Manufacturing Systems Technology

Prof. Shantanu Bhattacharya
Department of Mechanical Engineering and
Department of Industrial and Production Engineering
Indian Institute of Technology, Kanpur

Module - 02

Lecture - 09

Hello and welcome to this module 9 on Manufacturing Systems Technology and this would be probably one of the concluding modules on how to do you know the curve fitting and complex topology etc. So, a quick recap of what we did in the last section or last module.

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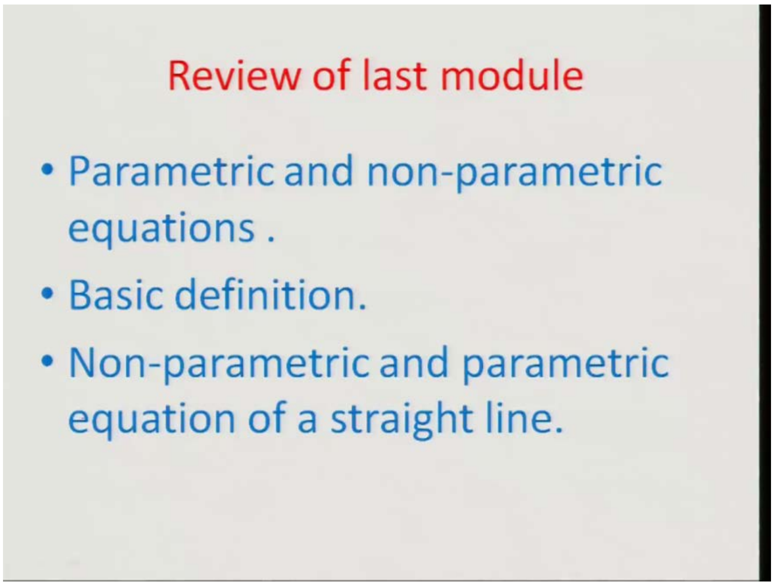

So, we talked about the various forms of representing equations as parametric and non-parametric forms.We further mentioned that these can be either implicit or explicit depending on the relationship being a direct one between two variables or relationship being an indirect one because of may be multiple equations,multiple variable kind of problem. So, we also learnt about how you know with parameterization,we can actually vary the curvelocally given a global relationship between two or more parameters like xyzso onsoforth which describes the 2D or 3Dcurve. So, particularly we studied the case of a straight line where introducing an extra parameter t.They would allow us to vary you know the fit of the curve between the various local small domains while the global domain of the straight line remains the governing basis for variations of such local domains.

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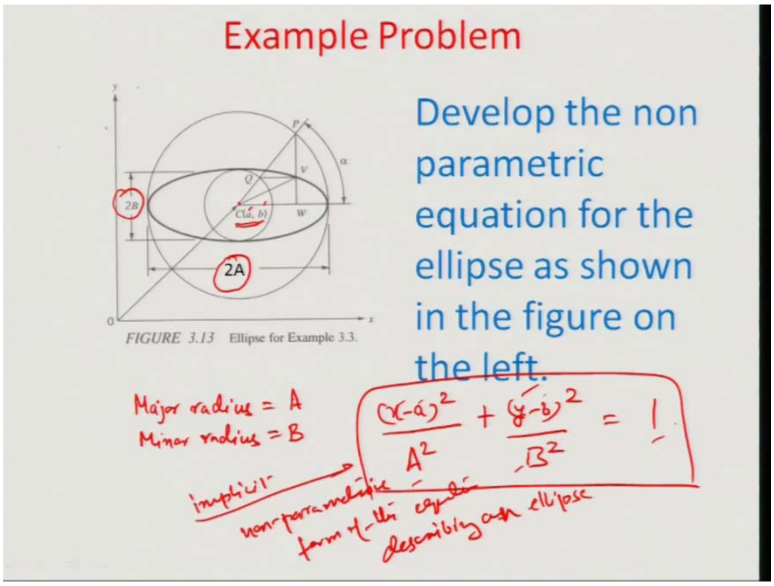

