

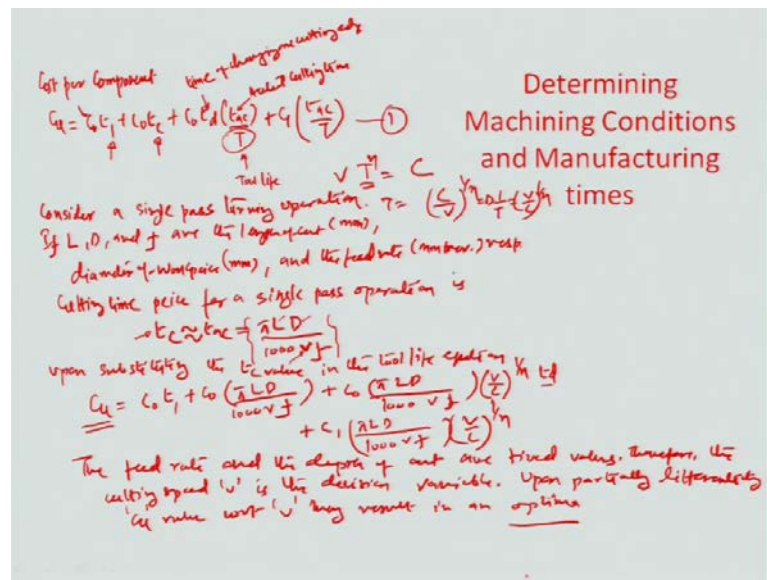
**Manufacturing Systems Technology**  
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**Module - 04**

**Lecture – 22**

Hello, we will just have the Manufacturing Systems Technology, Module 22.

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So, last lecture we had tried to understand how we do the optimization of the cutting velocity or feed and the depth of cut etcetera. And in that respect what we had formulated was that, there is a particular way to do the optimization by either considering the minimum cost model or the minimum pass time model or the maximum production rate model. So, we were just about trying to understand, what is the minimum cost model and there I think we have formulated in expression which was talking about the cost per component as the equation  $C_0 t_1$  plus  $C_0 t_c$  plus  $C_0 t_d$  times of  $T$  by capital  $T$  plus  $C_1 T$  by capital  $T$ , where the  $t_c$  is actually the actual cutting time, the capital  $T$  here is the tool life.

And; obviously, there are different cost like the liberal overhead cost  $C_0 t_1$  is actually the unproductive time or the non productive time,  $t_c$  is the cutting time, so on and so forth and  $t_d$  is actually the total tool changing time per cutting edge of the particular tool

and we had in detailed explained that how this cost is bestowed upon in a particular situation.

So, let us consider now single pass turning operation. So, if  $L$  is the length of cut in millimeters,  $D$  is the diameter of work piece again in millimeters and  $f$  is the feed rate and millimeter per revolution respectively. Then, the cutting time per piece for a single pass operation is represented by this formulation here  $t_c = \frac{\pi L D}{1000 v f}$ ; obviously, the cutting velocity being in the parameter, which is to be sort of optimized.

So, we want to create a maximum velocity kind of a condition and this is actually a standard formulation  $t_c = \frac{\pi L D}{1000 v f}$  the actual cutting time of a certain single pass turning operation.  $D$  is the diameter of the work pieces,  $L$  is the length of the cut that we are considering and in a way, how much material has been removed and  $f$  is the feed rate of millimeters for revolutions. So, for corresponding to one revolution, what is the feed at which the tool is proceeded, so it is the standard formulation.

So, if you substitute this value of time of cutting in the equation given earlier here equation 1, let us look at how it changes. So, we have upon substituting the  $t_c$  value in the tool life equation, the cost per piece  $C_u$  can be written down as  $C_0 + C_1 t_c + C_2 \left(\frac{\pi L D}{1000 v f}\right)^n$  plus  $C_0 \left(\frac{\pi L D}{1000 v f}\right)^n$  times of capital  $T$  value, which is governed by the tool life equation. So, the  $T$  really can be represented as  $C_2$  by  $v$  to the power of  $1/n$  or other words  $T$  can be  $v^{1/n} C_2$  to the power of  $1/n$ .

So, I can say straight away represent this here as  $v^{1/n} C_2$  instead of this  $T$  at the denominator here times of  $t_d$ , where the time  $t_d$  reflex the time to change the cutting edge of the particular tool. I have detailed about this  $t_d$  etcetera, when we talked about this particular equation time of changing one cutting edge plus the total cost per tooling, which has been represented as  $t_s C_3$  by  $t$  times of  $C_1$ . So; obviously, we substitute the value of  $t_s C_3$  again back here as  $\left(\frac{\pi L D}{1000 v f}\right)^n$  times of  $1/n$  which is  $v^{1/n} C_2$  to the power of  $1/n$  and this is how the overall cost equation would come out to be.

So, in most of the situations the feed rate and the depth of cut are fixed values therefore, the cutting speed  $v$  is the decision variable now. So, upon partially differentiating the  $C_u$  value with respect to the variable, the decision variable which is  $v$  in this particular case may result in some optima. So; obviously, we have to differentiate it equal to 0 in that case, so let us actually do this differentiation and see what it turns out be or what it

means actually in this particular case.

