BioMEMS and Microfluidics
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Lecture – 30

Hello and welcome back to this thirtieth lecture of bio microelectromechanical systems.

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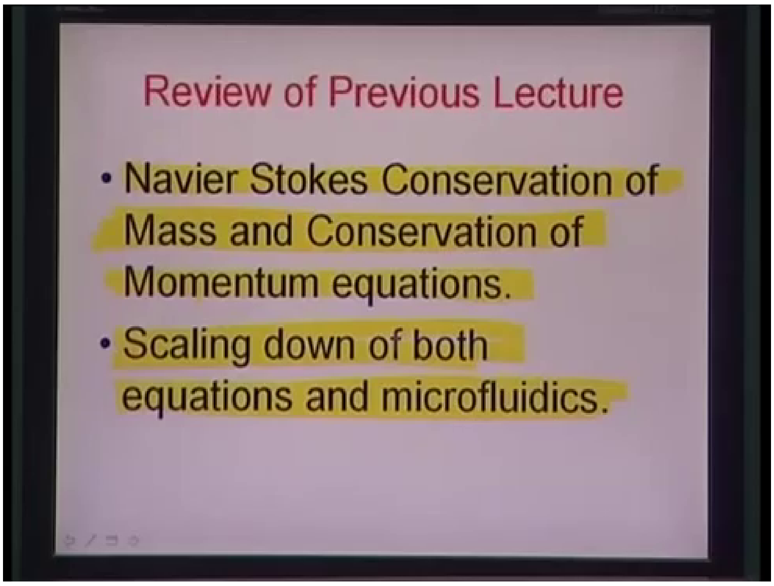

Review of Previous Lecture

- Navier Stokes Conservation of Mass and Conservation of Momentum equations.
- Scaling down of both equations and microfluidics.

Now, just quickly preview what we did last time; we talked about these famous Navier-Stokes conservation of mass and conservation of momentum equations and tried to derive them from the elementary control volume concept; and then, actually tried to go ahead and scale down both of these equations; and tried to figure out how they apply to microfluidics.

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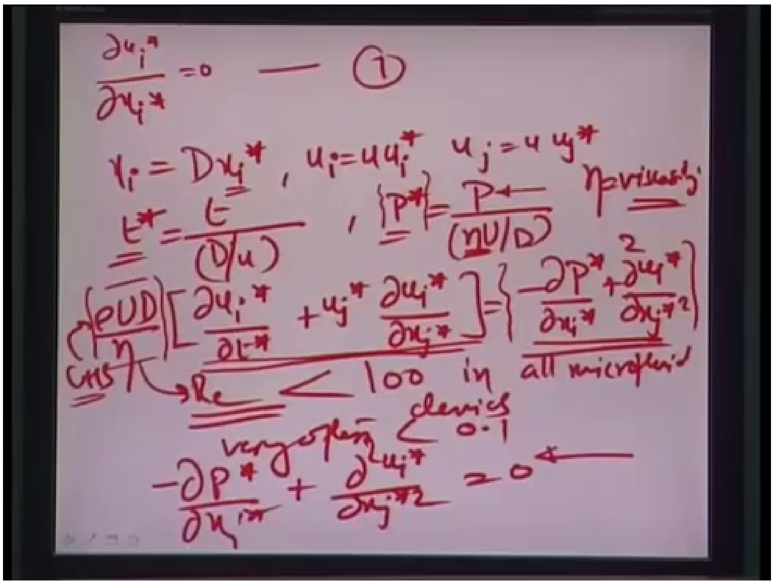

$$\frac{\partial u_i^*}{\partial x_i^*} = 0 \quad \text{—} \quad ①$$

$$x_i = D x_i^*, \quad u_i = u u_i^*, \quad u_j = u u_j^*$$

$$t^* = \frac{t}{(D/u)}, \quad P^* = \frac{P}{(\eta U/D)} \leftarrow \quad \eta \rightarrow \text{viscosity}$$

$$\left(\frac{\rho U D}{\eta}\right)\left[\frac{\partial u_i^*}{\partial t^*} + u_j^* \frac{\partial u_i^*}{\partial x_j^*}\right] = \left\{-\frac{\partial P^*}{\partial x_i^*} + \frac{\partial^2 u_i^*}{\partial x_j^{*2}}\right\}$$

const → Re < 100 in all microfluidic devices

very often < 0.1

$$-\frac{\partial P^*}{\partial x_i^*} + \frac{\partial^2 u_i^*}{\partial x_j^{*2}} \approx 0 \leftarrow$$

Let us actually go ahead and just reiterate a little bit of what the conclusions were out of this scaling down. We tried to first take the conservation of momentum equation. And, the scale down version of this equation came out to be $\frac{\partial u_i^{\square}}{\partial x_i^{\square}}=0$. And, this essentially indicated… And, let me just reiterate that, the way that these expressions came into picture – x_i would be related to a scaled distance at that particular scale times of $x_i^{\square}$; where, $x_i^{\square}$ is a dimensionless number. And similarly, u_i and u_j – both were equated to a scaled velocity at that particular scale times of $u_i^{\square}$. And, the same scale of velocity times of $u_j^{\square}$ respectively. And, also we saw that, the time star of the non-dimensional cum quantity in time in this particular scale would be presented as the original time divided by the distance by u – the velocity. $x_i = D x_i^{\square}, u_i = u u_i^{\square}, u_j = u u_j^{\square}, t^{\square} = \frac{t}{(D/u)}, P^{\square} = \frac{P}{(\eta U/D)}$

