Manufacturing Systems Technology

Prof. Shantanu Bhattacharya

Department of Mechanical Engineeringand

Departmentof Industrial and Production Engineering

Indian Institute of Technology, Kanpur

Module - 02

Lecture - 10

Welcome to this module 10 on Manufacturing Systems Technology,a quick recap of the last module.

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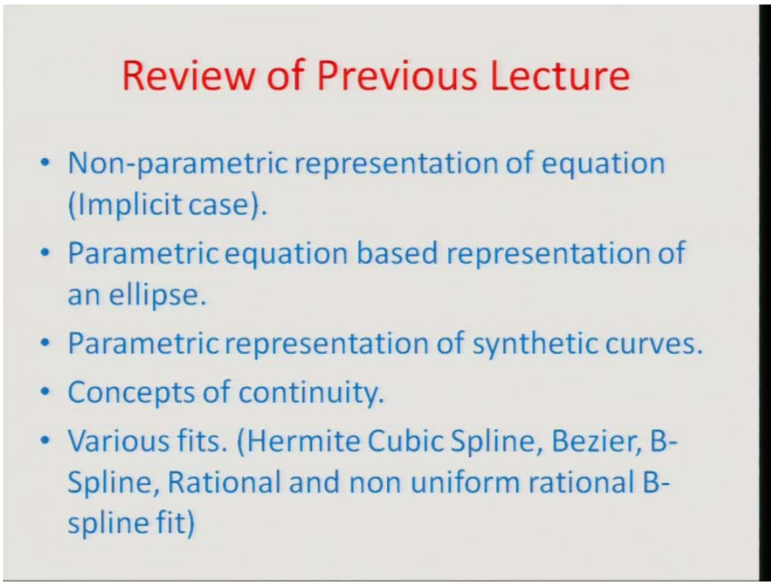

We were talking about or discussing about the non parametric representation of equations of a straight line followed by an ellipse. And then,we discussed about the implicit case,in which an ellipse can be represented or formulated a parametric equation in terms of an angle alpha,which varies between 0 and 360 degrees. And then, basically also talked about how a complex topology can be split up into different synthetic curves with a precedence relationship between the different elements of the curve and discussed concepts of continuity in this particular module.

Further I mentioned that probably, today I am going to talk over a little bit details about how this fits of the various sections of the curves can be made and also how the variation of the curves along a certain domain, a local domain can be made by changing the different parameters related to the fit that is in question.

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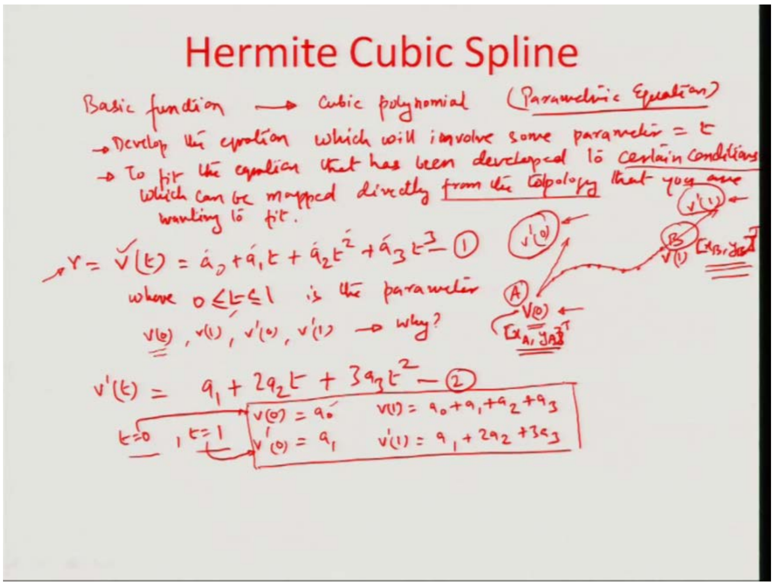

So,let us look at the Hermite cubic Spline fit or how to plot a Hermite cubic Spline curve. So, what are the basic assumptions in such a system?The first thing that we probably need to find out is the basic function,which in this case terms out to be a cubic polynomial. Obviously, what we are proposing is a sort of a parametric equation,which is more flexible as you have seen before that you can really vary it over a local domain given a certain global rangewithin, which the curve is there.

And, what are all this needed is that first of all we need to sort of develop,theequation which will involve some parameter.Let us say we call this parametert, which will have a cubic form of course,and the next step is to fit the equation that has been developed to certain conditions,which can be mapped directly from the topology that you are wantingto fit. So, let us say we have an equation a cubic polynomial function v t equals a 0 plus a 1t plus a 2 t square plus a cube t a 3 t cube, where t varying between 0 and 1is the parameter.