So, we would like to just analyze a few more example problems today.In this matter one of them is this ellipse. So, you can see that there is an ellipse in this particular you know equation, where we want to develop the non-parametric form of the equation and also, the parametric form. So, if we look very closely at the ellipse, there are various parameters which are mentioned here.One is this major radius a of the particular ellipse and the minor radius b. So, the major radius, this particular case is capital A and the minor radius is capital B. So, also important is this point C which is actually the geometric center of the particular ellipse where there are location coordinates small a and small b representing the c.As you may reckon from first level coordinate geometry,the equation for such an ellipse would be x minus small a square by square of the major radius plus y minus small b square by square of the minor radius equal to1. So, this becomes then implicit non-parametric formulation of the equation describing an ellipse.

So, obviouslyimplicit is because there is a clear cut relationship between x and y in terms of some nodes ab small or AB capital,and1 and non-parametric because again the governing relationship is directly between x and a, x and y without involving any extra parameters.

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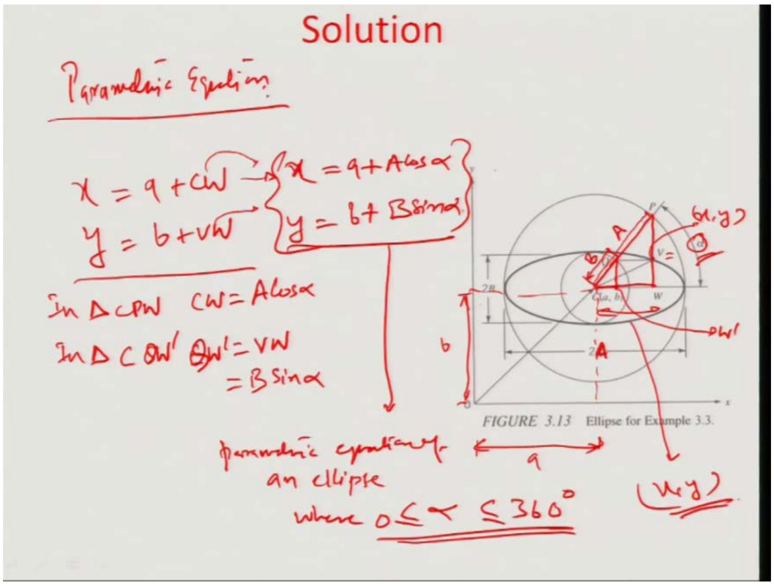

Let us now plot the parametric equation of the same ellipse and if we again look very closely here if we are talking about an array of points,let us say xy somewhere on the periphery of the ellipse or somewhere on the let us say the trace of the ellipse.The xy can be defined in terms of the already existing location coordinates of the point c or the centroid cwhich is at a distance of let us say a small a from the origin and small b particularly in the y axis from the origin. So, we can define the x coordinate of the point vat this particular v as equal to really you know a plus cw which is this particular length.That is how you can define the x coordinate of v.Similarly,y can be defined as the expression b,where somewhere around here plus the amount of distance which would move by translating w all the way to v. So, it is plus vw. So, that is how x and y can be defined and let us see if we can in process of defining this c and w involve an extra parameter alpha.

For example, in this particular case where alpha is an angle,we can vary between 0 and 360 degree to define the whole ellipse.So, obviouslyit can be because if we look at cw, so in triangle cbw we can say that cw is nothing, but the major radius which is actually capital Ain this particular case as can be geometrically visible here.This is a,so can be geometrically visible here. So, a times of cos of alpha and therefore, the simultaneous the x coordinate can be defined by small a plus capital A times of cos of alpha.Similarly in a similar manner if I look at the triangle,much shorter triangle,let us callthis point w dash. So, we are looking now at a triangle.In triangle c q w dashif I were to look at the total amount of displacement q w dash, that would be equal to obviously vwand qw dash. Further in this particular triangle can be mentioned as the smaller or the minor radius of the ellipse be sin of alpha, ok.

