Manufacturing Systems Technology

Prof. Shantanu Bhattacharya

Department of Mechanical Engineering and

Industrial and Production Engineering

Indian Institute of Technology, Kanpur

Module - 03

Lecture - 13

Hello and welcome to this module 13 of manufacturing system technology. If I may recall last module we were discussing about the Bezier effect particularly, and we talked about how a characteristic polynomial can be generated, it is also known as the Bernstein polynomial and you know in that aspect what we also discussed was that how you know instead of the earlier complex requirement of the slopes at the ends on the points of the ends in the Bezier fit. What is needed are only 4 different points are crossed which you can maintain a curve fit only on the base point values and not the slope values. We also elaborated about how different curve sections can be connected to each other using a C 1 continuity where the radius of curvature the center of curvature may be different, but there should be 1 tangent at the point of intersection of both the different curve segments. So, we are going to discuss a practical problem today of the Bezier curve fit.

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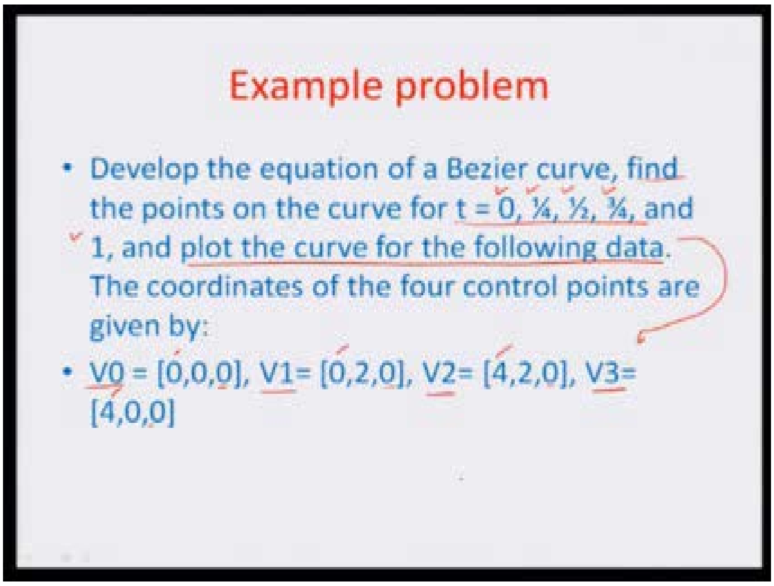

Let us look at this particular problem, where we want to develop the a equation of a

Bezier curve want to develop the equation of a Bezier curve given the values of the t, the different values of the t as illustrated here on the curve, and the different values that we are investigating to formulate the points are t equal to 0, t equal to 1 fourth half, 3 fourth and 1, and we want to plot the curve for the following data points. So, you have point v 0 point v 1 v 2 and v 3 and as it appears all the z are 0. So, therefore, it is actually a 2 dimensional case, it is the 2 dimensional curve that we are plotting. So, the first point is 0 to co ordinate axis x and y v 1 is 0, sorry 0 0 co ordinate axis x and y, v 1 is 0 2, v 2 is 4 2, v 3 is 4 0 so and so for.

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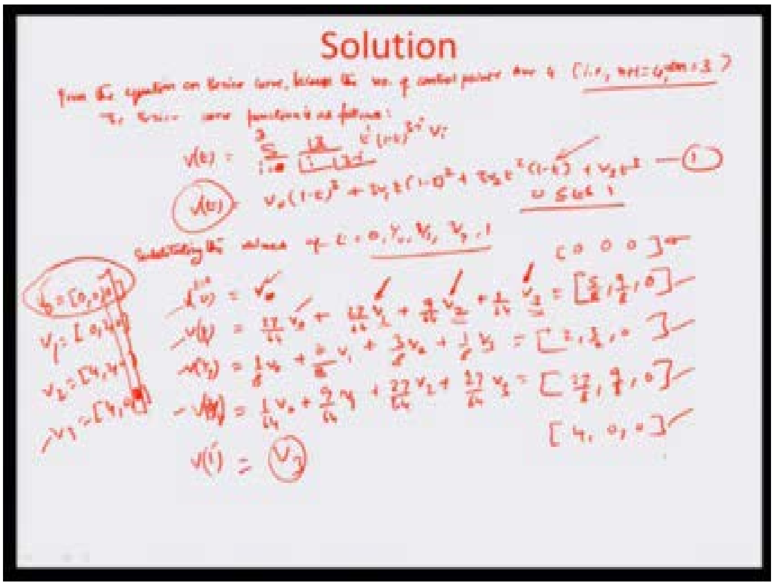

