Manufacturing Systems Technology

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Module - 01

Lecture - 06

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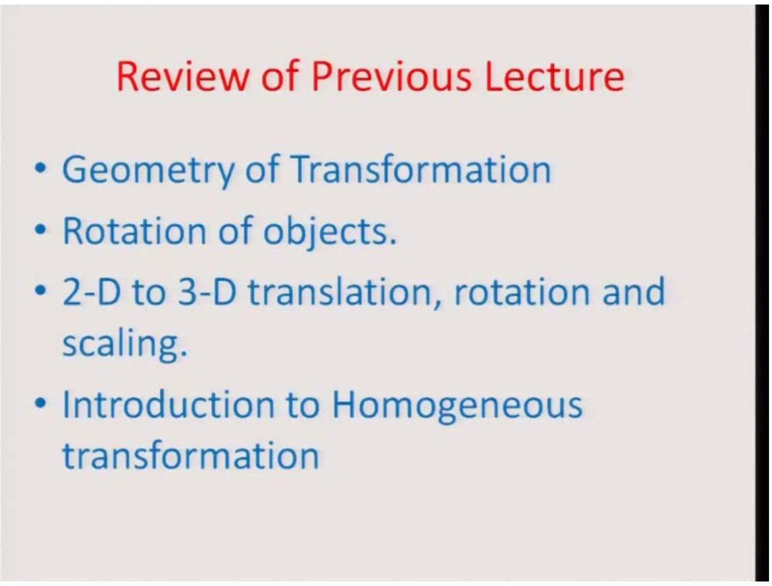

Hello and welcome to this module 6 on Manufacturing Systems Technology.A quick recap of what we did in the last module.We talked about geometric transformation, specifically with the transformation for a rotation of objects in two-dimensional space.Then, we also discussed about how this 2d transformation can actually convert into a three-dimensional transformation,and we studied this in context of translation rotation and scaling and also introduced briefly why it is needed for homogeneity of you know multiplication of matrices to describe the whole transformation process rather than having an addition and multiplication separately as was done before for the translation and multiplication for rotation and scaling.

So, we wanted to homogenize the whole thing in the interest ofintensive computing or less inexpensive computing and more easier computingso onso forth, and we almost sort of you know created a need for homogenous transformation process.

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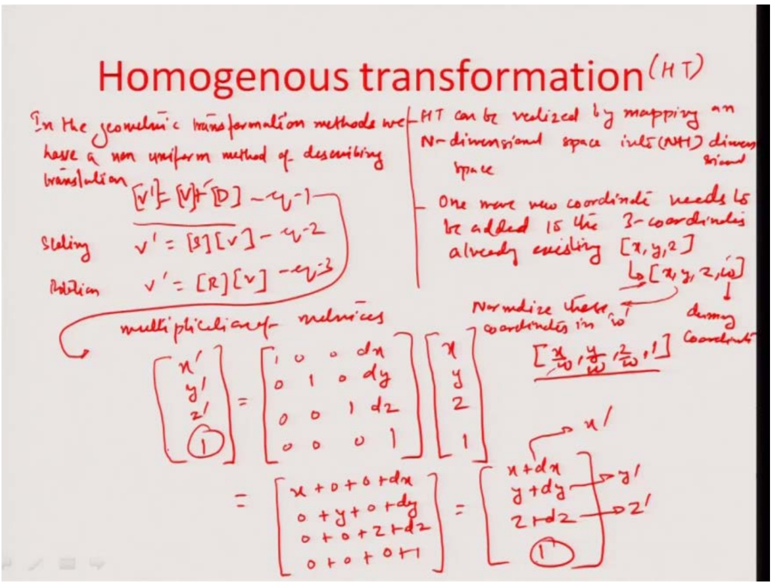

So, what is actually homogenous transformation? So, in the geometric transformation methods we have a non-uniform method of describingtranslation which is actually v dash equal to v plus d distance matrix root scaling, which is v dash equal to some scaling matrix times of v and rotation about some axis which is again v dash matrix into rotation times v. So, the non-homogeneity is coming from the fact that while equation 1 is an addition equation,equation 2 and equation 3.They are multiplication equations. So, homogenous transformation can be realized I will call this ht in short,realized by mapping n dimensional space into n plus 1dimensional space.Now, what does it mean really?It means that one more coordinate needs to be added to the three coordinates already existing. So, that xyz can be converted as xyz and some dummy coordinate w.We call it intentionally dummy coordinate because it is only something which is added for the sake of clarity of calculations,ok.

So, obviously now if I looked at how to convert this equation 1 into a multiplication of matrices; I would say by adding on a new coordinate 1 or w or anything. In fact you know what we can do is,we can scale these coordinates and normalize these coordinates in wwhich forms x by w, y by w, z by w and 1. So, this is how you can actually look at this.It is more or less a scaling issue and w is the scale factor that can be utilized to map these coordinates in this manner. So, I can say that this can be translated by a matrix 100 dx 010dy001dz and 00 01 and this you multiply with x y z and 1.Let us look at whether it is formulating what we need really.