And, the pressure star, which is again a non-dimensional quantity in pressure in this particular scale was represented by – of the term p in the absolute value of pressure divided by eta u by D. As we all know eta is the viscosity; it is actually scale independent. So, it remains constant even if you go to the micron scale. You have to assume though that, the equations are that of continuity equations or continuous maintain for the viscosity to be independent of time and a constant property associated with the medium. So, because of micro-flows are mostly in this domain, where the continuum is still assumed and all the modeling is done using the continuity equations. Therefore, it is almost always obvious to assume the viscosity to be constant at that particular scale. And so, therefore, the pressure star – that means p star, which is essentially the dimensionless number in the Navier-Stokes equation would be represented as $P^{*}=\frac{P}{(\eta U/D)}$. So, we derived the second equation – the conservation of momentum equation and you figured out that,

$$\frac{\rho uD}{\eta}\left[\frac{\partial u_i^{*}}{\partial t^{*}}+u_j^{*}\frac{\partial u_i^{*}}{\partial x_j^{*}}\right]=\left\{\frac{-\partial P^{*}}{\partial x_i^{*}}+\frac{\partial^2 u_i^{*}}{\partial x_j^{*2}}\right\}$$

$$\Re=\frac{\rho uD}{\eta}$$

And, this we found out last time – the scaling down. And, the Re is more or less less than 100 in all microfluidic devices; very often less than 0.1. So, almost always gives you an opportunity to write down minus del p star by del x i star. So, the right-hand side of this particular equation plus del 2 u i star divided… Or, del x j star square will be equal to 0, because the left-hand side here is essentially dominated by the lesser magnitude of the Reynold's number.

$$\frac{-\partial P^{*}}{\partial x_i^{*}}+\frac{\partial^2 u_i^{*}}{\partial x_j^{*2}}$$

So, irrespective of whatever this quantity here is, the overall size of the quantity in the LHS because of the low value of Reynold's number is very small in comparison to this quantity inside the brackets. So, because it is overall negligible, we can assume this RHS part of the equation to be equal to 0.

So, what are our conclusions from this particular statement? So, one conclusion, which almost always comes out is that, at this particular scale, the momentum equation becomes time independent. Therefore, it is almost always obvious to assume that, if suppose you have two or three flows, which are introduced into a same micro-channel and we assume a low value of Reynold's number, due to which the left-hand side of the this continuity equation or momentum equation becomes 0; the obvious conclusion would be that, the pressure-driven flow – number 1; and, the number two is that, it is time independent. So, therefore, if you could actually run past both these different types of flows from the outlet and back into the inlet and that, you reverse the flows in time, they should be able to emanate as original inlets to the particular channel. So, this is illustrated in this particular example here. If you see again, I would just like to reiterate and this slide was shown before.

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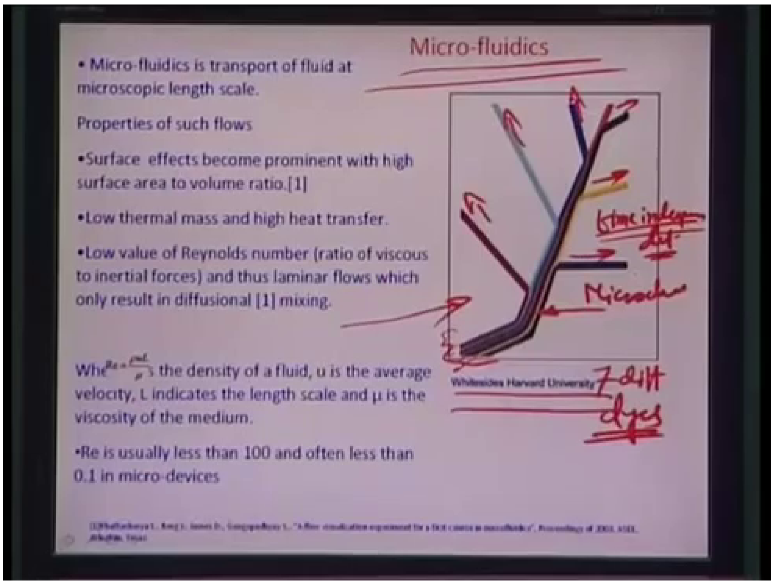

Micro-fluidics

- Micro-fluidics is transport of fluid at microscopic length scale.

Properties of such flows

- Surface effects become prominent with high surface area to volume ratio.[1]
- Low thermal mass and high heat transfer.
- Low value of Reynolds number (ratio of viscous to inertial forces) and thus laminar flows which only result in diffusional [1] mixing.

Wh $Re = \frac{\rho u L}{\mu}$ the density of a fluid, u is the average velocity, L indicates the length scale and μ is the viscosity of the medium.

- Re is usually less than 100 and often less than 0.1 in micro-devices

There in microfluidics, very often there are situations like this here, where you are seeing; where, you have these different dyes, which are flowing into this particular channel. You have about 1, 2, 3, 4, 5, 6, 7 different dyes here. And, as you see, as this flow happens past this micro-channel here and goes for a certain distance, you are able to extract the dyes independent in the same color as they were introduced originally at the other ends. Therefore, it is really… The momentum equation is really time independent. Assuming that, these were to be true; this is actually a simulation result done by Whitesides group at Harvard. If you assume this flows to be flowing in the reverse direction; then, you should be able to get these flows out as independent dyes or as independent colors. So, this is in fact the essence of the scale down form of the momentum equation in this particular scale.

So, the domain of microfluidics therefore, is a very very novel and interesting domain wherein it is really really very hard for flows laid out together without any diffusional effects to mix. However, because of this particular deliberation that has been just shown in the last slide; and, the time independents of the momentum equation – all the mixing, which happens at the micron scale takes place because of a concentration gradient; and, it diffusional in nature. So, basically, that kind of gives as a field that, it may be kind of impossible sometimes to flow two fluids together in a micro-channel and mixed them. If the two flows have equal concentrations and there is no concentration gradient, which is established between the flows. That is number 1. And, number 2 is that, it becomes a very interesting paradigm in microfluidics to design intelligent systems, which could be actually… Suppose there is also existence of diffusion in the parallel flows, which could be actually promoting diffusion and reducing the diffusion time in a manner that, it becomes lesser than the residence time of the flows; and, the flow starts mixing. So, these are two interesting paradigms in microfluidics.