So, you want to develop a equation using the Bezier curve fit. So, from the equation on Bezier curve, because the number of control points, in this case are 4 that is n plus 1 equal to 4 or n equal to 3, you have already learned about this in the last class where it actually described and talked about the Bezier fit. So, given this the Bezier curve function can be described as follows v t equals sigma i varying between 0 and 3 factorial n by factorial i factorial 3 minus i that is 3 c 1, and n the maximum notation which is equal to 3 in this particular case corresponding to the fourth point t to the power of i 1 minus t to the power of 3 minus i v i, that is how it is defined, and if you expand this you can have the value of v t as v 0 1 minus t cube plus 3 v 1 t times of 1 minus t square plus 3 v 2 times t square times of 1 minus t plus v 3 times of 3 cube given t varies between 0 and 1.

So, substituting the values now and they are corresponding to the different values of t 0 1 fourth half 3 fourth and 1. So, simply have v 0; obviously, should be equal to the substitute v 0, you just solving this equation for the various values of t v 1 fourth corresponding to t equal to 1 by 4 can be obtained as 27 by 64 v 0 plus twenty seven by 64 v 1 plus 9 by 64 v 2 plus 1 by 64 v 3. Similarly corresponding to t equal to half the equation gets modified to 1 eighth v 0 plus 27, am sorry 3 by 8 v 1 plus 3 8 v 2 plus 1 8 v 3. So, all we are trying to do is to put the value of t in this particular equation here corresponding to v t to find out the various equations. Similarly v 3 fourth corresponding to a value of v equal to add the point t equal to 3 by 4 can be represented as 1 64 th v 0 plus 9 64 th v 1 plus 27 64 th v 2 plus 27 64 v 3, and that is actually nothing but the co ordinates of the points.

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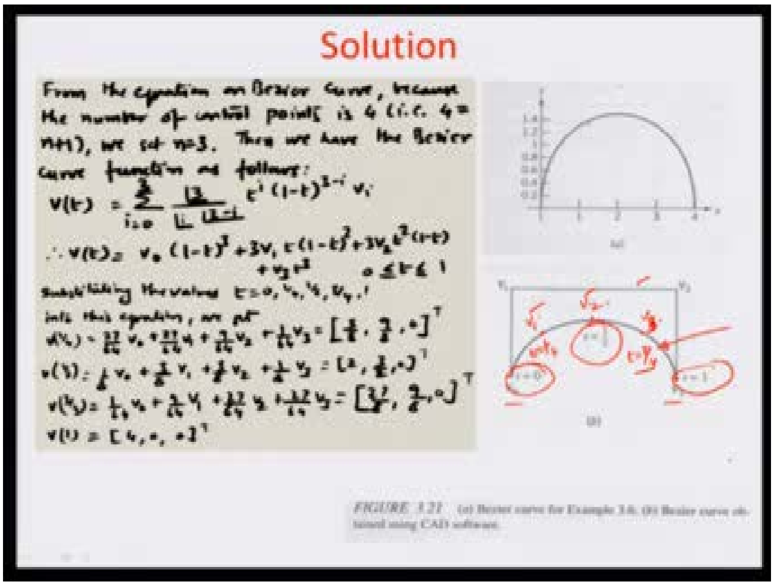

And that have been substituted and found before that has been found by substituting the values corresponding to v 0, v 1, v 2 and v 3, and these are the values which have been obtained earlier from the question. Remember we had talked about this 4 control points, which were the bases for creating this Bezier fit, am just going to write this here v 0 was given as 0 0 0 right. So, there is an x y and z co ordinate here which should leads the 0 0 zero; obviously, if I substitute the value of v 0 corresponding to all the x co ordinates, you obtain x co ordinates at these points corresponding to t equal to 0 1 fourth 1 half 3 fourth.