The Hermite cubic Spline tries to define the curve being estimated more meaningfully in terms of certain end conditions and these n conditions can match with the conditions of the topology that you would like to map.In this particular case,it sohappens that,the basic premise is that the curve; that is being formulated between two such points point,let us say A and point Bon the topology corresponding to the parameter t equal to 0 and a function v, which has been indicated here and v 1.

So, in a way on the topology if you have a xyz coordinate system you can plot, what really is the x and y dimensions or coordinates on that topologyand this can be mapped by any measuring system like CMM etcetera.And then, also these slopes at both the ends,which are important for formulating this particular fit, we call it v dash 0 and v dash 1. So, the n conditions which are more like a measurable conditions are the points along both the ends, which you need to fit and the slopes at those points,which are sort of the global domain for the whole curve starting between A and Bas can be seen in this particular figure.

So, how do we develop this connectivity or how do we develop this set of you know different points,which gives this cubic relationships between the point A and the point Bon the topology is the main question. So; obviously, the n conditions that we probably need to use are v0, v1, v dash 0and v dash 1.Why are we choosing these n conditionsalso is quite important and it has to be addressed.So, let us look at that how we can really do a curve fitting of this particular equation.

So, the best idea is to be able to keep on varying t between 0 and 1as has been illustrated here and try to find out, what are these different parameters a 0, a 1,a 2 and a 3given the end conditions, which are already known to you. So, v 0,which is the end which is representing the end A is corresponding to some coordinate point xa,ya and let us say this is measurable data from coming out of some CMM or something.

Similarly, xb, yb some coordinate point and you know the slopes are also the measurable data at both the ends of the topology.We will come back to address this issue that how difficult it is to really measure the slope and one of the reasons why Hermite Cubic Splinefit may not be preferred in comparison to some other fits like Bezier etcetera, where only the end points are needed and the variations are needed to sort of map the topology.

So, here let us say we first of all try to calculate the v dash t,which is essentially a 1 plus twice a 2 t plus 3 a 3 t square.And then, we try to put the n conditions corresponding to t equal to 0 and t equal to 1and find out, what are the different n conditions.We already know the set xa, ya,which is corresponding to the n condition v 0. Similarly the set xb, yb,which is corresponding to the n condition v 1and this data is as I told you earlier measured data,which can be used for fitting to that topology, which you are measuringthis particular piece of the curve synthetic curve.

So, here v dash 0is actually equal to a 1and v 0 from this equation you know, let us say equation 1 and equation 2, the v 0is equal to a 0.And similarly, if we go on the other side and do v 1,v 1 comes out to be equal to a 0plus a 1 plus a 2plus a 3and similarly, v dash 1 comes out to be equal to a 1 plus twice a 2 plus thrice a 3. So, at the two n conditions corresponding to t equal to 0 and corresponding to t equal to 1,we have now sort of four equations, where mind you this v 0, v dash 0, v 1 and v dash 1they are all known variables.

So, the v 0, v dash 0, v 1 and v dash 1they are all known variables and we want to represent the unknowns, which are the a 0 to a 3 in terms of the known variables.So, what we do is we try to solve these equations, which have been formulated,these are linear equations.

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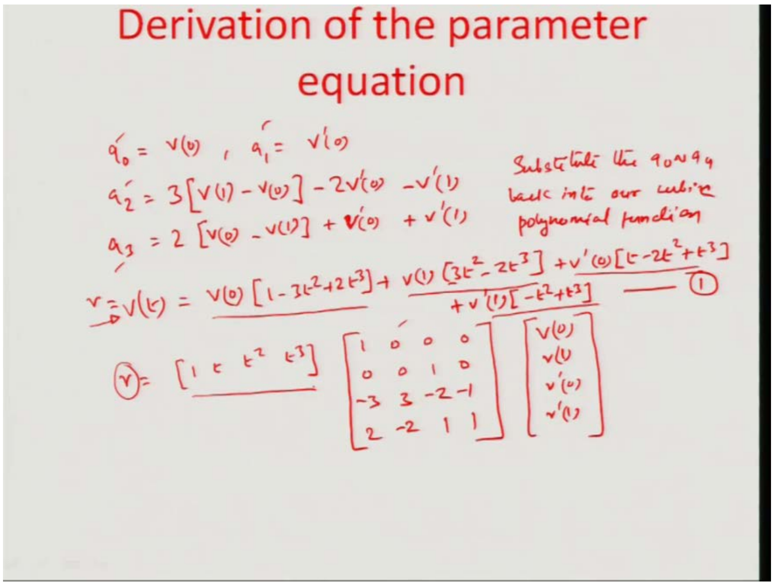

And come up with a solution,which is written in a manner that a 0is actually equal to v 0,a 1 equals v dash 0, a 2equals thrice v of 1minus v 0minus twice v dash 0minus v dash 1and a 3 equals twice v 0minus v 1plus v dash 0plus v dash 1. So, once these parameters a 0 to a 3 are known,we can actually substitute these back into our cubic polynomial function, sothat you could fit the particular polynomial to that topology which is in concern.