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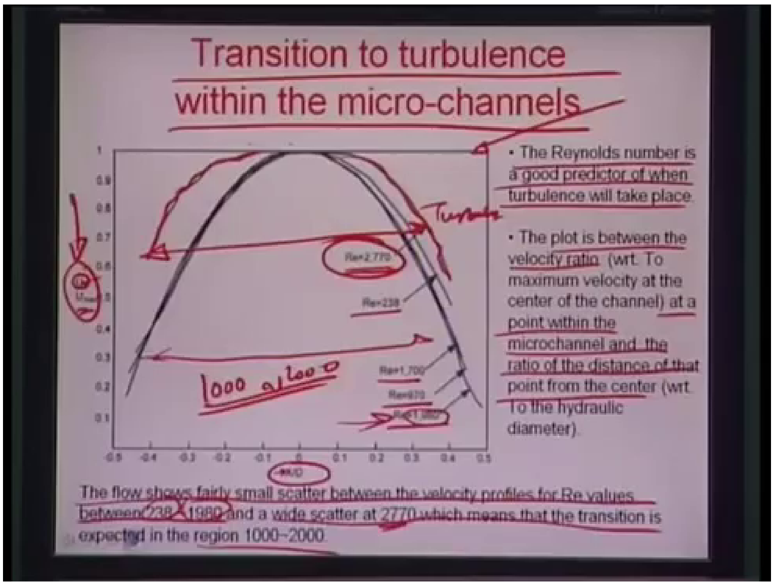

Transition to turbulence within the micro-channels

- The Reynolds number is a good predictor of when turbulence will take place.
- The plot is between the velocity ratio (wrt. To maximum velocity at the center of the channel) at a point within the microchannel and the ratio of the distance of that point from the center (wrt. To the hydraulic diameter).

The flow shows fairly small scatter between the velocity profiles for Re values between 238 – 1980 and a wide scatter at 2770 which means that the transition is expected in the region 1000–2000.

So, I would like to go ahead and actually see some of the effects of Reynold’s numbers on the different flows particularly within the micro-channel. And, I call this slide – transition to turbulence within the micro-channel. So, if you see here, there are different Reynold number values at which we have plotted the u by $u_{maximum}$, which is essentially the velocity at a certain position divided by the velocity at the center, which is the maximum velocity. And, this is plotted with respect to x by dx, is the position at which the velocity – local velocity is u; and, D is essentially the diameter of the particular channel. So, as we see here, the Reynold’s number definitely is a very good predictor of when the turbulence will take place. The plot is between the velocity ratio at a point within the micro channel; and, the ratio of the distance of that point from the center.

And, the conclusions from this are that, if you see this particular plot here; it is basically at a very high Reynold's number value – 2770. You see a fairly large scatter in the velocity. So, therefore, you can see that, if you see, this scatter can be also represented by this span of the parabola and inverted parabola. And so, if the span is much more in this case as compared to the low Reynold's number cases from 238 to almost 1980 when the scatter is not that high; and, which also shows that, probably at 2770, there is a transition. Or, between 2770 and 1980, there is definitely a transition into the turbulent zone. So, therefore, there is no control really on u after this higher Reynold's number values reached; whereas, at a lower Reynold's number value, there is still some control and you can really say that, the flows have not yet turned into eddies or vortexes; or, it has not really turned into turbulent flows. So, that is what this graph would indicate. So, therefore, the flow shows fairly small matter between the velocity profile for Re values between 238 and 1980. And, a white scatter at 2770, which means that, the transition is expected in the region 1000 to 2000, which is also true otherwise that, there is a transition definitely between Reynold's number 1000 and 2000 whether it is the macroscopic counterpart or the micro-channel.

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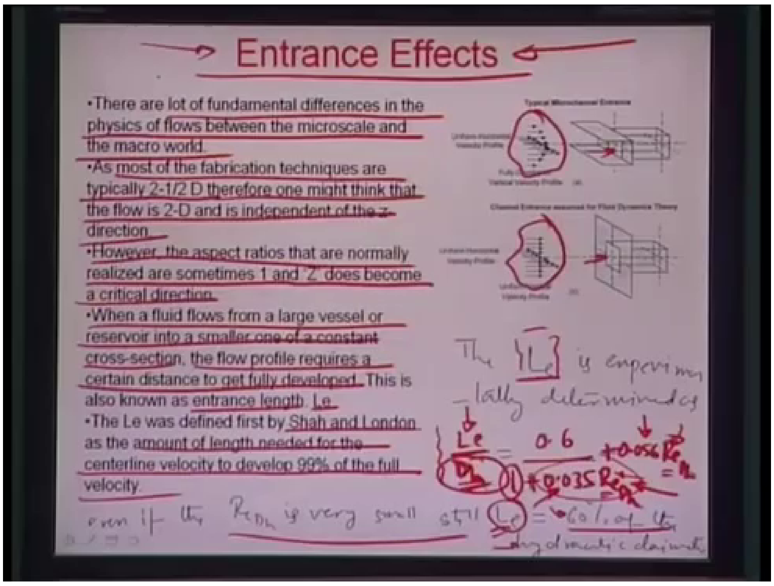

Entrance Effects

- There are lot of fundamental differences in the physics of flows between the microscale and the macro world.
- As most of the fabrication techniques are typically 2-1/2 D therefore one might think that the flow is 2-D and is independent of the z-direction.
- However, the aspect ratios that are normally realized are sometimes 1 and 'Z' does become a critical direction.
- When a fluid flows from a large vessel or reservoir into a smaller one of a constant cross-section, the flow profile requires a certain distance to get fully developed. This is also known as entrance length. Le
- The Le was defined first by Shah and London as the amount of length needed for the centerline velocity to develop 99% of the full velocity.

The {Le} is experimentally determined as

$$\frac{L_e}{D_h} = \frac{0.6}{1 + 0.035 Re_{D_h}} + 0.056 Re_{D_h}$$

even if the Re_{D_h} is very small still L_e = 60% of the hydraulic diameter

So, one more interesting paradigm to be valuated in microfluidics is related to the effects, which can come at the entrance of a micro-channel. So, as we know that, as a channel is approached by a flow of a constant velocity and pressure-driven flow; at the moment after the flow enters the channel, there is always sometime, which the flow takes to get fully developed, where this parabolic profile of the flow gets created within the micro-channel. Now, in cases… – within channel; in cases of micro-channels though, this length which is also known as the entrance length, where the flow profile or whatever the flow profile takes would get developed fully really also is a kind of function of the Reynold's number. And, it is very very critical to design micro-channels by assuming what really would be the entrance length over and above which the flow is fully developed within the micro channel. So, let us explain this event by saying that, there are lot of fundamental differences in the physics of flows between micro and macro scales.

