Manufacturing Systems Technology
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Module - 02
Lecture - 08

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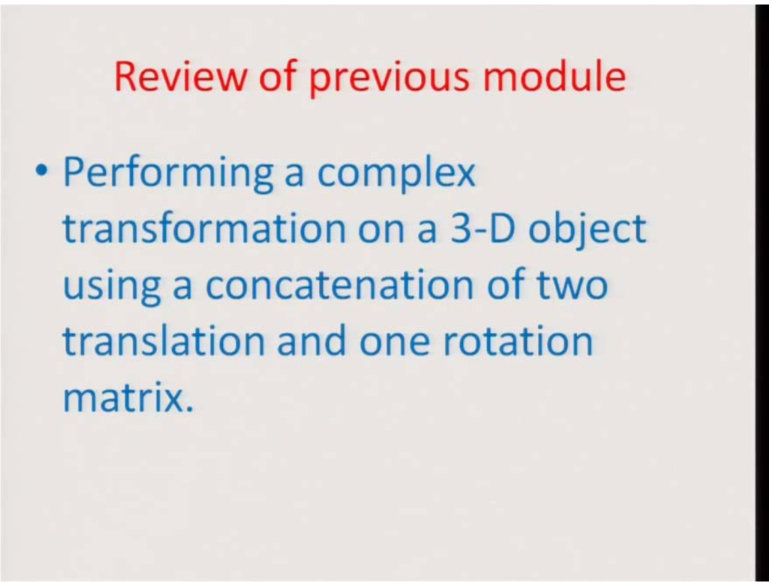

Hello and welcome to this module number 8 on Manufacturing Systems Technology.A quick recap of the last module,we actually discussed in the last module how to perform a complex transformation process on a 3D object with respect to an orthogonal coordinate system using a concatenation of two translations and one rotation matrix.The other important aspect that I would like to cover today is that you know geometries can be plotted using simple lines and arcs and curves and regular shapes, but they cannot be really estimated when the topology is that you are bornmapping is very complex in nature.

So, in this particular module I would like to look at how to represent such curves and what are the flexibilitieswhich are available when you try to force fit a polynomial function to a certain region of the curve,so that on a very local basis I could have a proper fit of a particular region.The essence of all this is that in a computer aided design, it is always important sometimes to be able to estimate the exact topology even if it is very complex. So, instead of having one whole regular geometric feature to be able to represent the whole topology, it is a very good idea to split up the whole geometry into small synthetic curves which are again joined end to end in a manner, andso that the whole topology can be mapped in a very accurate manner,ok.

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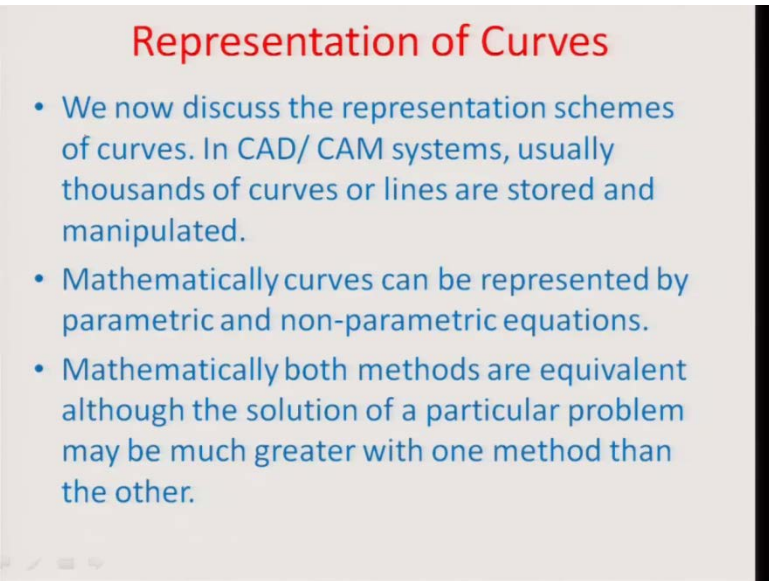

So, we now discuss the representation schemes of the curves that we have been talking about and obviouslyin CAD/CAM systems, usually thousands of suchcurves or lines are storedand manipulated,sothat you can have various objects of all different complexities topologically being mapped as in this particular case we are studying.

