Manufacturing Systems Technology

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Module - 02

Lecture – 11

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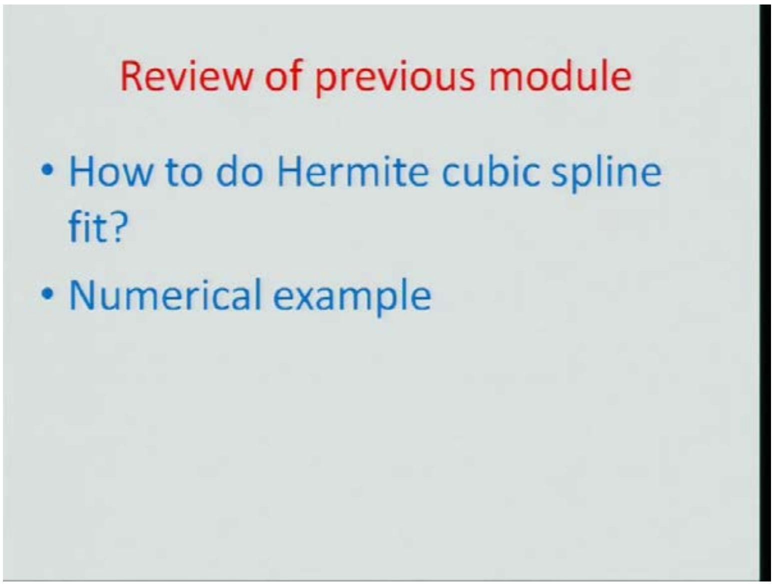

Hello and welcome back to this 11th module on Manufacturing System Technology. We were discussing in the last lecture how to do Hermite cubic spline fit and you know situations when you have multiple points and you have at least 2 end points to which a certain line or certain expression is to be fitted. And also, not only you have the points on the ends but also slope at the ends.

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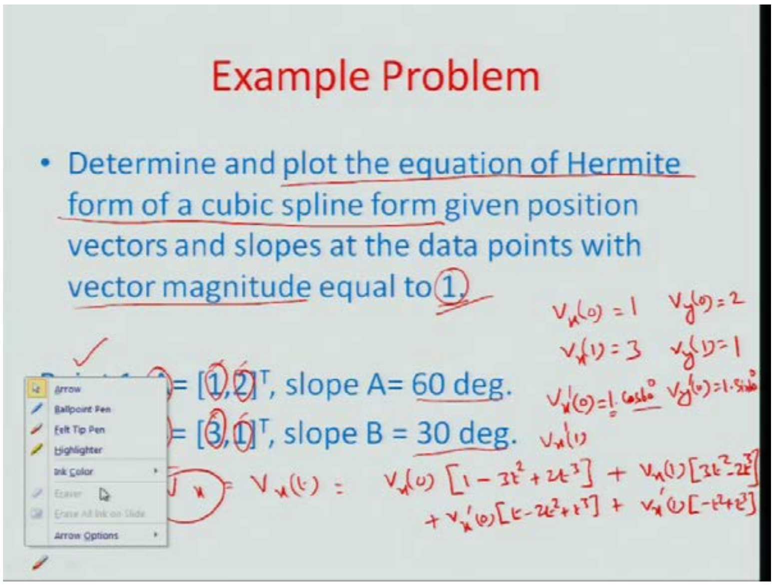

We would try to formulate the numerical problem where, we wanted to do actually see a realistic situation as given here where, there are 2 points: point 1 and point 2 whose coordinates and the slopes on those particular points are given. Also it is given that the vector magnitude of the slope will be equal to 1. So, it is a unity slope or unit slope on both the end points and we want to find out how to plot the equation of Hermite form of a cubic, Hermite cubic spline fit in this particular situation.

So, what we try to do here is that, we want to illustrate how to obtain the cubic spine in hermits form. And for simplicity, we know that if we really considered the equation which is used for this purpose, V x t equals V x 0 corresponding to 1 end point times of 1 minus 3 t square plus twice t cube. And I am just borrowing this equation of the derivation that we have done while actually looking at Herbit cubic spline fit, time plus V x 1 times 3 t square minus twice t cube plus V dash x 0 times of t minus 2 t square plus t cube plus V dash x 1 times of minus t square plus t cube.