So, here the first variable comes out to be x plus 0 y plus which is 00 z plus dx.The second variable comes as 0 plus y plus 0 again plus dy,and the third variable comes out as 0 plus 0 plus z plus dzand the forth comes as 0 plus 0 plus 0 plus 1. So, essentially what evolves is x plus dx y plus dy z plus dz and 1 which is nothing, but x dash or x prime y dash y prime z dash and 1 as such keeps you know as a dummy variable in that even if you do this homogenous transformation. So, this is sort of right way to convert the addition of matrices into the multiplication just by looking at, or changing the space from n dimensions to n plus 1 dimensions or introducing a new coordinateto the matrix space.

You can actually convert that into a homogenized multiplicative process. So, therefore, all you need to do is to add this coordinate throughout whether it is rotation or whether it is scaling and formulate you know a set of new equations,where you can all by multiplication do the transformation. So, that is called homogeneity.

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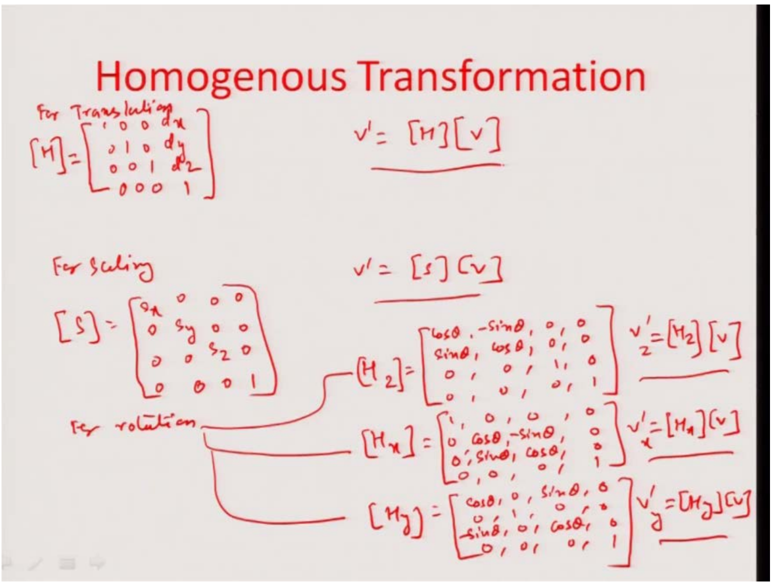

So, thereforenow if you really summarize it for the addition of matrices,you write a matrix 100dx 010dy001dz0001as the transformation matrix for addition. So, you can call this as a matrix for translationfor scaling the matrix can be just in the same manner, you have to change the order of the matrix. So, you have the diagonal matrix sx sys z and 1and remaining all are zeroes.So, that is how it will be for scaling and obviously, for translation v dash now would be equal to the matrix h times of the matrix v for scaling the v dash would be equal to matrix s times of the matrix v, and for rotation and it is plane based again. So, you have now rotation in three directions,that is rotation along z which could be represented by the matrix cos thetaminus sin theta 00sin theta cos of theta 0 0 0 0100001.

Similarly, you have hxand you have hy.Hx could be 10 0 0 0 cos of theta minus sin of theta 0 0 sin of theta cos of theta 0 and 0 0 0 1and similarly,hywhich is actually cos of theta0 sin of theta 00100minus sin theta 0 cos of theta 0 0 0 01as the rotation about the y axis. So, therefore, the rotation vectors of vector equation or the transformation equation for the rotation can be,if it is about z we can say this is v dash is hz times of v.If it is about v,we can call it v dash z,If it is about the x, we can call it v dash x is h x times of v and if it is about the y,we can call it hytimes of v and these are the three transformation equations apart from the ones which we have already told for translation and scaling.

So, this completes the three-dimensional transformation process number 1, number 2.It is a homogenous transformation that all the transformations are continued by doing matrix multiplicationand therefore, the complexity of the computationis; expense of the computation issubsequently reduced because of that.

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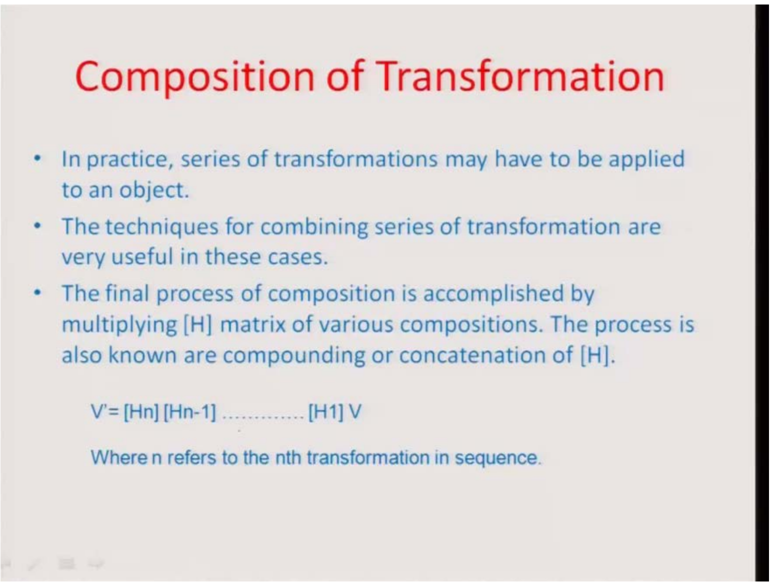

