Manufacturing Systems Technology

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Module- 08

Lecture- 48

Hello and welcome to this Manufacturing Systems Technology last module, module 48. As I already discussed earlier we had done a lot of studies regarding Kanban planning deterministic, in situations where there would be an estimation of stocking needed or even in situations where there are sub assemblies and you have to do a Kanban balancing on the sub assembly stage. Today we will be looking in to another aspect of lean manufacturing which is about how to probabilistically estimate the overall level of Kanban with a cost per view. So, that you have a cost in mind and which really would like optimize, and based on that probabilistically can you be able to predict in the number of Kanban, so the Kanban levels circulating in a system.

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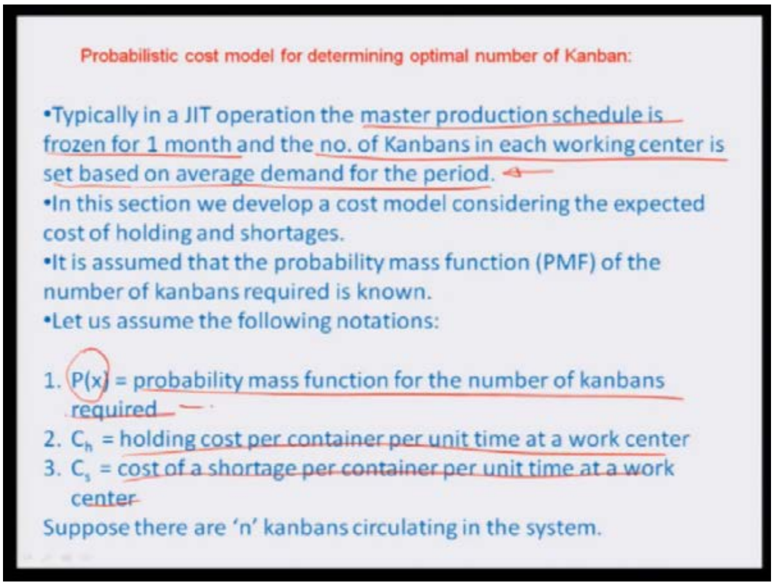

So, for doing that typically in a JIT operation the master production schedule, is frozen for about a month. And the number of Kanbans in each working center is based, based on the average demand of the period. So, in this section we will develop a cost model now, so considering the expected cost of either holding or shortage. You have to understand one thing that if supposing the level of Kanban which is flowing within the system is more than the level of Kanban which is needed by the system always there will be a hold up of the material. And this hold up would be inducing unnecessary costs to the system, if you really want to make it a leanest possible manufacturing system any hold up should be a penalized through a sort of a cost per unit which is being held up in that situation. Similarly, if the number of Kanbans flowing within the system are lower than the overall demand of the Kanban, there is a possibility that there will be a shortage of the material and such shortages are also penalized with some kind of a cost factor in to picture. So, if we keep this in per view and say that the operating level of Kanban is let us say some particular value N; and then we say that if supposing the overall Kanban level is above N we are having a situation of hold up.

So, we associate a cost CH per unit box or per unit container associated per unit time at a work center associated with that hold up and then if supposing its less than N and the overall number of Kanban is actually whatever is flowing is actually less than the demand which is there, then use always going to be a shortage associated and then the cost involved would be taken as CS which cost of shortage per container per unit time at a work center. So, having set all these and also may be generating a probability mass function for the number of Kanbans which are needed as P X, which should be able to somehow correlate all these and optimize all these for the minimum cost point where we can calculate the operating number of Kanbans.