That is how you represent the V x and you already have formulation where you know that V x 0 is actually equal to 1; and V x 1 is equal to 3 from the 1st x-axis coordinate points of the point A and B. Similarly V y 0 equals 2 and V y 1 equals 1 from the y-axis coordinates of the point A and B. Further we also know that V x dash 0 the slope at 0 is 1 cos of 60 degrees, where 1 is the magnitude of the tangent; and of course, cos 60 is the x component of the slope or x component of the tangent. And similarly V y 0 which is 1 times of sign of 60, and in this particular case again you have V x 1, V x 1 dash equals to you know you have 30 degrees as the slope at the point corresponding to x equal to 1.

The other end of the particular fit. So, it is 1 cos of 30 degrees and V y dash 1 corresponding to 1 sign of 30 degrees. So, given so many different parameters here, can we really put it back into this equation fit and see how it goes.

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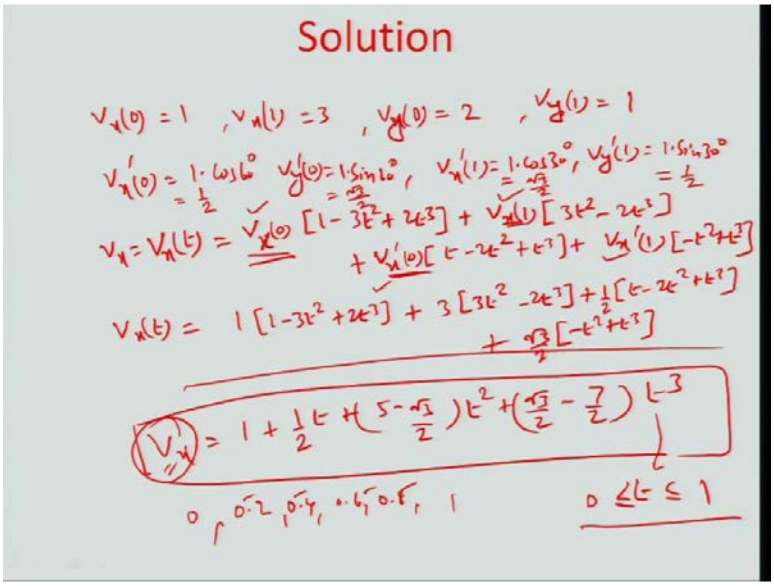

So, let is just do that in the next step, we have just go to write down all the parameters once again. So, V x 0 is 1, V x 1 equals 3, V y 1 let is put it 0 in the same sequence this is equal to 2 V y 1 equals to 1. And similarly you have V x dash 0 equals 1 cos of 60 degrees, V y dash 0 equals 1 sin of 60 degrees; and similarly V x dash 1 equals 1 cosine 3 degree and V y dash 1 equals 1 sin of 30 degree. Is given this expression you want to just solve for V x which is equal to V x as function of t, parametric form; V x 0 times of 1 minus 3 t square plus twice t cube plus V x 1 times of 3 t square minus twice t cube plus V x dash 0 times of t minus twice t square plus t cube plus V x dash 1 times of minus t square plus t cube, about from earlier. So, just putting the values the different values of V x 0, V x 1, V x dash 0, and V x dash 1. We get V x t in the function of t parametric form equals 1 times of 1 minus 3 t square plus twice t cube plus 3 times of 3 t square minus 2 t cube plus the V x dash half times of the coefficient in t. Which is I am sorry, which is t minus 2 t square minus 2 t square plus t cube plus V x dash 1 which is root 3 by 2 times of minus t square plus t cube.

So, if you want to algebraically solve this further we are left with V x an expression for: V x 1 plus half t plus 5 minus root 3 by 2 t square plus root 3 by 2 minus 7 by 2 t cube. So, this is how you can calculate the value of V x given a certain value of t now. If t varies between 0 and 1 and the increments of let us say 0 point 2, 0 point four, 0 point 6, 0 point 8 and so on and so forth, 1 on one side and 0 on other. So, you actually have the mapping of the function V x and all these different values to exasperate the local domain in this whole global function and that way you can formulate a good fit. So, this is on the x coordinate though, we have to do the same job for the y coordinate.