So, when we talk about the mathematical representation of a curve, a very simple representation can be just in terms of how the coordinates are related to each other.For example, let us say if you have a straight line.We are talking about a straight line y equal to x plus 1.So, obviously there is a relationship governed between x coordinate and y coordinate of all points which will lie on that particular straight line and that governing relationship would be y equal to x plus 1.So, it is a very simple way of looking at just by looking at a relationship between different coordinates.

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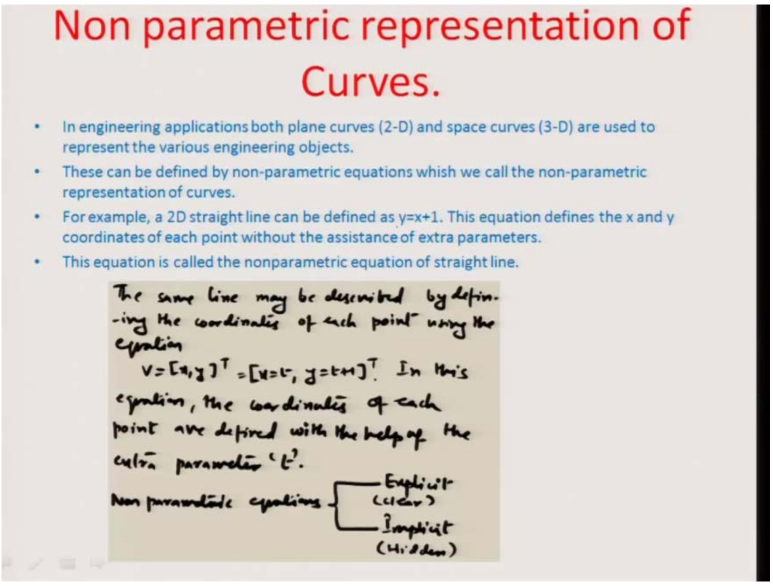

This representation in mathematics is known as the non-parametric representation of describing a curve in a similar ground.We can actually talk a little differently.We can say that instead of having axy description;let us say that there is a parameter,an extra parameter t which is somehow function of which actually becomes the x coordinate. So, it may be simply a relationship like x equal to t, but supposing the curve is a non-linear curve, when we are talking about let us say a parabola or some other non-linear curve,where it can be a square of t or it can be a cube of t.Therewould be slight differences in case of straight line.It may be appearing to be one and the same thing, but for different set of curves when it comes from a linear to a non-linearmode, it may appear to be different if you are just changing these parameters order from 1 to 2.

So, what you assume is that let us say in the same straight line case where we talked about a non-parametric equation y equal to x plus 1,we assume x to have a parameter equal to t.Obviously, y would also have a parameter t plus 1andso, if you vary thist from some limits,let us say limit 0 to 5or 0 to 1 or 0 to 3.Simultaneously, y would also be limited within that domain. So, what I am trying to say is that instead of looking at the whole global picture of simple x and y relationship,parametrically we are trying to associate an extra parameter which we are trying to vary locally,sothat we can have an idea of the local variation of the straight line at some level.Obviously, in case of a straight line,the geometry is regular, it is linear.Both those variations would be one and the same. So, the non-parametric equations can be further divided into two different cases.One is a clear cut case just called the explicit non-parametric form of the equation;the other is a hidden form which is also known as implicit. I will justexplain what this implicit and explicit representationis in the non-parametric domain are really.

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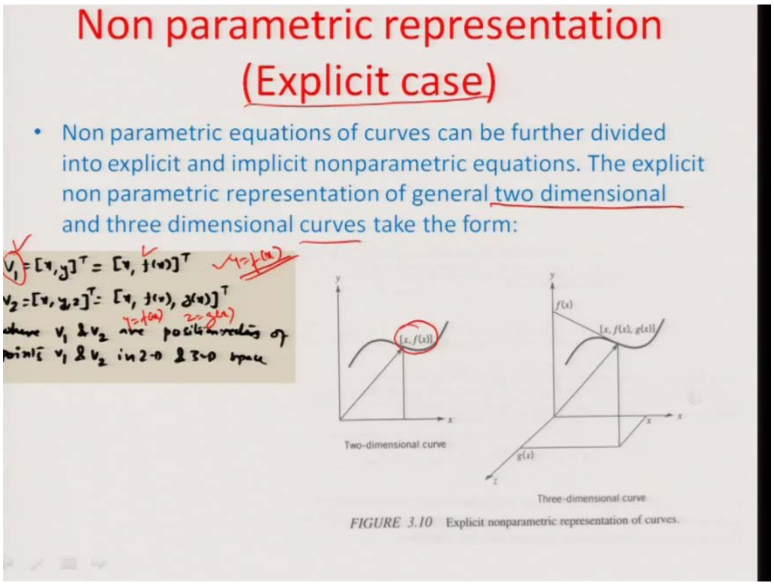