And, so most of the fabrication techniques – as you know typically to two and a half D. And, therefore, one might think that, the flow in 2D is independent of the z direction. However, the aspect ratios that are normally realized that, sometimes 1 and z becomes a critical direction for this kind of flows. And, when a fluid flows from a large vessel or a reservoir into a small one of a constant cross-section; which means that, the velocity of approach of this particular flow is almost constant and it approaches this small channel here. The flow takes or the flow profile requires a certain distance to get fully developed. So, this is also known as the entrance length. So, various people have tried to experimentally and otherwise calculate what this entrance length would be.

And so, the best experiment that was proposed by Shah and London was that, the amount of length needed for central line velocity to develop 99 percent of the full velocity is actually given by this expression here L_e – L_e is the entrance length – divided by Dh is equal to… So, D_h is essentially the diameter of the particular channel is equal to 0.6 divided by 1.0 – 0.035 Re – Reynold's number at that particular diameter or cross-section. And, plus 0.056 Re – Reynold's number at that particular cross-section.

$$\frac{L_e}{D_h} = \frac{0.6}{1 - 0.035\,\Re} + 0.056\,\Re$$

So, entrance length is really then a function of the Reynold's number very critically; and also, the cross-sectional diameter of the particular micro-channel. So, typically, Le is normally about 60 percent of the hydraulic diameter in case the Reynold's number is low, because then there is – one is prominent over this term 0.035 Reynold's number or 0.056 Reynold's number.

However, in cases where this Reynold's number becomes a little bigger, the entrance length effects are felt more appropriately. This is the hydraulic diameter – cross-sectional diameter or the hydraulic diameter. So, is cases where the Reynold's number becomes little bigger, the contribution from this term and this term is significantly enhanced. Therefore, the entrance length becomes less than 60 percent – half the hydraulic diameter. What does it mean really? It means that, if suppose in case of a micro-channel, when the Reynold’s number is comparatively low, the entrance length has significant amount in terms of 60 percent of the hydraulic diameter supposed to macro scale with the R e is more, where this is much much lesser than 60 percent. So, this kind of shows that, whenever you want to design micro-channel, you almost have to be careful always about selecting the entrance length before the perturbations of structures that you would like to create within the micro-channels to cause certain phenomenon – physical phenomenon would start to be placed. So, you need to wait up till the entrance – the whole entrance length, which is needed for developing the flow fully is transverse or traversed before the flow can get into a region where significant differences can be made to it by changing the physical form or shape of the micro-channel. So, definitely, entrance effects are very very critical as far as microfluidics are concerned. So, this kind of brings us to an end of the kind of theoretical analysis that was needed to understand microfluidics.

So, I would like to go ahead now today and try to get into a little more different area as to what would happen really if the continuum assumptions fail. And essentially, then would be talking about things, which are more related to intermolecular forces. And, scaling from microns to nanometers, it is very important that, we consider now an altogether different approach, which is not driven by continuity; and, which is more driven by intermolecular forces between the molecules, which are present in a very small nanometer cube size – probably control volume. And, after that, after looking into that aspect, really I would like to go ahead and start the topics of the actual technology that microfluidics can be put to and then can be directly applied to as in bioMEMS wherein we would start realizing these active devices like micro-valves, micro-mixers, micro-pumps, etcetera. And, the idea is that, after we demonstrate all these on a fundamental level, we will go ahead and assemble these units together on a single bioMEMS platform, which would do something useful and important. The last part of the course would be probably dedicated to viewing some of the papers and articles, which have been in this area, where these small fundamental concepts are introduced and integrated together to form integrated systems. We should do total chemical analysis.

So, let us actually go ahead and see what happens when the scale changes to a little bit lower. And, as I told you before in one of my earlier lectures that, continuum assumptions fail beyond a certain scale. Let us say for example, if you have a small control volume, which is about 1 nanometer cube in dimensions; and, you assume or you try to find out what is the density or velocity of molecules, which are within this 1 nanometer cube. So, there is a tendency of the molecules to rush past in the boundaries of these very randomly with time as a resulting in change in density with time, maybe a change in viscosity with time. So, all these properties, which were assumed to be kind of averaged out properties as in case of continuum mechanics or continuum fluid mechanics would actually boil down to a time dependent property when you go to this particular scale. And so, therefore, whatever presumptions – maybe a Stokes equations made in assuming that, the density is constant, the flow is in compressible or even if the velocity or this really only dependent on space. And, even if it is dependent on time, it does not change at a certain instant of time. So, these assumptions were made in order to derive the Navier-Stokes equations. However, that is not true at that small scale of 1 nanometer cube or so. And therefore, inter molecular forces would play a major role there in order to figure out what is going on in terms of interactions between the molecules, etcetera.

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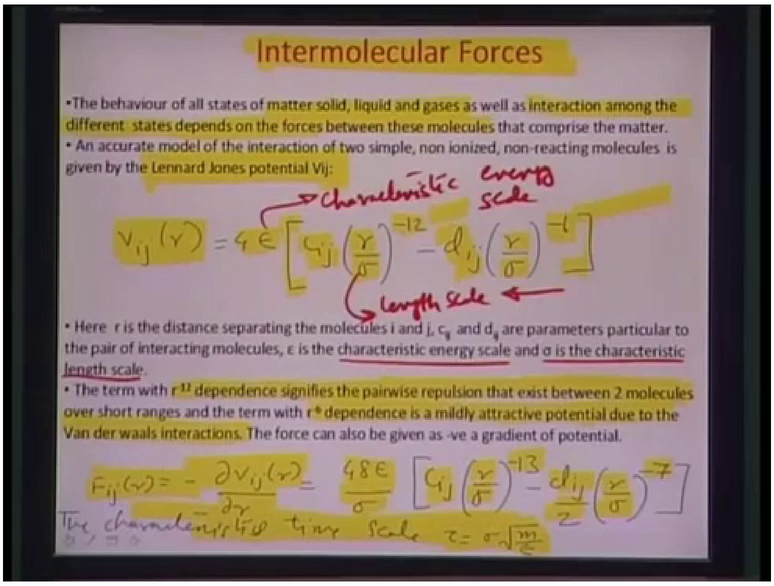