So, therefore, y can be represented as b plus small b sin of alpha. So, these two then formulate theso-called parametric equation of an ellipse,where the alpha now can be varied between 0 and 360 degrees to produce the whole trace of x and y on the variable point on the ellipsesurface. So, in a way that is how you represent both the implicit parameter non-parametric and the parametric form of equation for a particular ellipses shape. So, today you know after doing all these different regular geometric problems, I would like to move on a little bit and give you an illustration ofhow to really represent a surface through synthetic curve.

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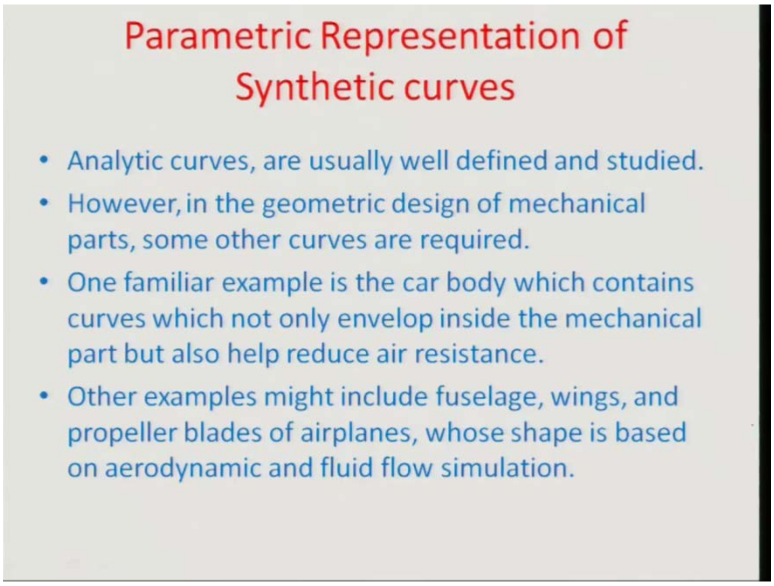

Now, what is the utility for synthetic curves?Let me just come to that explanation first. So, you know that in modern day engineering, there are a huge variety of surfaces which have complex topology.For example, there can be you know the propeller blades of an airplane and the fuselage which is for extremely complex shapes, the wings of an airplaneso onso forth.For example,you know in flow assistance in aerodynamic flow assistance for compared to moving bodies like let us say automotives or even airplanes,alot of complex topologies are involvedwhich is otherwise not very easy to plot using regular describable geometric parameters like lines, arcs, curves, circles, ellipse,parabola, hyperbolaso onsoforth. So, therefore, the question of representing them more accurately comes into existence when we talk about synthetic curves to define such geometric parameters.

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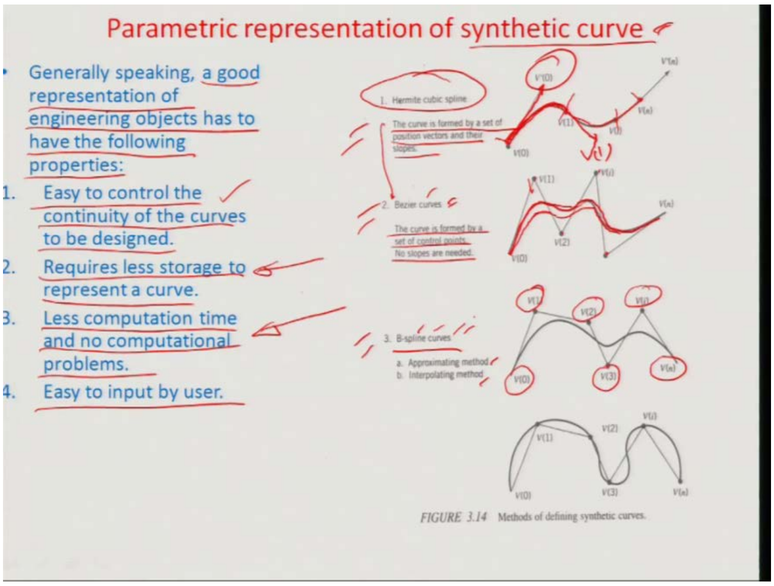