So, let us look at an explicit case first of the non-parametric representation. So, let us say for a two-dimensional general and you know curve that we are talking about there may be a representation v1 in a manner that you know the y coordinate is varying as a function of x,and you are representing this by looking at v1as xfx as the fundamental way that you know the coordinate frames are varying of a certain curve,ok.So, therefore, there is always a relationship between y and x in the manner given here y equal to fx. So, it is a very clear cut relationship which is available.

Simultaneously for a three-dimensional curve also, you may assume a third dimension z and say that in one case, it is related with respect to a function f of x. So, y varies as fxand in another coordinate z, it varies another function g of x.Again there is a clear cut relationship between the x and the y and x and the z as given in this particular example. So, this is actually called an explicit or a clear definition in which you can define a particular non-parametric equation for a curve, ok.

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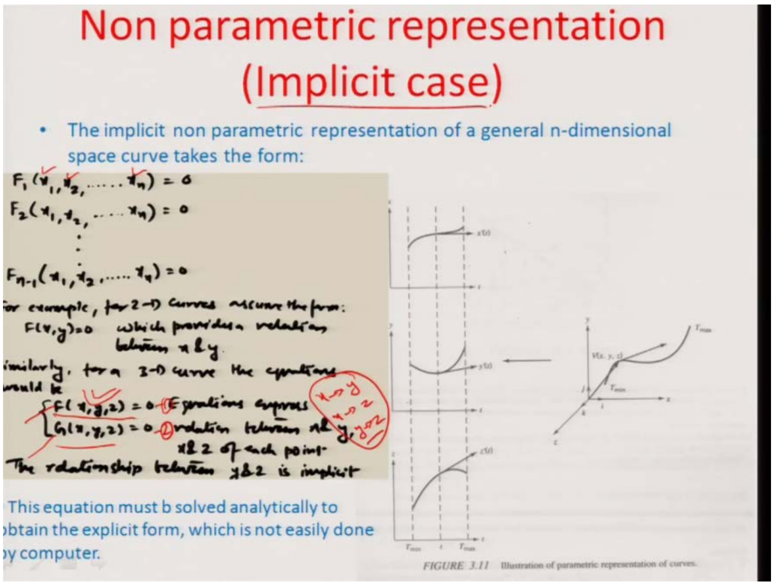

So, in the implicit case for example, in this particular case as you can see, there are let us say you know curve defined by all sort of variables between x 1 to x nand let us say you know only on a three-dimensional case,we have two functions f xyz is equal to 0and g xyz equal to 0. So, there is not enough clarity that how x varies with respect to y or how x varies with respect to z, or as a matter of fact how y varies with respect to z.The clarity is not available, although all the information which is necessary for assuming a sort offunctional relationship is available within these two equations 1 and 2.Such a state of description of a curvewould be known as an implicit representation of the non-parametric form of the curve.

So, what we learntso far is how do you non-parametrically define a curve with respect to the x and y coordinates.You have a clear cut case,where the y coordinate or the z coordinate as in a three-dimensional curve is completely a function of the x coordinate and another case of information is around, but it is in a hidden manner you have two different functions, where there is a relationship between x,y and z being indicated and you will have to infer from that how x varies with respect to y or how x varies with respect to z, or even as a matter of fact y and z what are the variations with respect to each others. So, that is the implicit way of describingthe non-parametric representation of the curves.