Intermolecular Forces

- The behaviour of all states of matter solid, liquid and gases as well as interaction among the different states depends on the forces between these molecules that comprise the matter.
- An accurate model of the interaction of two simple, non ionized, non-reacting molecules is given by the Lennard Jones potential Vij:

$$V_{ij}(r) = 4\epsilon\left[c_{ij}\left(\frac{r}{\sigma}\right)^{-12} - d_{ij}\left(\frac{r}{\sigma}\right)^{-6}\right]$$

characteristic energy scale

length scale

- Here r is the distance separating the molecules i and j, c_{ij} and d_{ij} are parameters particular to the pair of interacting molecules, ε is the characteristic energy scale and σ is the characteristic length scale.
- The term with r^{12} dependence signifies the pairwise repulsion that exist between 2 molecules over short ranges and the term with r^{6} dependence is a mildly attractive potential due to the Van der waals interactions. The force can also be given as -ve a gradient of potential.

$$F_{ij}(r) = -\frac{\partial V_{ij}(r)}{\partial r} = \frac{48\epsilon}{\sigma}\left[c_{ij}\left(\frac{r}{\sigma}\right)^{-13} - \frac{d_{ij}}{2}\left(\frac{r}{\sigma}\right)^{-7}\right]$$

The characteristic time scale $\tau = \sigma\sqrt{\frac{m}{\epsilon}}$

So, intermolecular forces – so, what are intermolecular forces really? So, if you look at the different states of matter, you can really categorize matter into solids, liquids and gases. And, these are all based on the interaction among the different states depending on the forces between molecules of these particular states, which comprise the matter. So, these interaction forces can be very well explained by a model of potential energy of two such molecular systems, which are close by – by a distance r. And, this is also known as the Lennard Jones potential model. Essentially, it is a non-dimensional equation; and, we have a scaled down factor there. So, you have a scale factor for distance essentially between let say two molecular systems. And so, without going to the details of derivation of such an equation, I would just like to illustrate, because most of the simulation work, which is at this nanometric scale as far as fluidics is concerned is more using this model of Lennard Jones potential.

So, let us say you have two systems: i and j; and, that means two molecular systems of this particular notation. And, they are separated by a distance r. So, the potential that would happen between two of these i and j systems, is essentially equal to $V_{ij}(r)=4\in\left[C_{ij}\left(\frac{r}{\sigma}\right)^{-12}-d_{ij}\left(\frac{r}{\sigma}\right)^{-6}\right]$. Let me just go ahead and explain what all these different terms are. So, epsilon is essentially the scaled energy. Now, this is the scaled energy – characteristic energy scale we call it – characteristic energy scale. And, sigma is the characteristic distance scale or the length scale. So, this is the length scale. By now, you are probably familiar, because we did a scaling down in the Navier-Stokes equation of conservation of momentum. I probably are aware of what these different scales mean. So, it is essentially a characteristic distance or a characteristic energy value at the scale at which all these different forces or potentials are considered. So, r by sigma essentially is a non-dimensional term. Similarly, V_{ij} by epsilon again is a non-dimensional term, because it is a comparison of the potential with respect to an energy scale; that means a value of energy at that particular scale in which all these experiments or all these equations being used for studying intermolecular behavior.

What is more interesting here is that, there are two different terms and dependencies on r by sigma. One is to the power of minus 12; another is to the power of minus 6. So, these – essentially this minus 12 term would signify the pair-wise repulsion that exists between two molecules. And, this repulsions as you all know are because of interelectronic repulsions between the two molecular systems. So, there are orbitals, which contain electrons, which has a probability of having an electron or having a bunch of electrons. And, when two of such molecular systems come close by, there is a repulsion between the electrons on both these systems, which would cause them to have a positive potential. So, it takes some work for getting them any closer than the characteristic distance r in which they are placed. So, that is what the r minus 12 dependence would signify.

Then, there is a r minus 6 dependence, which is a mildly attractive potential. And, this is due to Van der Waals interaction forces. So, what are really Van der Waals forces? They are basically attractions between formulated between two such molecular systems between the electrons on one and the nucleus on another; and similarly, the electrons on the other and the nucleus on the first one. So, that is what Van der Vaals interaction forces are and they are principally attractive in nature. However, they are very weak forces, because essentially, the attraction between the nucleus on the first and the electron on the second has to go through an electronic layer; I mean it has to seal through an electronic layer; it is sealed by an electronic layer, which is in between. So, therefore, r by sigma to the power minus 6 dependency is a pretty much weak force in comparison to the minus 12 dependency, which is a strong force. So, therefore, we can easily say that, the potential is contributed by the pair-wise repulsion and the pair-wise attraction given by these two terms here – minus 12 and minus 6.

Also, what is interesting for me to tell you is that, as you know, force in between these two systems – i and j, is also represented by a negative gradient of the potential between the two systems. So, we can calculate F_{ij}

$$F_{ij}=\frac{\partial V_{ij}(r)}{\partial r}=\frac{4\delta\in}{\sigma}\left[C_{ij}\left(\frac{r}{\sigma}\right)^{-13}-\frac{d_{ij}}{2}\left(\frac{r}{\sigma}\right)^{-7}\right]$$

where, r is the intermolecular distance between these two molecular systems. Now… So, in this particular scale, I can also assume that, the corresponding time scale would be represented by this quantity here – sigma root over m by epsilon; m by epsilon again is mass per unit energy, which has the dimensions of square of velocity. And so, velocity – a typical characteristic velocity at this scale could be represented by the mass at this scale divided by the energy at this particular scale. And so, therefore, if you under root the mass per unit energy, it gives you the velocity scale. And, dividing the distance sigma by the velocity scale would give you an indication of the time scale – what kind of times or characteristics times would exist in this particular scale.