So, I would like to now show how to represent using synthetic curves the complex topologies or shapes. So, before doing that let us say that you know first of all a good representation of engineering object has to have at least the following properties. So, what are those properties?One of them is easy to control.You are having curves to control the topology.For example, let us say when we are talking about Harmet cubics line kind of a curve.You can see there are various segments of these curves which are sort of interconnected along these particular points. There are small curves which are representing the actual topology.

Now, the question is how much would be the variability in this local domainstarting for all these different points,so that there can be a good shape given to the overall fitment of these curve components to a particular topology. So, it should be easy to control the continuity of the curves.It should not be that between these two points.There is some discontinuity which exists between the curves.

Similarly, it should require lesser storage particularly data storage when you are talking about representing of the curves.Obviously, they should have less computation time and absolutely no computational problems as far as you know the individual components of these curves are concerned and then,of courseeasy to input by the user. So, these are the properties that should be there in the synthetic curve,so that they can match the complex topology cases that we are considering. So, there are at least three different kind of you know formulations which are there, and essentially it falls downto fitting of local sections of the whole complex topology for you know governing the whole nature or array of such curves which are representing the complete surface or complete topology.

So, they are Hermite cubic,Spline-Bezier curves and B-Spline curves and briefly the Hermite cubic,Spline fit is sort of a family of curves which is formed by a set of position vectors and their slopes at least the end position vectors,and the end slopes are important for varying the topology along this particular domain of the curve.The Bezier curve is formed by a set of control points, no slopes are needed. So, in away it is handy for example, in this case you can see there are different control points over which this complex surface topology has been developed by just variation of these control points.The idea here in the Bezier curve is that practically speaking the slopesat certain points are very difficult to measure.For example, how will you measure this slope at v dash 0 or for example, v dash 1at this point? So, Bezier has an advantage over Hermite in that.It does not need theseslopes.It only needs this position vector on the points itself.

Finally, we see plain curves where variation of these points would also change the convexity concavity part of the curve.For example, you know if you compare with Bezier,the difference between this plane and Bezier is that if this point were to vary may be the curve will vary in terms of the bulge without really having a cross-over from a convexity into a concavity, but in this particular case you may have that additional advantage of varying it locally even in its curvature you know apart from the other parameters.

So, we will justinvestigate some of these curves spacing as we go along, but for you know plotting a surface one part of the course is the engineering curve that is used to plot the surface.The other part is how we are able to connect such curves to each other in a continuous manner or in some kind of a relationship,sothat there is a sort of complete mapping you know without any breaks or without any discontinuity of the surface which is a concern.

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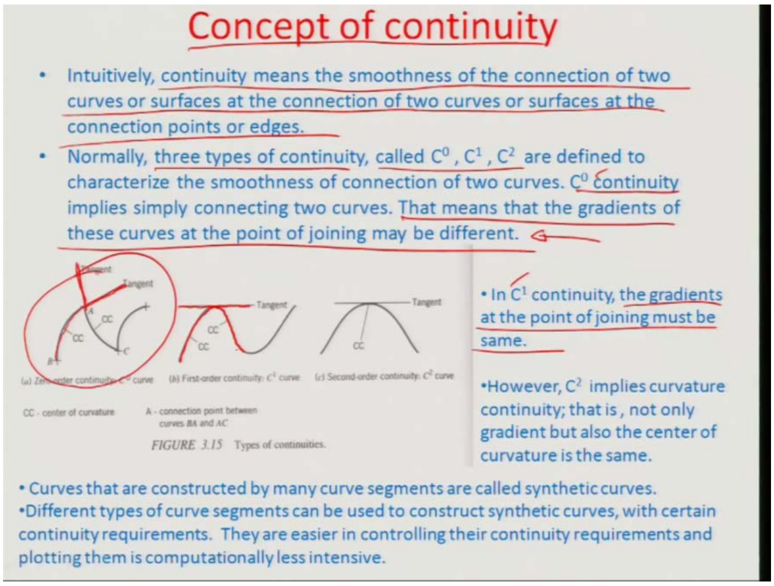