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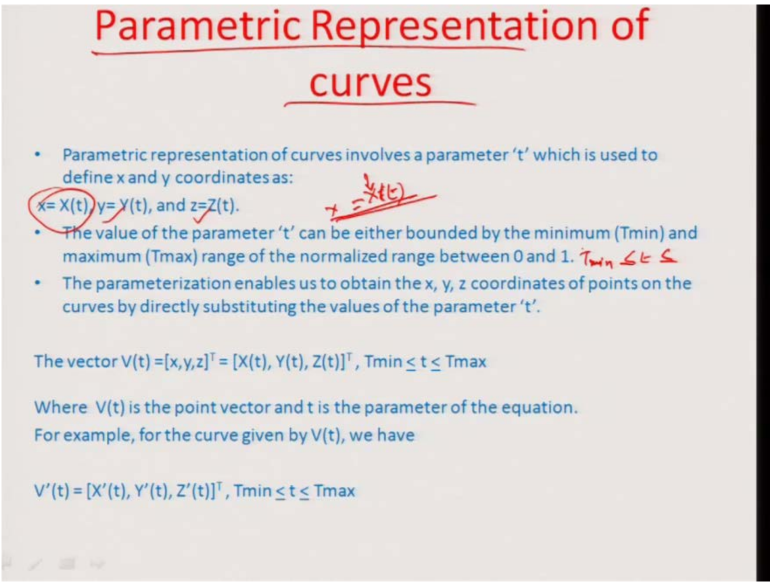

So, having said that let us look at the corresponding parametric representation. I think I had already mentioned that if I involve another extra parameter t to define all the points,for example as you can see here the x coordinate is varyingas a function of t, right. So, x is varying as a function of t some function capital X of t.Similarly, y is varying again as a function of t andso is z varying as a function of t and then, we say that we associate a range for this t value.Let us say the t varies between some t minimum and t maximum. So, what we can do is that instead of moving over the whole domain of xyz,you can actually now limit the value of t on the parameter in the manner,so that on a very local basis you can describe what is going on as a relationship between xyz between that parametric domain varying between t minimum and t maximum. I will just come to an example problem where we show how this parametric equation can be developed.

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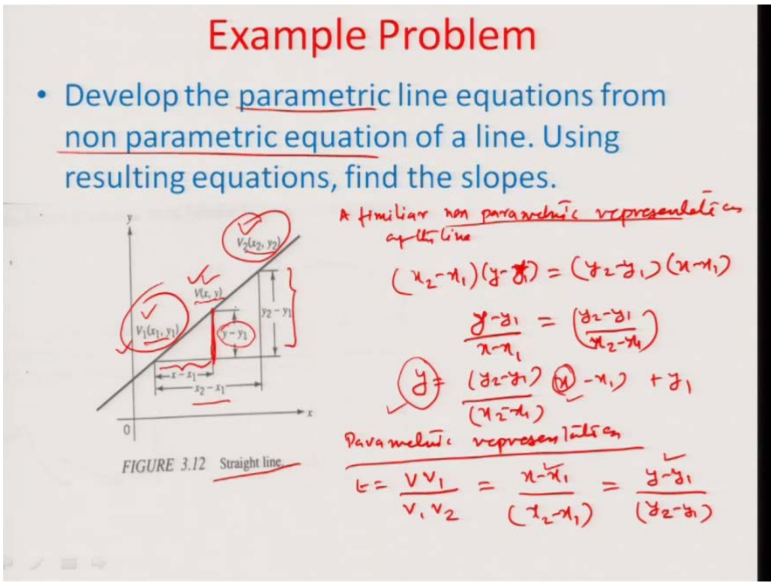

Let us say we are looking at a straight line here for example, in the xy plane and it is given by two points v1 x1y1 and v2 x2 y2 as the two ends of a straight line.Further we are also having a situation where we are describing a third point here vxy which is a variable point between v1 and v2.If we want to develop the non-parametric equations and the parametric equations of this straight line,how do we actually approach the problem?