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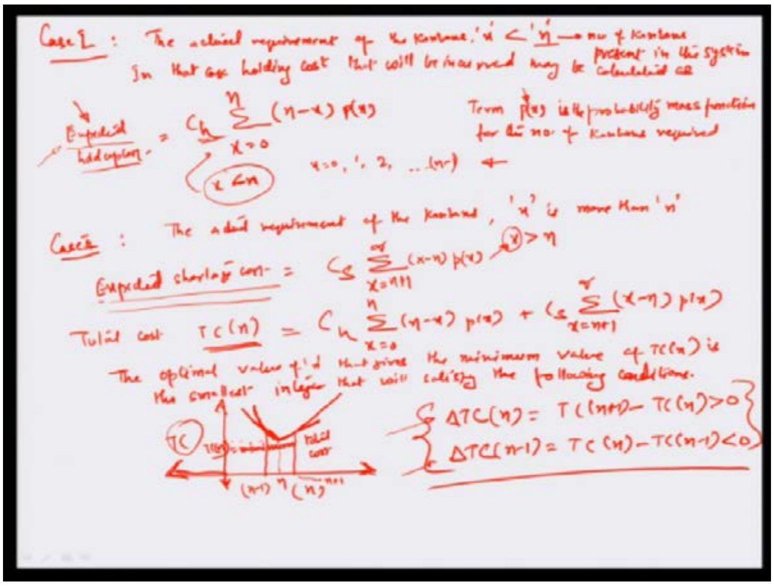

So, in case one; the actual requirement of the Kanbans we just x lets say, is less than n which is the number of Kanbans present in the system, on the Kanban level of the particular system. So, in that case the holding cost that will be incurred may be calculated as Ch times of sigma x varying between 0 to n, n minus x times of p x. The term p x is the probability mass function for the number of Kanbans needed. So, supposing the actual requirement x, is less than n as in this particular case where every time there is a short fall x could be 0, x could be 1, x could be 2 so and so forth; x could be 1 n minus 1, less than n. So, whenever this condition is being met your suggesting that the numbers are always lesser than the actual level which is present, which leads to the holdup cost. This is actually also the expected hold up cost.

The probability mass function is very very common term which is used in distributions and the expected mean or the expected cost is provided always through the parameter which you are considering; like for example, in this case x is the number of Kanban which is the variable parameter in question times the probability mass function for x number of Kanban in to the system. So, that is how the expected cost or the expected mean is always defined in probability distributions.

So, I am not going into the detail of proving this how this summation of products would be the expected cost, it is very common place statistical equation. However, I would like to use this particular thing to estimate what are the cost associated. So that we can optimize them to work on a Kanban level which we should be able to operate. So, that minimum cost is incurred on this. So, that is case one. Case two: is a case when the actual requirement of the Kanbans, x is more than n. So, x is more than n and obviously, because the actual requirement is always more than the level which is present which is n, there is always going to be a short fall in the supply. Because the requirement is more than what is present of the supply. So, this a case of shortage, the material fall short because of that.

So the expected shortage cost in a similar manner, would be Cs times of sigma x; obviously, would vary between n plus 1 and let us say it goes all the way up to infinity. And the expected cost would be defined then in terms of x minus n, the total amount of shortage times of the probability of having the operation level at x value. So, that is the probability mass function for the parameter x, and this is the expected shortage cost incurred because of that value of x always greater than n, meaning requirement more than supply or shortage.

So, the total cost let us called Tc for the operating level of the Kanban which is n, where n in one case may be falling short of the actual requirement, in other case may be falling short of the more than the actual requirement. And may necessarily include shortage cost in one case and hold up cost in other case, is the total cost can be a summation of both these expected hold up cost and expected shortage cost; because x can vary anywhere between 0 all the way to infinite. So, there can be any level of Kanban that can be maintained in a particular systems. So, the Tc n here comes out to be equal to Ch sigma x varying between 0 and n, n minus x px plus Cs sigma x varying between n plus 1 to infinity x minus n p x. So, having said that the optimum value of n, that gives the minimum value of Tcn is the smallest integer that will satisfy the following conditions.