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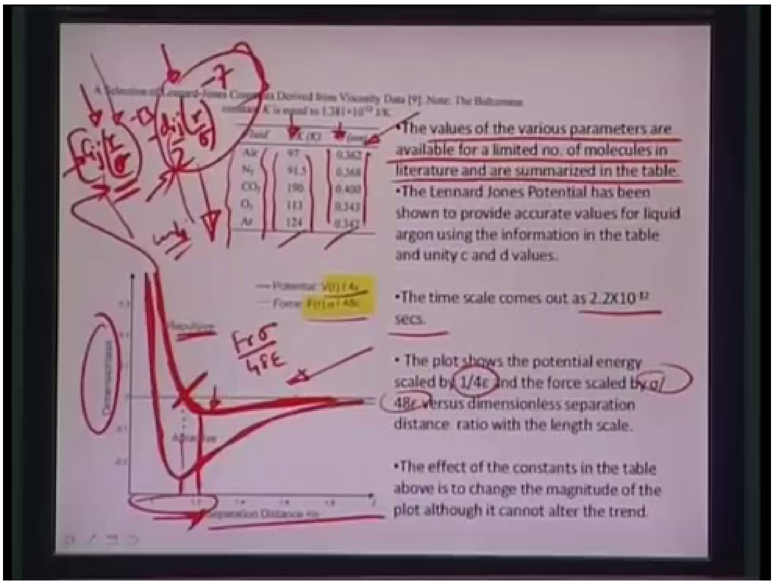

A Selection of Lennard-Jones Constants Derived from Viscosity Data [9]. Note: The Boltzmann constant K is equal to 1.381×10⁻²³ J/K.

Fluid	ε/K (K)	σ (nm)
Air	97	0.362
N_2	91.5	0.368
CO_2	190	0.400
O_2	113	0.343
Ar	124	0.342

- The values of the various parameters are available for a limited no. of molecules in literature and are summarized in the table.
- The Lennard Jones Potential has been shown to provide accurate values for liquid argon using the information in the table and unity c and d values.
- The time scale comes out as 2.2X10 ^{12} secs.
- The plot shows the potential energy scaled by 1/4ε and the force scaled by σ/48ε versus dimensionless separation distance ratio with the length scale.
- The effect of the constants in the table above is to change the magnitude of the plot although it cannot alter the trend.

So, what is also interesting here is that, if you really plot the dimensionless parameters v r by 4 epsilon and f r by 48 epsilon times sigma, which is essentially if you just see back in this equation, nothing but this whole dimensional term in one case; the force divided by 48 epsilon sigma. And, this whole dimensional term on another case, which is V_{ij} – the potential function divided by 4 epsilon. So, these are two non-dimensional terms. And, what is interesting here to find out is that, essentially, there are two curves, which come out: one is for the potential function. And of course, these dimensionless numbers are plotted with respect to the separation distance r by sigma. So, what we find out here is that, the behavior in both the cases are pretty much similar except the fact that, the force value has deeper well in comparison to… The potential value has a much deeper well in comparison to the force value here.

What is also interesting here is to note that, beyond a certain distance – let us say this particular distance, where it really has the peak point; the forces kind of try to gain prominence of the Van der Waals attraction. So, this region here is the repulsive force. So, if you see, the dimensional quantity in this particular example, $\frac{Fr\sigma}{48E}$ is continuously going down; which means that, if you look at the dimensional quantity here, which is let us say $C_{ij}\left(\frac{r}{\sigma}\right)^{-13}-\frac{d_{ij}}{2}\left(\frac{r}{\sigma}\right)^{-7}$. So, this going down would signify that, this term here – the second term here is starting to dominate; and, which means that, this is the pair-wise repulsion; this is the pair-wise attraction and the Van der Waals attraction. So, the Van der Waals attraction is continuously dominating or it is trying to dominate. But, essentially, it is being superposed by the higher value of r by sigma when it is below a certain separation distance. So, after it crosses a certain characteristic separation distance, this component here really predominates, which is actually essentially this point here, where the pair-wise attraction becomes more than the pair-wise repulsion. After which again, there is a slow prominence sort of slow gain of the first term, where it asymptotically goes all the way to 0. So, if you draw the potential function really, it also shows similar behavior.

The point where the forces have really changed from being perfectly repulsive to slightly an attractive here, the potential function is actually dipping down to its minima at that particular point, where they are probably close to zero forces; there are no forces of any attraction or repulsion. And, the attraction repulsion is kind of balanced to each other. Both the terms in the dimensionless quantity are equal to each other. So, if you really look at these – Reynolds, this Lennard Jones functions in case of diatomic gases particularly… or in case of certain gaseous states, the values of various parameters available for limited number of molecules have only been studied and summarized. And specially, they have been done so more for gases wherein you can find out the characteristic energy scale per unit Boltzmann constant; and also, the characteristic distance scale.

Mind you – there are only very few systems in the world which have been really modeled and studied for finding out these different energy scales or distance scales, etcetera. Some of them are illustrated here in the table, like for example, in case of air, the energy scale would be… Typically, energy scale connect Boltzmann constant k would typically be equal to about 97 kelvin. For N_2 – nitrogen, it would be about 91.5; for CO_2, it is 190; oxygen, it is 113; Ar – 124, so on, so forth. And, in terms of distance scale in nanometer range, air molecules have probably a characteristic distance of 0.362, which goes all the way to about 0.3424 organ. So, these have been of course, calculated, assumed by assuming the C_{ij} and d_{ij} parameters to be both unity. That is how these values have been really calculated. So, the times scale for these kind of energy and distance scales come out to be about 2.2 pico seconds. So, these are some of the characteristic dimensions, which are available at the atomistic scale that we are talking about.

The plot again here as you are seeing has been shown only for these few systems. What is interesting here, what I did not discuss before is that, it is basically the potential energy scaled by a factor of 1 by 4 epsilon and the force scaled by sigma by 4 epsilon. And so, the effects of these constants in the table above here really is to not change the basic characteristic; but, to actually shift it by adding a DC bias. So, the whole curve can shift up and down if these per characteristic parameters – epsilon by k and sigma would change without really changing the behavior of the trend that, the dimensionless number would follow with respect to r by sigma. So, that is all about Lennard Jones potential models.