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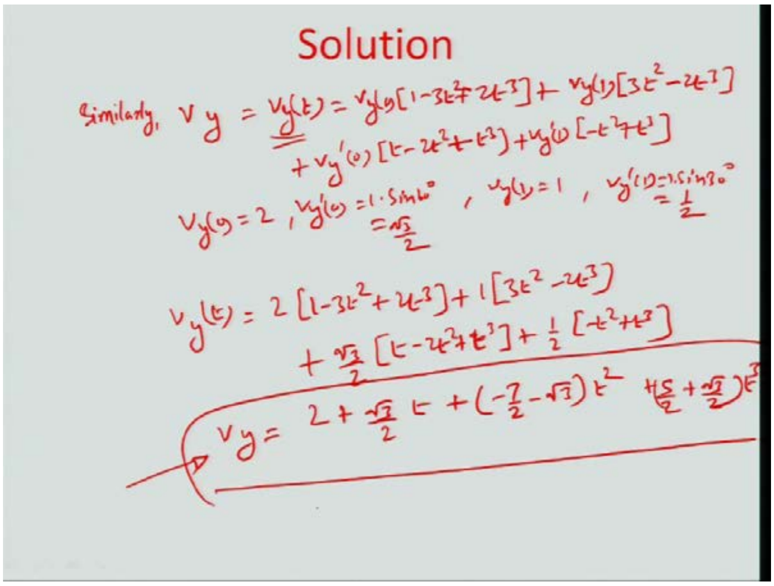

So, similarly, we can write V y as V y t which is actually V y 0 and in a similar manner times of 1 minus 3 t square plus 2 t cube; and this equation really has been borrowed from the previous module. Where we arrived at how this equation comes into existence; V y 1 times of 3 t square minus twice t cube plus in a similar manner V dash y 0 times of t minus twice t square plus t cube plus V y dash 1 times of minus t square plus t cube. That is how you calculate the V y t, and in this particular module again substituting the different values of V y 0 to be 2 V y dash 0 to be 1 sin of 60 which is root 3 by 2 again V y 1 to be equal to 1 and V y dash 1 to be equal to sin of 30 which is actually half.

So, we have to now, we actually get another equation of similar type in y. So, you can say that V y is the function of t becomes equal to 2 times of 1 minus 3 t square plus twice t cube plus V y which is 1; V y 1 is 1 times of 3 t square minus twice t cube plus V y dash 0 which is root 3 by 2 times of t minus twice t square plus t cube plus V y dash 1 which is half times of minus t square plus t cube. So, again simplifying in a similar manner as before the V y comes as a function of t as twice plus root 3 by 2 t plus minus 7 by 2 minus root 3 t square plus 5 by 2 plus root 3 by 2 t cube. That is how we can be derived at and so this can report the y value for the function where the x value was earlier reported by this particular expression. So now, if you really want to verify whether what we are saying is true, let us look at the end conditions. I am just going to write down both the equation again in the next page.

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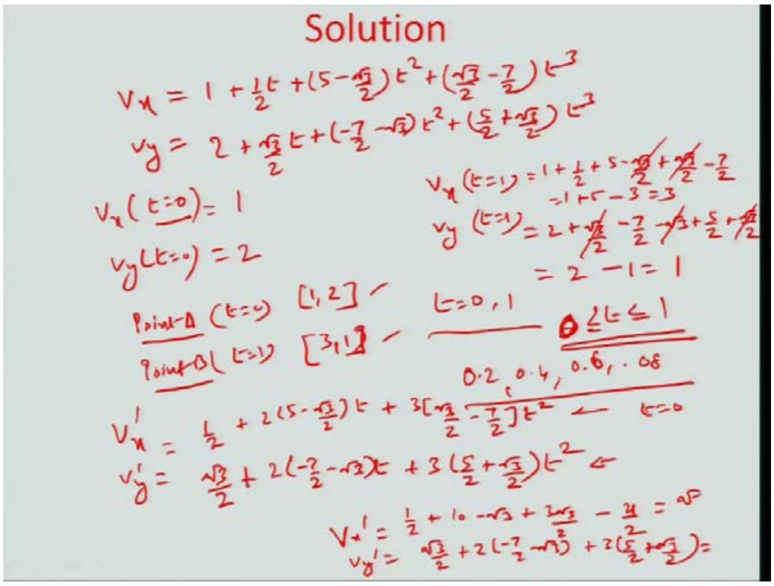