So, now we can easily say that if you had a series of transformations to an object for example, let us say you wanted to rotate the object, you wanted to linearly translate the object, you wanted to scale the object all together. You all need to sort of make a series of multiplier matrices,let us say these are the n transformations that you have done on to vector v. So, you are multiplying it with h1h2 h3so onso forth to hn. So, there are different transformation matrices which come out because of rotation because of scaling because of whatever. So, this is a generic equation which comes up now forany kind of transformation which can be computationally performed on a computer by you know as many number of transformations to a initial possible coordinate system v.We can predict the final coordinate system in this way.

So, now if you had that situation where you blow up an object or let us say scale an object or maybe you know translate an object or may be assemble an object with another one,so you are moving it in space and trying to mix and match two different objects.All you need to do is this transformation generic equation that has been pointed out here and this is also known as compounding or concatenation of the transformation matrix h. So, it is all mathematically now become very simple that all you need is to know what are all the transformation matrices,and all you need is to know what is the initial coordinate space, and you will get the dimensions for the coordinate space. So, let us actually now do practical example problems where we try to do this transformation.

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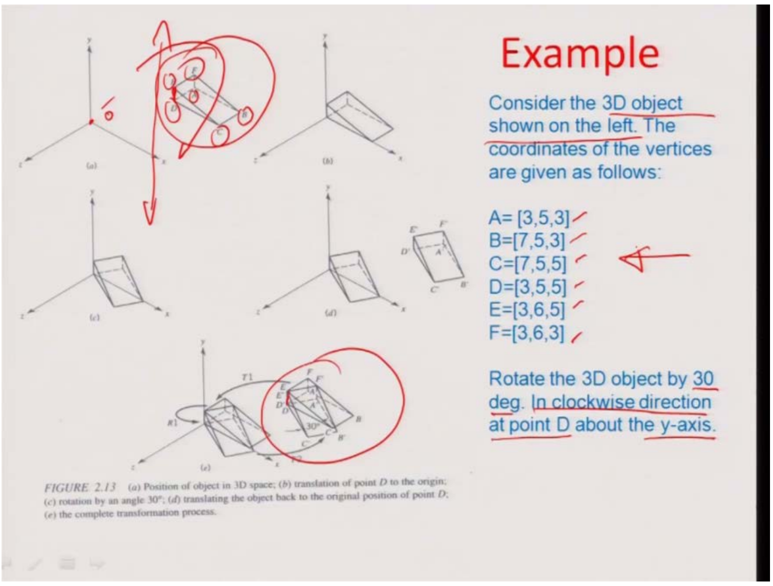

As shown in this example problem here, these are 3D objectwhich is shown on the left figure somewhere here. So, this is the objectand there is an orthogonal coordinate system xyz which is having an r edge around this point oand the object is not placed at the origin, ok.It is away from the origin and we know what are the existing vertices of this particular object by looking at the different points a b c d e and f.You can see these points being marked on the figure hereand these coordinates are already known to you.You have to rotate this object by 30 degreesand that is too in the clockwise direction. So, seeing if it is and this is the rotation is about the point d and around the y axis,so therefore what you are trying to do is to sort of rotate this object by 30 degrees around this y direction in the clockwise as seen from the positive side.

So, basically you are trying to rotate in this manneraround the y axis. So, you can see herein fact how the rotation has happened. So, the object is rotated from the position ab c d ef to a dash, b dash, c dash, d dash, e dash and f dash.Obviously, d the line along which the rotation has to take place are same, meaning there by d dash and e dash are same as d and e.The remaining all are rotated and you have to calculate the coordinates of the final position of the object given the initial coordinates of all the points a b c d and essentially, it is a series of transformations that you have to do.Obviously, you cannot rotate the objectas it is on the position that it is in because you know the rotations all happen when the objects are centric to the origin.

So, therefore you have to somehow be able to bring back theobject first to the origin and then, from there you would perform all the transformations,so that you know the object, the final coordinates can be mapped. So, essentially you first bring this object to the origin and then, perform the rotation, add the origin as you can see in step b and c and then again translate the rotated object back in space to the initial point d that it was in order to do this transformation.

So, in the next module I am going to sort of you know work on trying to tell you how this transformation would be possible mathematically, and how you can actually solve this transformation.