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Determining  
Machining Conditions  
and Manufacturing  
times

$$\frac{dc_u}{dv} = \frac{c_0 L D}{1000 f} \left( -\frac{1}{v^2} \right) + \frac{c_1 L D t d}{1000 f c_1^{1/n}} \frac{d}{dv} \left[ c_1^{1/n} v^{1/n-1} \right]$$

$$\rightarrow \frac{c_0 L D}{1000 f} \left( -\frac{1}{v^2} \right) + \frac{c_1 L D t d}{1000 f c_1^{1/n}} \frac{d}{dv} \left[ (v)^{\frac{1}{n}-1} \right] = 0$$

$$v^{1/n} = \frac{c_0}{\left( \frac{1}{n}-1 \right) (c_0 t d + c_1)}$$

$$v_{min} = \left[ \frac{c_0}{\left( \frac{1}{n}-1 \right) (c_0 t d + c_1)} \right]^n$$

Upon substituting the value of cutting speed in the tool life equation we obtain the optimal tool life (T<sub>min</sub>) for minimum unit cost

$$T_{min} = \left( \frac{1}{n}-1 \right) \frac{(c_0 t d + c_1)}{c_0}$$

So, upon differentiating  $dc_u$  with respect to  $dv$  from equation for the cost drawn earlier, we get  $\frac{c_0 L D}{1000 f} \left( -\frac{1}{v^2} \right) + \frac{c_1 L D t d}{1000 f c_1^{1/n}} \frac{d}{dv} \left[ c_1^{1/n} v^{1/n-1} \right] = 0$  and from this particular equation, we have left with a final form  $v^{1/n} = \frac{c_0}{\left( \frac{1}{n}-1 \right) (c_0 t d + c_1)}$ .

In other words the  $v_{min}$  happens to be equal to  $\frac{c_0}{\left( \frac{1}{n}-1 \right) (c_0 t d + c_1)}$ . So, upon substituting the value of cutting speed in the tool life equation, we obtained the optimal tool life  $T_{min}$  for minimum unit cost as  $T_{min} = \left( \frac{1}{n}-1 \right) \frac{(c_0 t d + c_1)}{c_0}$  this just corresponding to the value the  $V_{min}$  by just substituting in the equation  $v$  to the power equals to  $c$ . So, that is how you arrive at the minimum time based on this formulation.

So, in the nut shell then we are left with the condition that has been given to us, where there is a minimum velocity at which you can operate in involving the different costs including the over rate cost, the cost of tooling really, the overall time to change the one particular cutting edge and the tool life index and the tool constant. So, of course, exponential, so all these parameters are really determining the minimum in the particular case when we are talking about the minimum cost model. The other model which is of

importance is really the maximum production rate model as I had earlier illustrated.

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### Maximum Production Rate Model

- Another criterion used to determine the optimal conditions is maximum production rate which is inversely proportional to the production time per piece, which is given by:

Time per piece  $T_p =$  non-productive time per piece + cutting time per piece + tool changing time per piece

$$T_u = t_1 + t_c + t_d \left( \frac{L D}{T} \right)^{1/n}$$

$$= t_1 + \frac{\pi L D}{1000 v f} + t_d \left( \frac{\pi L D}{1000 v f} \right)^{1/n}$$

Upon partially differentiating  $T_u$  w.r.t.  $v$ , equating to 0 and solving the  $v$  we obtain:

$$v_{max} = \left[ \frac{C}{\left( \frac{1}{n} - 1 \right) t_d} \right]^n$$

So, this is again another optimal condition and for the maximum production rate, which is inversely proportion to the production time per piece. Obviously, the maximum rate would correspond with minimum time per piece, which we are talking about. So, we now try to find out what is that optimal model that can be illustrated. So, let us look at what really we mean by time per piece. So, again we consider some representation here to you for the time per piece. So, what could the essential components of the time per piece. So, it will be of course, a non productive time per piece plus the cost of machining time per piece plus the tool changing time per piece as the overall total time that is applied for doing one particular single past turning operation on the system.

So, let us now substitute the usual course of values in terms of the tool life time, the cutting time and the actual cutting time. So, we obtained  $t_u$  to be equal to  $t_1$  plus  $t_c$  plus  $t_d$  times of  $t_s c$  by  $t$  this represents the frequency at which the tool is been changed; obviously, the cutting of the time for changing one cutting  $h$  that permit as remain as such same this is actual cutting time, this is the non productive time. And so we just substitute the value here by having again the formulation  $\pi L D$  by  $1000 v f$  which would described in the earlier cost model,  $L$  is the length and millimeter these the diameter of the work piece in millimeter,  $v$  is the cutting velocity in meters per minute and then  $f$  is basically the millimeter per revolution feet that we need to consider.

And so this would be again argument by the actual cutting time  $t_d$  times of  $t_a c$  the

actual time of cut again represented approximately by the same  $t_c$  value divided by the tool life from the Taylor equation 1 by capital T. So, we see  $v$  by  $c$  to the power of  $1/n$ . So, upon partially differentiating  $T_u$  with respect to  $v$  equating to 0 and solving the  $v$  we obtain  $\frac{\partial T_u}{\partial v} = \frac{\pi L D}{1000 f} \left( \frac{1}{v^2} - \frac{\pi L D}{1000 f c} \frac{1}{v^{n+1}} \right) = 0$ .

And from this condition we can actually arrive at a maximum velocity condition as  $c$  divided by  $\frac{1}{n-1} \times t_d$  to the power of  $n$ . So, again for the maximum production rate model, the total amount of tool life from the Taylor's equation can come out to be  $\frac{1}{n-1} \times t_d$ . So, you have a velocity from the tool life the maximum velocity corresponding to the maximum production time and the time corresponding to the maximum production time represented in this situation.

So, you have learnt now, so you have an optimization criteria based on cost with the minimum cost criteria per piece and then the minimum time criteria or maximum production criteria per piece and in both aspects you have certain end conditions which are defining the cutting velocity, which would be the optimum range of velocity of operation of the particular turning process.

So, I am going to delve into a little more into a practical example and show you how such machine selection decisions can be made or the machining conditions can be chosen for doing this maximum production rate or the minimum cost per piece model in the next module. So, this brings us to the end of the current module.

Thank you.