So, V x in our case was obtained as 1 plus half t plus 5 minus root 3 by 2 t square plus root 3 by 2 minus 7 by 2 t cube; and V y was obtained again as twice plus root 3 by 2 t plus minus 7 by 2 minus root 3 square of t plus 5 by 2 plus root 3 by 2 cube of t. So, add the outside let's just find out what would be the V x and y 0, by substituting the value of t equal 0. And similarly find out what is V x at t equal to 1 and so on V y at t equal to 1. So, we get here by substituting t equal to 0 that V x becomes 1 and V y becomes 2 and if you substitute 1 and then, this expression becomes 1 plus half plus 5 minus root 3 by 2 plus root 3 by 2 minus 7 by 2; which eventually falls down to 1 plus 5 minus 3 is again 3. And the other expression falls down to 2 plus root 3 by 2 minus 7 by 2 minus root 3 plus 5 by 2 plus root 3 by 2 putting the value of t equal to 1. So, these strike off and you left with 2 minus 1 which is 1. So therefore, this replicates the n conditions. You already know that point 1 corresponding to equal 0, was add the coordinate point 1 2 and point B at t equal to 1 was at the coordinate point 3 1. That is how we initially started with as seen here.

So therefore, whatever fit we have done through this whole process, is actually yielding us the data particularly for the n conditions; corresponding to t equal to 0 and t equal to 1. So obviously, for a value of t varying between 0 and 1 and as I illustrated before point 2 let us say increments of 2, point 4, point 6, point 8 so on and so forth. You should able to fit this equation appropriately.

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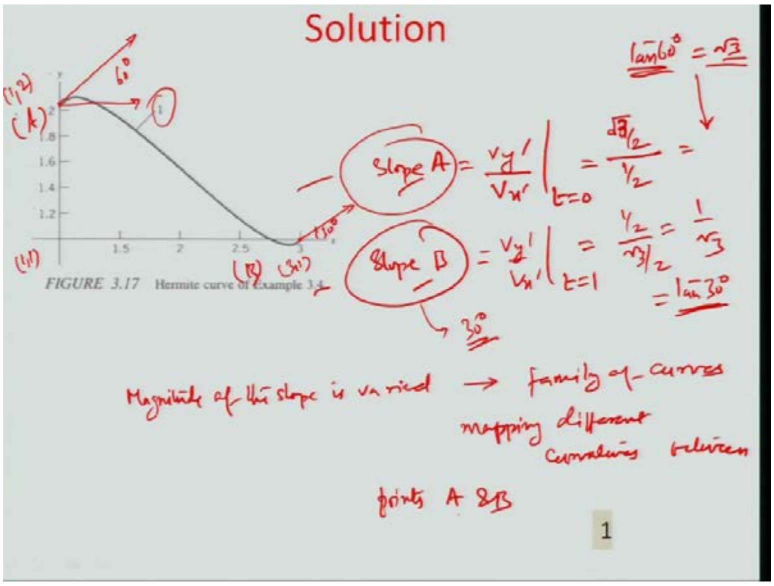

This is how you can actually do the fit; you can see the values of different values of t with slope of 1 you now taken, and this is how the fit results from the slope. So, here of course, if I look at this slope at the point A, which again you know by the Hermite cubic spline phenomenon can be written down as V y dash by V x dash at the point t equal to 0. So, I can actually physically calculate what is the value of the slopes by looking at what is our V dash x and V y dash from this expression. So, the slope becomes 1 by 2 plus twice 5 minus root 3 by 2 t plus thrice root 3 by 2 minus 7 by 2 t square and similarly this becomes root 3 by 2 plus twice minus 7 by 2 minus root 3 t plus thrice 5 by 2 plus root 3 by t square. And if I substitute the value of t equal 0 for example, V x dash becomes equal to 1 by 2 and V y dash becomes equal to root 3 by 2. So, therefore, in this particular expression V y dash by V x dash comes out to be root 3 by 2 by 1 by 2 which is root of 3. Which is actually at the slope at A is corresponding to tan of 60 degrees which is root 3. And similarly we can calculate, what is the slope at B by looking at the V y dash by V x dash expression corresponding to t equal 1.