And then obviously, there as to be v 1 which would correspond to only v 3, then n value and value v 1 was at that particular obtained question obtained us or given as 0 to 0 v 2 was given as 4 to 0 v 3 has been given as 4 0 0. So, one thing very interesting about the Bezier fit, here the assumption basically was that the first and last points are on the curve itself is true here, because obviously, the x y z co ordinates have put into this equations to solve for the various v zeros to v ones becomes 0 0 0. So, this is matching as you see with the first point v 0. So, v 0 and corresponding to v corresponding to t equal to 0 and the point v 0 are the same. So, that is the last point. Similarly v corresponding to t equal to 1, and the v 3 the last point are same. So, it can be a as it as well replicated as 4 0 0 on the values of the co ordinates. So, now we substitute the x co ordinate of v 0 x co ordinate of v 1 x co ordinate of v 2 and x co ordinate of v 3 and calculate this thing and we obtain the value as 5 8. Similarly if I plug in the value of the y co ordinate of v 0 y co ordinate of v 1 y co ordinate v 2, and y co ordinate of v 3. We get the value 9 8, and if you plug in the values of the z value of v 0 v 1 v 2 v 3; all of them are 0. We know that we are talking mostly about a plain curve, in this case all the values of the z co ordinates as you can see here as 0, and only the x and y are varying. So, this becomes 0.

Similarly corresponding to v half the co ordinates become twice 3 by 2 0. Similarly for v 3 by 4 the co ordinates become 27 point 27 by 8 9 8 and 0. So, that is how you have mapped different values of t through, which you can plot now. And if you really plot all these together you get a curve like this right, which correspondence to probably some value of t here equal to 0 t at the value t equal to half, and then t equal to 1 again which is at this particular point. And obviously, the t 1 fourth and t 3 fourth are 2 points on this and between these 2 points v 0 and v 3, there are several other points like probably in this particular case v 1, v 2, v 3, and the v 4 point.

So, these different points corresponding to you know the 4 different points, which are also governing this particular curve, and the displacement of this points which may or may not be on the curve will actually control the topology of this curve very much, and that is how without really taking care of any slopes across any portion of the curve in this particular case the only the variation of the control points, you are able to vary the topology. So, that is how you can do the Bezier fit; for example, in a cads situation or in a cad problem which can map a lot of variety of topologies between such control points in question.

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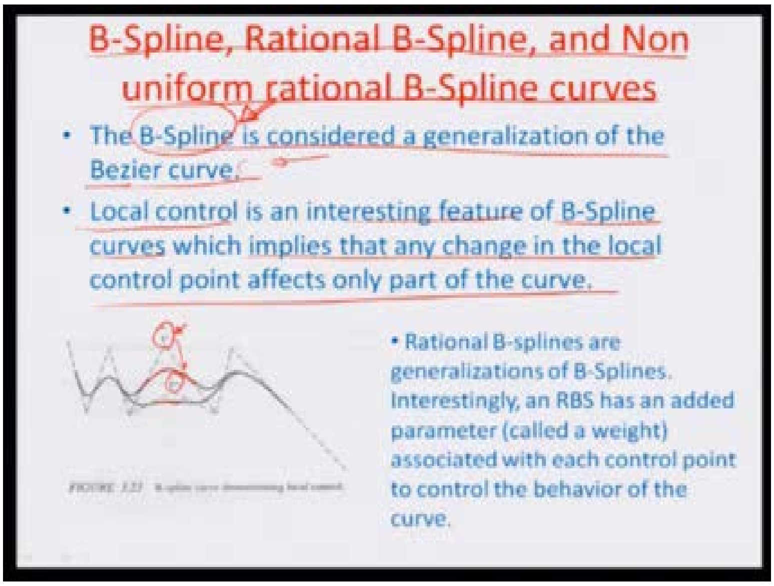