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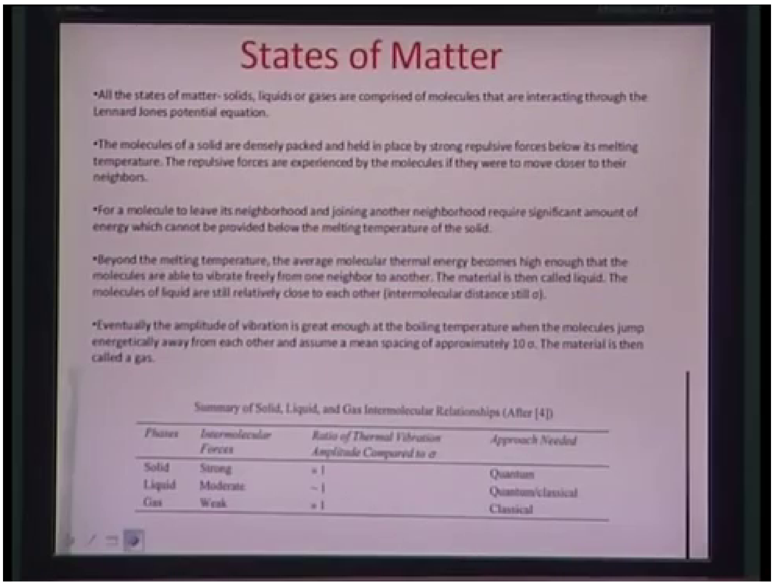

States of Matter

- All the states of matter- solids, liquids or gases are comprised of molecules that are interacting through the Lennard Jones potential equation.
- The molecules of a solid are densely packed and held in place by strong repulsive forces below its melting temperature. The repulsive forces are experienced by the molecules if they were to move closer to their neighbors.
- For a molecule to leave its neighborhood and joining another neighborhood require significant amount of energy which cannot be provided below the melting temperature of the solid.
- Beyond the melting temperature, the average molecular thermal energy becomes high enough that the molecules are able to vibrate freely from one neighbor to another. The material is then called liquid. The molecules of liquid are still relatively close to each other (intermolecular distance still σ).
- Eventually the amplitude of vibration is great enough at the boiling temperature when the molecules jump energetically away from each other and assume a mean spacing of approximately 10 σ. The material is then called a gas.

Summary of Solid, Liquid, and Gas Intermolecular Relationships (After [4])

Phases	*Intermolecular Forces*	*Ratio of Thermal Vibration Amplitude Compared to σ*	*Approach Needed*
Solid	Strong	$\ll 1$	Quantum
Liquid	Moderate	~ 1	Quantum/classical
Gas	Weak	$\gg 1$	Classical

So, if you look at the states of matters really; what are the differences in terms of forces; so, in a solid, we can assume all the molecules being densely packed and held by strong repulsive forces. This pair-wise repulsion is very high in some solids. And, as a result of which, putting a particular atom from its position out of that is highly difficult, because it is being repelled from all these sides by equal amount of high repulsive forces. So, it remains in its state very firmly bound in where it is without really being able to get liberated from that particular state. So, moving an atom from one neighborhood to another becomes an immense problem. So, for a molecule to kind of leave its neighborhood and join another neighborhood, you would need to provide energy to the system. And, this energy would be in terms of thermal energy.

So, essentially, if you have bond vibrations, which are happening between this molecular systems, after a while the energy – the kinetic energy of the molecules would be sufficient enough for it to leave one neighborhood to another. And, that is also known as the melting temperature or the melting point of the particular solid, where the molecules are just about capable to move between neighborhoods. And, it has sufficient amount of kinetic energy for doing that. And, beyond the melting temperature of course, the average molecular thermal energy becomes high enough, so that the molecules are able to vibrate freely, go over large distances and between one neighborhoods to another. And, if the distance that they are traversing or the distance scale that they are traversing is still sigma; they are also called liquid state material.

But, then if the distance becomes almost an order of magnitude more like about 10 sigma or so, they are known as gaseous states. In that kind of a situation, where the intermolecular distance is more than about 10 sigma; sigma being the distance scale in the solid scale – solid state; these are essentially then known as the boiling temperature. And, the species are known as gaseous species. So, in a nutshell, there are three different configurations to look at solid state. The atoms are firmly bound; there are high – huge amount of repulsive forces to an atom in its pocket from all its neighbors. And, movement between neighborhoods becomes a possibility, becomes an impossibility. In gaseous state or in a liquid state, just about when kinetic energy is applied, has been able to lift it to a state, where the atom can move from one neighborhood to another; the distance is moved still about same range sigma. But, the moment just becomes possible; it is known as liquid state or the melting point or the material. And, at a point when this sigma increases to about 10 sigma or so and there is total decontrol on the molecule from its current location, it reaches the boiling point; it is known as the gas phase.

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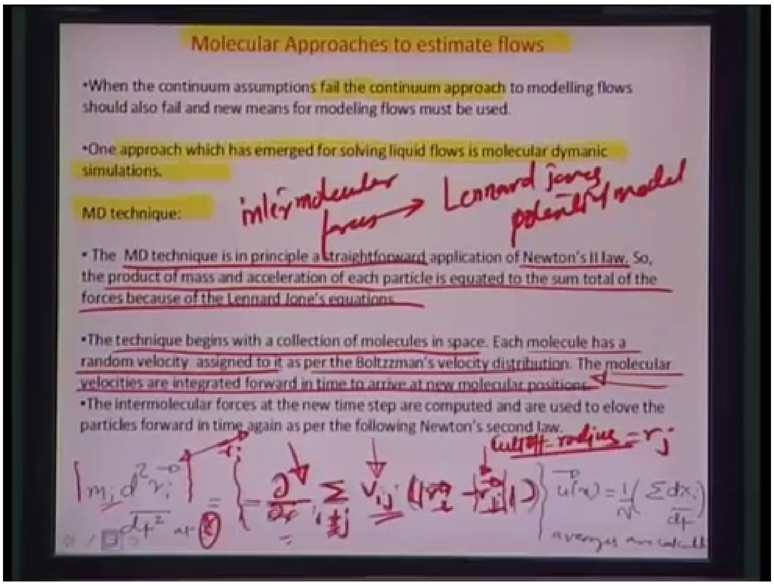

Molecular Approaches to estimate flows

- When the continuum assumptions fail the continuum approach to modelling flows should also fail and new means for modeling flows must be used.

- One approach which has emerged for solving liquid flows is molecular dymanic simulations.