So, we want to now first investigate this concept of continuity for plotting such curves intuitively.Continuity means the smoothness of the connection of two curves or the surfaces at the connection of two curves or surfaces at the connection points or the edges.Normally three types of continuity are existingand they are called c0,c1 and c2continuity.C0 continuity as can be seen here implies simply connectingtwo curves that you know the two tangents of the two curves should intersect to each other at one of the points which would mean that the curves are in physical touch with each other, and physically they are continuous. So, the gradients of these curves at the point of joining may be different, ok but they are essentially intersecting.That is the basis of c0 continuity.

The c1 continuity again has the gradients at the point of joining as the same. So, you can see the tangent at the point of joining of this section of the curve is slope wise similar to the tangent at this section of the curve. So, that is how c1 continuity is defined.Now, c2curvature continuity would mean that typically not only tangent, but the center of curvature would be uniquely similar in case of first order continuity.The center of curvatures may be different for example, in thesecases you have two different center of curvatures for plotting these two curves with only a condition of the tangent or the slope being similar at the line at the point of intersection, but here in this case in the second order, you can see that not only the tangents are similar, but also the curvature is similar.That is how you do the c2curvature continuity,ok.

So, there is essentially c0, c1 and c2, three different forms of continuity which can ensure that two such sections or two such synthetic portions of the curve can be connected to each other in some relationship or some governing relationship which can actually map the whole surface continuously without any breaks, ok.

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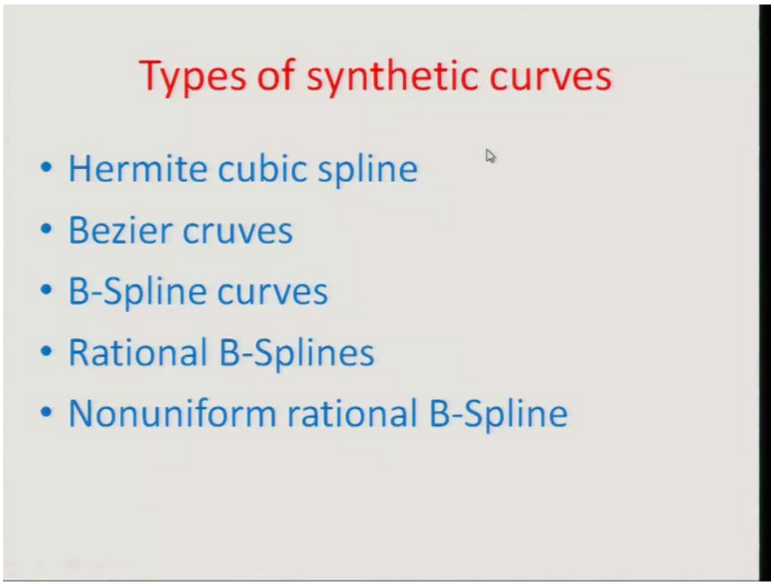

So, in a nutshell the different types of the synthetic curves that we would like to just explore are the Hermite cubic, Spline, Bezier curves,B-Spline curves, the rational B-Spline and non-uniform rational B-Spline.We would give emphasis to the first two kinds with a description of the third kind.In the interest of the timewould sort of keep the rational B-Spline and non-uniform rational B-Splinefor just you know reference,so that you know if you are interested as a reader you can go through that. So, we in the next module would like to take up these two different Hermite cubic spline and Bezier curves in some details,and try to do some geometric fit with these two curves.This brings us to the end of this module and I would like to take up the various curve fittings,the actual curve fittings starting next module.

Thank you so much.