So, there are many other kind of fits which are possible, and I think I had mentioned about the B spline rational B spline and non uniform rational B spline curves. So, as for as the B spline is considered generalization of the Bezier curve is done in a manner to obtain a B spline, where a you know there is some degree of local control which is there. So, for from the only difference between the actually Bezier curve in the B spline is that local control becomes an interesting feature of the B spline curves, and it can imply that any change in the local control affects only a part or region of the curve. So, for example, suppose in this particular case there was a point v and dash, and it goes to a another point v double dash.

So, there is some kind of a change in the bending radius or center of curvature of this particular curve because of the control or because of the change over of this control point from v dash to v double dash. So, this feature is not available in a Bezier, because if you have only one control point changing it may change the overall curve geometry overall curve topology, and you cannot have a sectional change in that corresponding region or or governed by that particular point or the Bezier, in fact the B spline function is actually plotted in this manner.

So, in fact manner I mean this is not being covered now, because here the mandate is to slowly move towards little bit higher level manufacturing from the cad process. So, the idea was to give you an insight on to how geometric transformations etc are used to really now map everything in terms of coordinates, and interestingly these are the coordinate systems which would formulate the output data for the cad. And they would be the manured over the computation manufacturing systems to actually start producing the parts based on the coordinate data coming out of the cad. So, that is what we are actually interested in right now.

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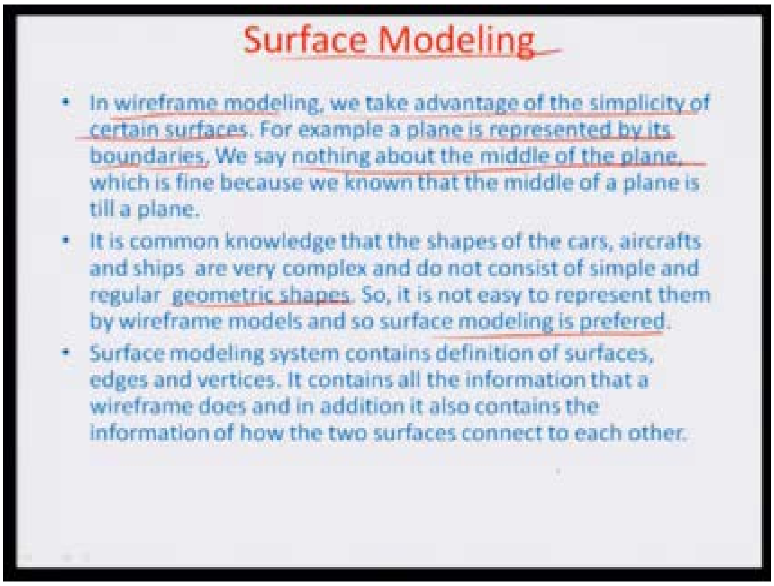

And before just producing ahead one small important section needs to be done, because so far we have talked about only 2 D curves, and obviously some illustration of 3 D curve, but you know a surfaces much more complex then only a curve, because of the surfaces is essentially an array of curves, and how to handle that array of curve becomes very important in terms of the level of computations in a 2 D curve. For example, the level of computation has been what you have already seen in the previous few lectures, but when you talk about a 3 D curve not only that an array of such 3 D curve we defines a surface the expansivity of the computations becomes enormous and you have to handle a very large volume of data.