So, therefore, vt the final polynomial can be written down as v 0 if I do some algebraic manipulation in a way that you know we assemble all the v 0's or the coefficients with the v 0's v 1 v dash 0 v dash 1as together. So, we have v 0 times of 1minus 3 t square plus twice t cube plus v 1 times of thrice t squareminus twice t cube plus v dash 0times of t minus twice t square plus t cube plus v dash 1times of minus t square plus t cube as the main equation,which is the fitted equation.

In this particular case you can see that the n conditions v 0v 1v dash 0and v dash 1are known again I am just sort of ascertaining this point and based on that you can develop an equation in cubic order,which can actually fit the real life surface of the real topology which is in particular question.So; obviously, we are doing a 2D mapping right now, but this can be extendable to the three dimension, where you also have the z component it will become a little more complex and we will take this up separatelyas a module.

But,if you are doing a two dimensional curve fit kind of a situation between two points it is very easy to go in this manner and formulate it fit. So, if I were to construct this equation for the r in terms of you know multiplying matrices we can say that r becomes equal to1 plus 1 t t square t cube times of some kind of a basis matrix times of the values v 0 v 1v dash 0and v dash 1and this basis matrix if you lay this equation out in a proper manner equation one out in a proper manner and try to separate out the multipliers comes out to be 1, 0, 0, 0, 0, 0, 1, 0minus 3, 3minus 2 minus 1, 2 minus2, 1, 1.

So, that is how you can actually do a fit with this basis matrix a matrix, which has all the essential slopes and the n conditions the n slopes and the n coordinates and the of the vector comprising of the various orders of the parameter t and I would just like to illustrate it in the next module, how sort of in the next example problem that how you can actually do a curve fit, how such a fit can be made.

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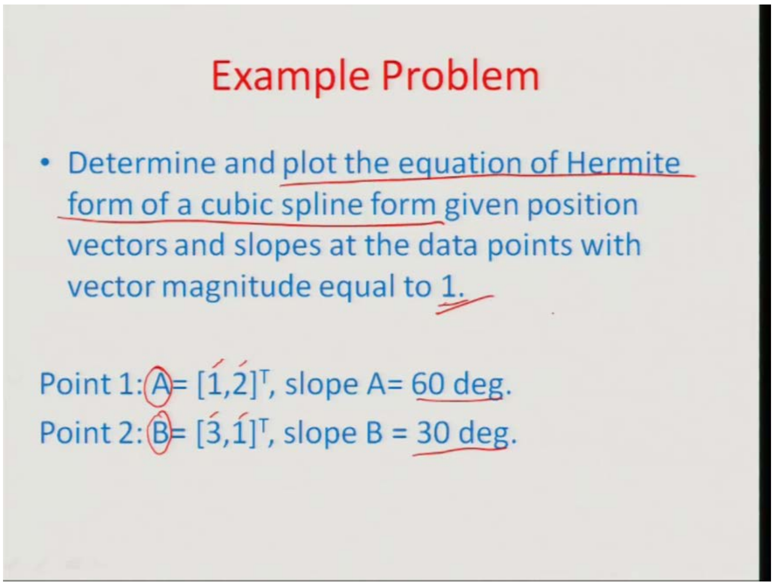

For example in this problem we want to determine and plot the equation of Hermite form of cubic Spline fit and we have been given position vectors and slopes at data points with vector magnitudeis equal to 1. So, the data points are point a of point 1 corresponding to A on the curve that I had illustrated earlier the x and y values are 1 and 2.And similarly, at B the x and y values are 3 and 1; obviously, the slopes are also given to be 60 degrees and 30 degrees at both the n conditions.

So, in this manner we have to fit the equation that we had formulated earlier to these n conditions and then, we will try to physically plot them and see, what is really the variability of the surface aspect.

So, if you vary the different end conditions as you will see in this particular case mostly it is the magnitude of the slope that will vary the curve there would be a family of curves, which are generated and the idea is, then that whatever topological mapping is needed can be done by looking at the topology and looking at this variation in the local domain of the curve and several such synthetic curves are plotted in unison to one another, sothat the whole surfacecan get mapped up, sowe are going to do this particular plotting and mapping in the next module.

Thank you.