tends to approach $p = 0.08$.

Sometimes it gets closer, sometimes it backs away from p .

Before the total sample size was 100, the occurrence ratio was below .08.

Between 100 and 150 it was 0.08 and thereafter above.

Principle:

We can say that the $p = d/n$ is an estimate of the constant probability of population p . How close the estimate will be depends on sample size n , the value of p and also upon chance.

So, let's look at what they are. So, the sample here is basically given the number of pieces which are the sample. And the d here is actually referring to the number of defectives, and obviously the Σn and here talks about the cumulative numbers and Σd here talks about the cumulative number of defectives at this way we want to find out what is the occurrence ratio of this population. So, for example, in the first sample of ten the defectives are zero, so obviously, the Σn is ten and Σd is zero, but in the next sample of ten we have actually picked up 10 plus 10 - 20. So, the sample size increases to 20 now Σn is 20. The number of defectives here is zero plus one, so that is about the increasing the defectives Σd is 1. And so when another 10 are drawn to the sample size would become 30 and the number of defects would become 1 plus 1 plus zero there is 2 and. So, this way we are plotting or we are trying to record all these data of the defectives in each draw are in each circumstance on the total number of samples in total defectives.

So, in our calculate, the occurrence ratio which is represented by the Σd by Σn assume as see here that the probability of the sample p is point zero and first case 0.00, 0.50 in second case 0.667, 0.075, 0.0600, 0.800, 0.0800. So, basically what you are seeing here is that as the value is increasing in size, the probability is slowly approaching you know the probability of the population. So, the proportion defective only tends to approach $p = 0.08$, sometimes it gets closer sometimes it backs away from p , before the total sample size was hundred for example, in this particular case the occurrence ratio was below the population size of 0.08. Between 100 and 150, let's saying

these two observations the p dash was replicated in both the cases point 8.8 and there after above.

So, principle that we sort of draw from all these is that we can say that p equal to d by n is an estimate of the constant probability of population p dash, how close the estimate is will depend on the sample size, and the value of p dash and also upon some chance causes that is how your explaining the probability of occurrence or likelihood of a minor defect in the sample. So, having said that there are certain laws of probability that we may need to discuss here because we will start calculating probability just in about few minutes.

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Probability laws

- Consider again the production line producing pieces with a constant probability .08 of the piece being a minor defective, and such that each piece is independent of the other produced.
- This means that the probability of the next piece being a defective is 0.08 and the piece being good is .92 irrespective of the preceding piece.
- Now let us suppose that we draw a sample of two pieces and inspect them.
- The outcomes are that the sample may contain 0, 1 or 2 defectives. Let us find the probabilities of this outcome or events.
- For the probability of the samples containing no defectives, we must have good pieces on both draws.

$$P(2 \text{ good pieces}) = (.92)(.92) = 8464$$

$$P(2 \text{ defective pieces}) = (.08)(.08) = 6464$$

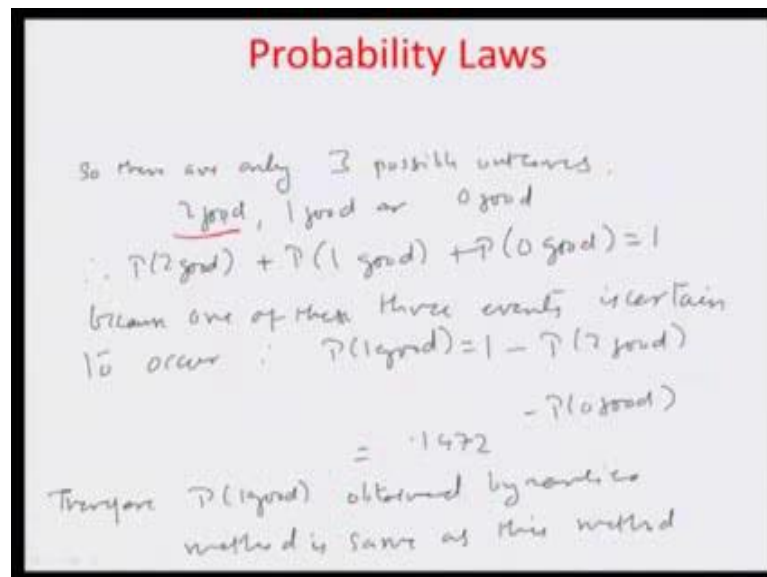
$$P(1 \text{ good piece}) = (.92)(.08) + (.08)(.92)$$

So, consider again the production line producing pieces with a constant probability 0.08 of the piece being a minor defectives and such that each piece is independent of the other produced, so this means that the probability of the next piece being a defective is 0.08 and the piece being good is 0.92 irrespective of the preceding piece whether it is a defective or a non defective. So, whenever you are drawing sequentially that is would be the probability every time or every draw you make. Obviously, you may fall within the point two nine two category and you may fall within the 0.08 category the percentage of being within the point note to 0.92 category is much more in comparison to that within 0.08 category.

So, now let us suppose that we draw a sample of two pieces and inspect them the outcomes are that the sample may either contain zero defectives that means, both of them

are within 92 percent category or one defectives one non defectives or both the defectives, this is what can be happening. And let us find out the probability of this outcome or events. So, for the probability of the samples containing no defectives, we must have good pieces on both draws. So, the probability of two good pieces would therefore, be 0.92 times of 0.92 these are two successive draws which we are talking about. So, it is about 0.8464. When we are talking about probability of two defective pieces again 0.08, 0.08, so 0.0064. And then you have talking about one good piece one defective piece. So, whether it is a first good piece second it doesn't matter it basically point nine two into point zero eight whole twice, so, that is about 0.1472.

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And so in general, so there are only three possible outcomes as you see here one two good one good or zero good, and therefore, it would be really a summation of all these probabilities. So, either it is two good or one good or zero good and summation as we can see also is actually equal to one; this being as we calculated earlier 0.8464, this being 0.0064, this being 0.1472. So, this all sums up to be a 1. So, that is how it is. So, because one of these three events is most certain occur therefore, probability of one good can be regarded as one minus probability of two good minus probability of zero good that is 0.1472 that is how the proportionate has been drawn here. So, therefore, probability of one good piece being obtained by earlier method is same as this method, and so whether you go by just laying out this logic and trying to calculate based on all these both the answers converge to the same values.

So, the law of probability is really sort of organized as far as the acceptance sampling in

this particular manner. Now I am going to close on this module in the interest on time, but in the next module, we are going to talk about several different issues of probability like for example, if there are mutually exclusive event which are going to happen then what would be the overall probability in terms of acceptance sampling, we would also be discussing about the example of dependence and equal likelihood some events occurring. And then again when we talk about large size distribution how we converges all these probability distributions towards the binomial distribution to estimate what is going on in terms of chances toss when we talk about acceptance sampling. So, all those theories we will get into in the next module.

Thank you.