So, what am going to do in is to give one case study, where we will talk about the hermit cubic fit polynomial to map a surface without readily going into all the local derivations involved, you know you have already done the 2 D curve before. So, by virtue of that you will probably be able to ah extrapolate that knowledge and be able to derive, but I will just mention the basic needs just to be give you an idea of the expanse of computation which is have which would happen, if you go from a 2 D curve to a surface case. So, let us about surface modeling little bit. So, obviously, you know that in wire frame modeling we take the advantage of this simplicity of certain surfaces particularly the regular geometries and a plane is represented by its boundaries for example,. So, this is a very, very regular geometry, we don not say anything in the wire frame model about the middle of the plane, which is find, because we know that the middle of the plane is still a sort of a plane, but obviously for complex shapes or irregular geometric shapes when we talk about may be a very complicated surface, it is not easy to represent. So, therefore, surface modeling is preferred in that particular aspect. So, let us look at surface modeling from a little more detailed ah view point.

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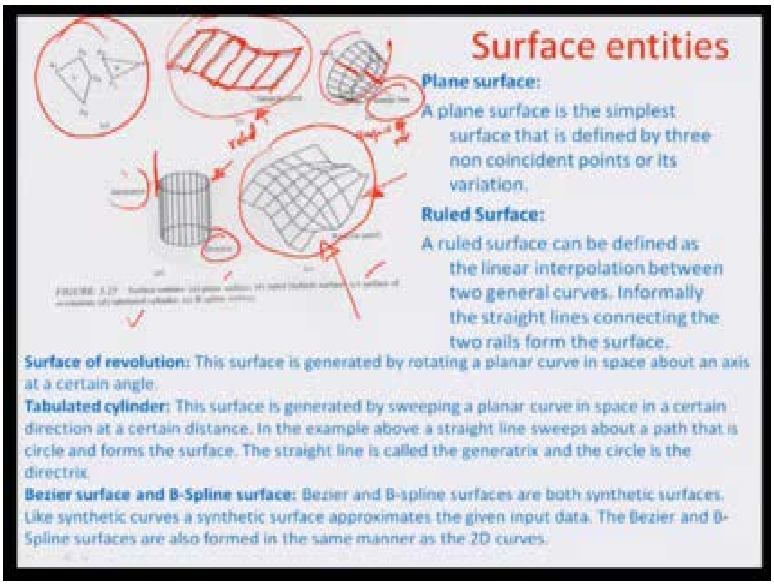

So, typically there are many ways of doing surface modeling one of them would be just by representing a certain, you know set of lines. Let us say in this particular case there are 2 curves which is represented by curve one, you know in this particular case. The curve 1 is somewhere like this, and the curve 2 is something like this. So, this is a general curve and the set of curves, and what you do is to simply a extrapolate the curve by making sections like, you know just like a sort of a slacked conveyor you know, and make different sections which represents the surface of the various sections of the curve and that is how the whole surface can be mapped in this particular manner.

So, this is a very crowd way of representing the surface then there is another representation of the surface, where we have a sweep line. For example, in this case there is a line here is one of the elements in your just rotating or sweeping it at an angle around an axes. So, there is another kind of you know surface generated probably, because of the revolution of this particular line along the axes. So, in this particular case the definition of the surface here is called a ruled or a lofted surface in this case, we call it surface of a revolution base creation of a surface, surface of revolution that are other ways like you can have a tabulated cylinder, where there is a something called a directris and a genetrics.

So, the genetrics is basically the element, which is needed to be generated and the directrics is a direction in which these element would moves. So, if it moves in a regular manner on a circle we get a surface. So, this is another way of doing you know classifying any product I mean any particular surface with respect to a genetrics or a or a regular feature or a line in a directric switch goes around. And then finally, there is something which is very complex which is called a surface fit which is represented here. So, if you look at the various surface entity is based on whether they are regular or irregular, you can have these all different kind of a representations from a planes surface to a ruled surface to a surface of revolution to tabulated cylindered to a B spline surface to represent such geometry.

So, most complex is obviously, this patch here which we called a Bezier patch or a B spline patch or hermit cubic spline patch, and am going to a actually now look into how to generate that patch in the next module. So, with this I would like to end the module here and in module 14, we will discuss more about the patch formulation and how you can map that as a surface entity.

Thank you.