So, if I substitute the value of t equal to 1 in these 2 equations here, the V x dash and the V y dash now changes because now, t is 1 and here we get 1 by 2 plus 10 minus root 3 plus 3 root 3 by 2 minus 21 by 2. Again the V y dash becomes equal to root 3 by 2 plus twice minus 7 by 2 minus root 3 plus thrice 5 by 2 plus root 3 by 2. So, these come out to be. So, this is equal to root 3 by 2 and this is again equal to 1 by 2 and this can be again extended here; as 1 by 2 divided by root 3 by 2 and it comes out be 1 by root 3.

Obviously, this corresponds to tan of 30 degrees and it obviously, means that at B these slope is corresponding to 30 degrees.

So, we are kind of mapping both the information that we had in the question related to the coordinates at the point corresponding to t equal to 0 and t equal to 1, and also the slopes at the point t equal to 0 and t equal to 1 in this particular expression. And as you can see here the whole map is between two such points here. Here the coordinates start from 1 1. So, you have 2 1 as 1 2 as 1 of the coordinates from which the fit starts. And here probably the slopes as you can see is actually is 60 degrees and the other coordinate is add the point 3 1 again. And here the slope is corresponding to 30 degrees. So, that is pretty much how with the magnitude of 1 of the slope you can have it curve fit. Now, the question is can I be able to vary this curve. So, that we can map different topologies and the solution for that is suppose the magnitude of the slope now, is and you can try it yourself. Magnitude of the slope is varied; you can have a family of curves. Mapping different curvatures between the point A and the point B.

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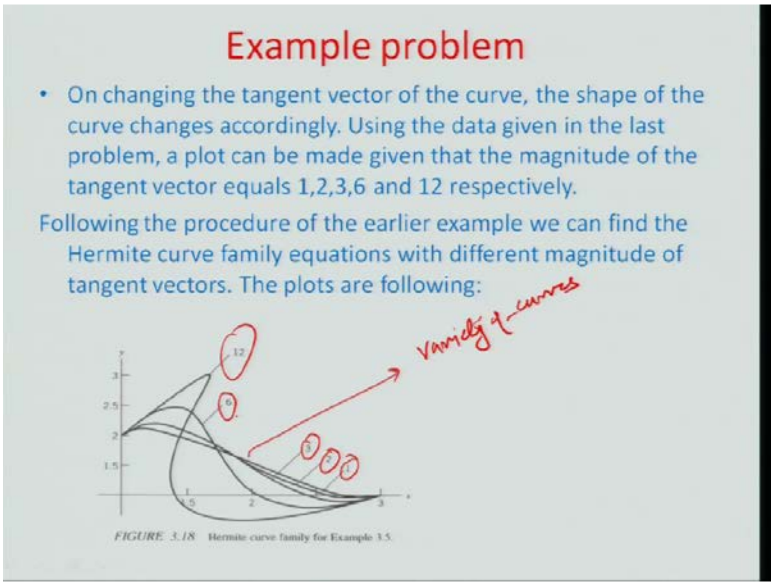

It can be looked at here the how I just the change of slope from 1 to 2 to 3 6 to 12, we have variety of topologies or variety of curves; you know. And so, the idea is to now be able to fit whatever curve you have to any of this family of curves and that would be the corresponding fit between the points 2 and 3. Suppose you have to define complex topology between the point A and B that is 2 1l; sorry, 1 2 and 3 1. Between these 1 particular curve may be corresponding to, let us say this slope equal to 6 would be found to be very close to the actual topology with less, with very less error and so this can rightly map the topology correctly. So, that is the whole basis of doing such a curve fit.

So, we this brings us to the end of the Hermite cubic spline fit, but the problem with Hermite spline fit as you know that the information needed is not only the end points but, also the slope. And the slope is not so easy to measure on the realistic scale when we are doing metrology. And so therefore, we move all together into the different domain of the curve fit which is called fitting of the Bezier function which will do in the next module.

So, thank you as of now.