Obviously, the if you look at the cost equation and the way that it goes, let us say we are plotting Tc as a function of n; and obviously, there is a certain level around which we are operating which is the working minima level for the integer n, where Tcn is the minimum cost, minimum total cost. So, the relationship that will be obeyed for two values n plus 1 and n minus 1 should be delta Tc at n equal to let us say Tcn plus 1 minus Tcn. That is the change of cost per unit the change of the actual number of Kanbans present, should be 0; greater than 0, meaning there by that this slope here write here should be more than 0. And similarly this particular slope here at the other end should be less than 0, that is the condition for the minima. So, we call delta Tc n minus 1 equal to Tcn minus Tcn minus 1 should be less than 0. So, that is how you defined the minima point corresponding to the integer n where the total cost Tc should be the minima.

So, if I now try to calculate these two expressions from the holdup cost and the shortage cost, I should be able to hit upon a situation when the problesty mass function may be correlated to the minima condition; and may be correlated to a relationship between n and x in a manner, so that I should be able to find out the operational level of x.

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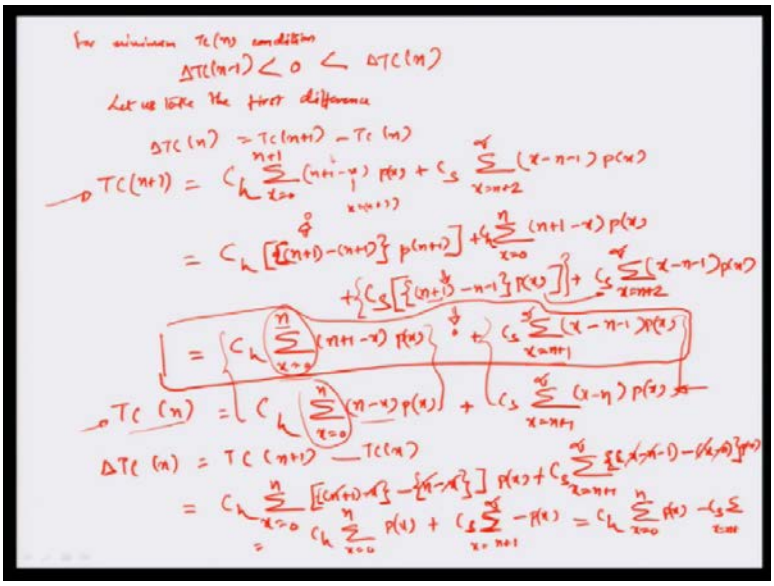

So, therefore, the minimum Tcn condition, so for minimum Tcn condition delta Tc n minus one should be less than 0 less than delta Tc n. And let us take the first difference; delta Tcn equal to Tcn plus 1 minus Tcn. And let us try to calculate what really is Tc n plus 1. So; obviously, Tc n plus 1 then would be the holdup cost Ch times of sigma x varying between 0 and n plus 1, n plus 1 minus x px plus Cs the shortage cost times of sigma x varying between n plus 2; 2 infinity x minus n minus 1 px. So, if I want to further solve this summation, I would like to place couple of things here which would be not changing the situation much.

For example, if I want to put the value of x equal to n plus 1 here in this equation, then this factor n plus 1 minus n minus 1 should be equal to 0. So, I can easily write this as Ch times of n plus 1 minus n plus 1 the value of x times of p n plus 1 plus sigma Ch x varying between 0 to n, n plus 1 minus x px plus Cs value here; which corresponds to addition of n plus 1. So, supposing if I want to add this term Cs times of x equal to n plus 1 minus n minus 1 times of px; plus Cs x varying between n plus 2 to infinity x minus n minus 1 px. It will really not make any difference because this is actually equal to 0 and so is this. So, having said that I am going to just re write this equation little bit in to more suitable for where we say that the first term become Ch x varying between 0 and n, n plus 1 minus x px; plus Cs varying between x equal to n plus 1. I am adding this subscript corresponding to n plus 1 equal to x you know, in this particular sigma 2 infinity x minus n minus 1 px; obviously, because this is 0, this term is 0 here it does not make any different if we had this to the overall shortage cost side or expected shortage cost side the equation.