So, a familiar non-parametric representation of the line can be let us say x2 minus x1 times of y minus y1 equals y2 minus y1timesof x minus x1.This is completely true and valid because if we look at the slope of the line from this variable point to one of the end points,it would be defined as y minus y1 by x minus x1.So,the slope does not change if you go from the local domain to the global domain of variation where you are having both the end points. So, this can be represented as y2 minus y1 by x2 minus x1. So, it is completely justified in writing in this manner now if I were to. So, this is a simple case of parametric form of representation, where I can say that y is varying with respect to x in a manner,so that y2minus y1 by x2 minus x1times of x minus x1plus y1. So, there is a variation, there is a completely valid relationship betweeny and x which is governing the whole equation or your straight line and this is a non-parametric representation.Can I now represent this in a parametric manner is the question.

So, let us say we want to just slightly change the way to represent this whole thing by adding a parameter here. So, parametric representation, sowe add a parameter t and define t in a manner,so that t is equal to vv1 by v1 v2.In other words,t is described by the fraction of x length that is x minus x1as a part of the overall length in the x direction in this 2 minus x1of the particular straight line,and there is a equivalence between this fraction and the way that y minus y1would be defined with respect to xy2 minus y1. So, that is the fraction of yfrom y1 with respect to the overall length y2 minus y1.So, obviously these two fractions are going to be similar if we assume a straight line like relationship. So, the xy would vary linearly between the points x1 y1 and x2 y2,so that always the fraction of the x length with respect to the overall x length should be equal to the fraction of the y length with respect to the overall y length if this point varies between x1y1 and x2 y2,ok.

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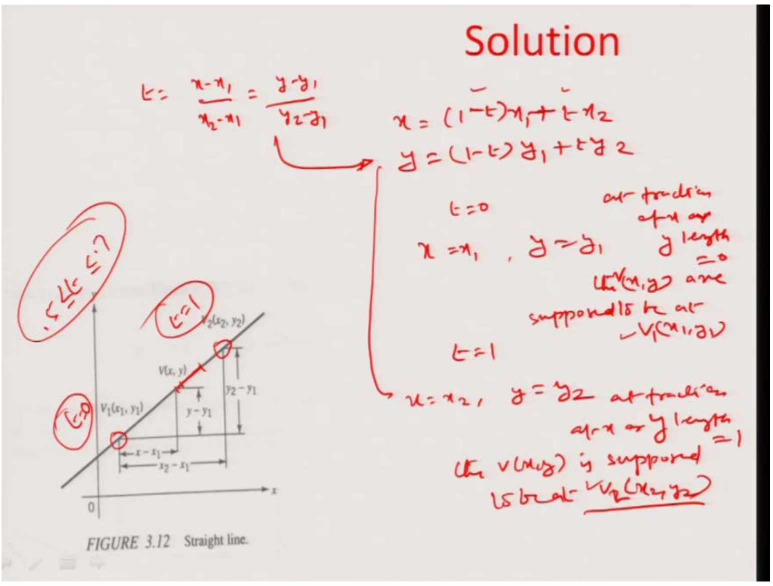

So, if we just look at the way that we can reassemble this equation,we can have the first equation just out of this relationship t which is the x fraction equal to the y fraction.The first relationship x can come out to be 1 minus tx1 plus tx2 and the second relationship y can come out to be 1 minus ty1 plus ty2from these equations. So, there is a flexibility that we have now because now if we vary let us say put t equal to 0 here,x becomes equal to x1and y becomes equal to y1,right which means that at the fraction of x or y length equal to 0,the points xy are supposed to beat x1 y1.Similarly, if t equal to 1,x becomes equal to x2and y becomes equal to y2from these relationships which means that at fraction of x or y length equal to 1,the vxyis supposed to be at v2x2 y2,ok.

So, the points really move between the initial point and the final point corresponding to t equal to 0 and t equal to 1.What is important for me to say is that just by merely varying this parameter between a local domain, let us say varying between 0.5 to 0.7, I can really zoom downthe parametric equation to a point which is corresponding to 50 percent of the fraction to 70 percent of the fraction. So, the parameterization of the equation enables me in a way to look at a geometric object locally, provided there is a global description given by a non-parametric form of equation of the particular curve. So, this is the power of such non-parametric representation.

Now, I am going to go to the next level and tell you that how to represent synthetic curves in a parametric and non-parametric manner,and there you will have a very good feelthat a very complex topology constituted of many small synthetic curves with some relationships of interconnects between each other,how they can be traced on a profile topologically,so that they can match the exact profile into question and that will be in the next module.

Thank you.