So, having said that now so we are left with a sort of equation where you have the index going between 0 n and on the holdup side and n plus 1 to infinity on the shortage side. And I can actually write this term in a little different manner, because now I also have the term Tcn which is equal to Ch sigma x varying between 0 and n, n minus x px plus Cs sigma x varying between n plus 1 to infinity n minus x px. And we are talking about delta Tcn term which is actually equal to tc n plus 1, which is this value write here which we obtained after these algebraic derivations minus of the value because of Tcn which is obtained through this particular equation here.

So, let me just write this down in a manner where this more convenient and see what results because of all these. So, we can combine; obviously, this two terms together, because the stigmas' are between 0 and n. And we can combine these two terms together in an algebraic manner and I can write this as Ch sigma x varying between 0 and n. And I take down the first term here n plus 1 minus x, and further subtract the corresponding Tcn value here. Which is n minus x times of px and on the shortage cost side similarly I take down the value varying x plus x varying between n plus 1 to infinity. And on one side I have x minus n minus 1 minus x minus n times px. So, here; obviously, the first term becomes equal to Ch sigma x varying between 0 to n times of px, and second term here tries to become equal to these kept canceled off and you left with only minus of px x varying between n plus 1 to infinity. So that is how a basically get the final form of the expression here as Ch x varying between 0 and n p x minus of Cs a sigma x varying between n plus 1 to infinity of px.

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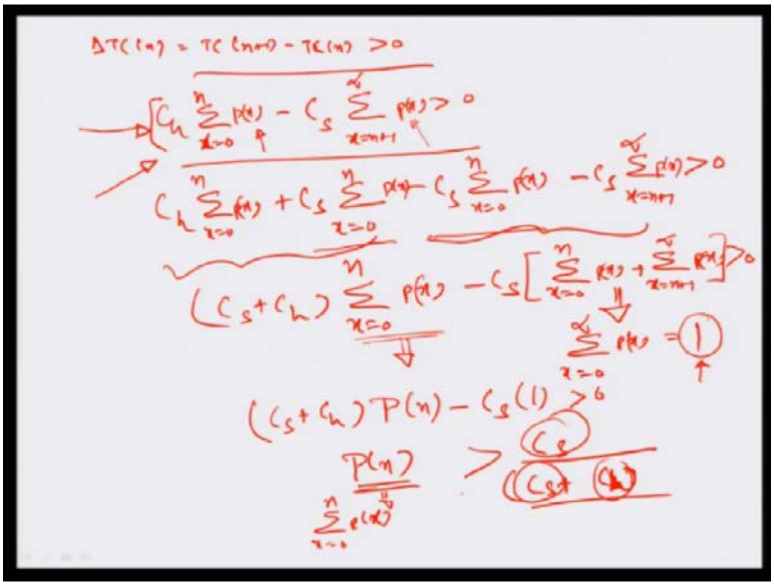

So; obviously, then the condition that we initially had delta Tcn which is equal to Tc n plus 1 minus Tcn should be greater than 0; it is a positive slope side, meaning there by that the full and final term which has been obtained from the last step here. That is Ch sigma n x varying between 0 and n px minus of Cs x varying between n plus 1 to infinity px should be greater than 0. So, we want to somehow change this equation form in a manner. So, that we have a more appropriate method of defining everything in terms of the probability mass function px. So, here what I want to do is to sort of subtract and add a term Cs sigma x varying between 0 to n px minus to the same term Cs x varying between 0 to n px, minus Cs sigma x varying between n plus 1 to infinity and put that condition that this equation as greater than 0.

So; obviously, these two being same quantity is if added and subtracted will not change the equation; however, we can actually combine these two terms and these two terms in a manner. So, mathematically we can write this has Cs plus Ch sigma x varying between 0 and n px, and on the other hand we can say here Cs sigma x varying between 0 and n px plus sigma x varying between n plus 1 to infinity px. So, that would actually mean sigma of px x varying between 0 to infinity which can be treated as 1. So, the probability that there is 1out of these so called infinite numbers of x where the Kanban level is needed. So, at least one of those would be needed, because it is 0 to infinity its large range and any of them all of them are integer numbers. So, in this whole integer range the probability having one possibility or one particular x would be exactly equal to 1.

So, having said that then the operating the probability mass function or the summation of all the probability mass functions between x equal to 0 all the way to the let us say to the value n, can be written as a cumulative distribution P of n and therefore, Cs plus Ch times of p of n minus Cs times of 1 is greater than 0. Or this particular value which is nothing but, the summation of all the probability mass functions where x varies between 0 and n should be greater than Cs divided by Cs plus Ch. So obviously, now we are seeing that it is very important to estimate what the shortage and holdup cost are and that would define the optimum cost level where you can operate corresponding to a certain level of Kanban n in the particular system.

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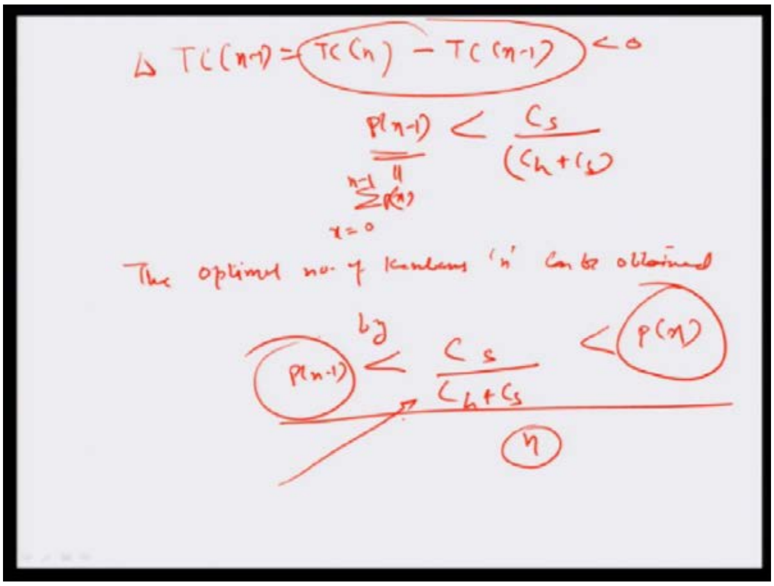

Having said that if I consider the other cost equation which was earlier taken to be, delta Tcn minus 1 which is nothing but, Tcn minus 1 or Tc n minus Tc n minus 1. If you should be remember this was just done initially, Tcn minus 1. So, the way that you can obtain another condition here, by putting this to be less than 0 is P n minus 1, same thing sigma P x where x probably varies between 0 to n minus 1. That is less than Cs divided by Ch plus Cs. So obviously, the optimal number of Kanban, optimal number of Kanban n can be obtained by P n minus 1 less than Cs divided by Ch plus Cs less than P n; and so whole idea is to look for n which will satisfy this particular condition.

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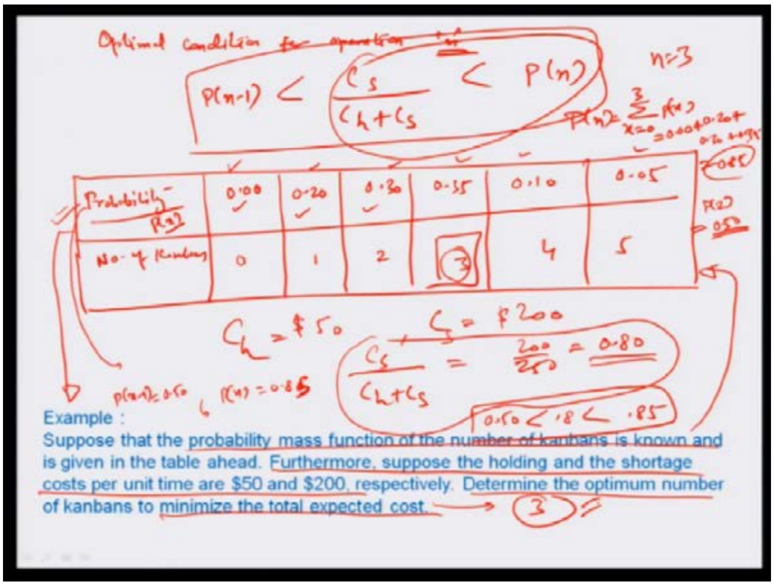

We will try to solve a problem example where will try to find out how you know this probability mass functions can be put together in terms of the shortage cost and the holdup cost. So that the end level the operating level of the Kanbans can be determined. Here we at the outside give a table; obviously, we know that the optimal condition for operation here corresponds to level of Kanban n obtained from the equation P n minus 1 less than Cs divided by Ch plus Cs less than Pn. And the probability distribution table that has been given here can be represented as let us say probability number of Kanbans and corresponding to a number of Kanban 0 the probability 0.00; that corresponding to 1, corresponding to 2, 3, 4, and 5 has been recorded as 0.20, 0.30, 0.35, 0.10 and 0.05.

So, in this context this data is already provided of the probability mass function. There is a way there is a full algorithm to calculate the probability mass function which is beyond the scope of the syllabus at this particular point of time, but supposing that the probability mass function of this Kanbans at a particular production process is known to be given by this particular table. And further more suppose the holding and the shortage costs per unit time 50 Dollars and 20 Dollars also are provided. So, you have to determine the optimum number of Kanbans at which the system should operate to minimize the total expected cost. So obviously, here you are talking about Ch value of 50 Dollars per unit time, and a Cs value of the shortage cost value of 200 Dollars per unit time. So, the Cs divided by Ch plus Cs value should be really equal to 200 by 250 which is actually about 0.80. And if I wanted to calculate the cumulative probability distribution function from this particular table.

So, corresponding to let us say the value of n equal to 3 the Pn, which is equal to sigma x varying between 0 to 3 of this probability mass function Px which has been defined in this table, should be equal to 0.00 that is the first value plus 0.20 the second value plus 0.30 the third value plus 0.35. So, this absolutely becomes equal to 0.85. So, you can see that corresponding to a n equal to 3 this condition here is obeyed and then you know if you operate at n equal to 2 for example, if this were changing to 2 then the probability mass the cumulative probability mass function P2 would then in that case be equal to 0.5. So obviously, the operating level of the Kanban here for the minimum cost would be 3 because the condition satisfied here says Cs divided by Ch plus Cs is less than Pn and greater than Pn minus 1, and the Pn here is 0.80 as you saw just about little bit before and Pn minus one here a 0.50.

So, typically this, this 851 sorry. So, the typically this Cs divided by Ch plus Cs .8 satisfies the condition 0.85 on one side and 0.50 on another side. So, the operating level here is 3. So, that is how you do typically the probabilistic way of Kanban calculations in a system. How you calculate this Px value, is again a separate a separate matter and this I think in this particular course will probably skip because beyond the course of this syllabus of this course to a calculate the probability mass function mythology or to discuss the probability mass function mythology. So, with that I think I would like to complete this section on the deterministic module of estimation of the Kanban and then subsequently this sort of brings us to the end of this part or this part of the manufacturing systems technology course there will be a future module in which we will probably plan the quality and also the material handling aspects regarding such manufacturing this completely you know, Computer Integrated Manufacturing Systems or Manufacturing Systems Technology.

So, with that I would like to close this course here. Thank you for being a patient in listening to me and all the lectures and I wish you good luck for your examinations.

Thank you.