MD technique: intermolecular forces → Lennard Jones potential model

- The MD technique is in principle a straightforward application of Newton's II law. So, the product of mass and acceleration of each particle is equated to the sum total of the forces because of the Lennard Jone's equations.

- The technique begins with a collection of molecules in space. Each molecule has a random velocity assigned to it as per the Boltzzman's velocity distribution. The molecular velocities are integrated forward in time to arrive at new molecular positions.
- The intermolecular forces at the new time step are computed and are used to elove the particles forward in time again as per the following Newton's second law.

cutoff radius $= r_j$

$$m_i \frac{d^2\vec{r}_i}{dt^2} = \left\{-\frac{\partial}{\partial r}\sum_{i\neq j} V_{ij}(|\vec{r}_i - \vec{r}_j|)\right\} \qquad \vec{u}(x) = \frac{1}{N}\left(\sum \frac{dx_i}{dt}\right)$$

averages are calculated

So, therefore, with this kind of a concept in mind, let us actually look at what happens when the continuum fails. So, we would like to do this molecular approaches to estimate flows particularly when there is a failure of the continuum approach. And so, on the other approach, which has probably emerged for solving liquid flows is called molecular dynamic simulations, which talks about all these intermolecular forces as I have been telling before – intermolecular forces; and, which can be established from the Lennard Jones potential model. So, the MD technique is in principle very straightforward; it is an application of just Newton's second law. And so, what is Newton's second law, the product of mass and acceleration is equal to the total amount of force that a particular system should have. So, same can be translated into molecular systems as well. So, however, we have to somehow ascertain how to find the acceleration of this kind of a molecular system.

We already know that, from Lennard Jones potential, we can find out the average force between two molecular systems: i and j separated by a distance of r. So, essentially, that is what we are doing here; the product of mass and acceleration of each particle is equated to the sum total of forces because of Lennard Jones equations. And, the technique really begins with a collection of molecules in space. Now, each molecule has a random velocity assigned to it as per Boltzmann velocity distribution. And, the molecular velocities are integrated forward in time to arrive at the new molecular positions. And, if you see the intermolecular forces really, which are equated to $\frac{m_i d^2 r_i}{d t^2}$; let us say time instant t is equal to minus d by d r; this is the negative gradient of potential – Lennard Jones potential – sigma i naught equal to j; that means these are systems – all systems of the likes i and all system of the likes j; there are two different molecular systems between which there is a force of attraction and repulsion. So, it is the negative gradient with respect to r of V i j.

$$\frac{m_i d^2 r_i}{d t^2} = \frac{-\partial}{\partial r} \sum_{i \to j} V_{ij}\left(\acute{r}_i - \acute{r}_j\right)$$

Here the assumptions that we use to simplify is that, if you consider a molecular system really, it is a huge amount or a huge number of molecules. 1 mole as you know is about 10 to the power of 23 molecules. And so, therefore, if suppose you are talking about a certain radius or if you are talking about a molecular system where there are thousands of these molecules, which are interplaying and there are intermolecular forces between these thousand of molecules; there would be a tendency of the computations to go extensively at a very high rate. And, that is sometimes… So, essentially that, the whole idea is that, in a molecular system like this, if you have one molecule and surrounding thousands or actually millions of molecules; there are intermolecular forces between this one molecule of interest and its neighborhood, which comprises of millions of such forces. And, it makes the whole system computationally extensive. And, the solutions may not converge with time and there maybe issues related to more number of computations. And, to solve this problem, you really need to consider an effective radius within which you can consider the interactions. So, this is called a cutoff radius.

If the radius is exceeded or the radius goes beyond the cutoff radius, we assume that, the molecular forces are rising out of molecules rising or which are outside this cutoff radius is 0. So, you kind of terminate the calculations at a certain intermolecular distance; beyond which you assume that, the distances are too large for the forces to be really effective in the overall calculations. So, therefore, this Lennard Jones function really changes into $V_{ij}\left(\acute{r}_i - \acute{r}_j\right)$; r_j is essentially the vector radius; where, there would be a force cutoff. So, this is the cutting radius or cutoff radius. And, any point below this would be effectively considered in the calculations here. So, the force… So, it is kind of a truncated solution. So, the truncated solution of the force between a radius – with a cutoff radius of let us say r j within this particular case is given by minus d y d r of sigma; i not equal to j; that means these are two different molecular systems: $V_{ij}\left(\acute{r}_i - \acute{r}_j\right)$. And, this is equated to by the Newton's second law $\frac{m_i d^2 r_i}{d t^2}$. So, this is the acceleration at time t.

The forward integration of this twice successively would give you the new position vector r I; from which you can again recalculate this; consider its criteria to define the new cutoff radius. And so, therefore, from position to position, you can have a very good estimate of the velocity, the position vectors and the acceleration vectors of these individual particles based on this force change of momentum approximation. And so, therefore, the averages of velocities let us say are calculated by if you have a knowledge of all positions by this forward time integration and you want to find the x velocity; so all the x positions – the d by d x i by d t; you make a sigma of all these and calculate an average velocity – an average density – an average of other properties, etcetera.

Mind you because these positions are changing with time, these quantities would also be changing with time. So, even though the continuum assumption is not evaluated in this particular case, the averages keep on varying based on the interactions – the molecular interactions in that small controlled volume. And so, therefore, and this appropriate… This is a very appropriate and a very accurate technique to simulate the motional properties of fluids at the nanometer scale. So, these are called molecular dynamic simulations. In fact, such simulations can be used to study a lot of a properties like combustion, fluid mechanics in general, then all sort of surface-based interactions with charged molecules or interactions between different charged molecules in a system, so on, so forth. And, they are very very useful as a technique to predict the position vectors, velocity, average properties, etcetera; time-variant essentially. So, this kind of gives you an estimate of how from continuum to a scale where continuum does not hold. You could change your approach to get different solutions.

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Micromixers, Microvalves
& Micropumps

Now, I would like to now go more towards the application side and describe these different kind of applications of these micro-scale fluidic techniques into realizing engineering products like micro-valves, micro-mixers, micro-pumps, and etcetera. So, let us study these one by one. So, would like to first start by looking at micro-mixers.

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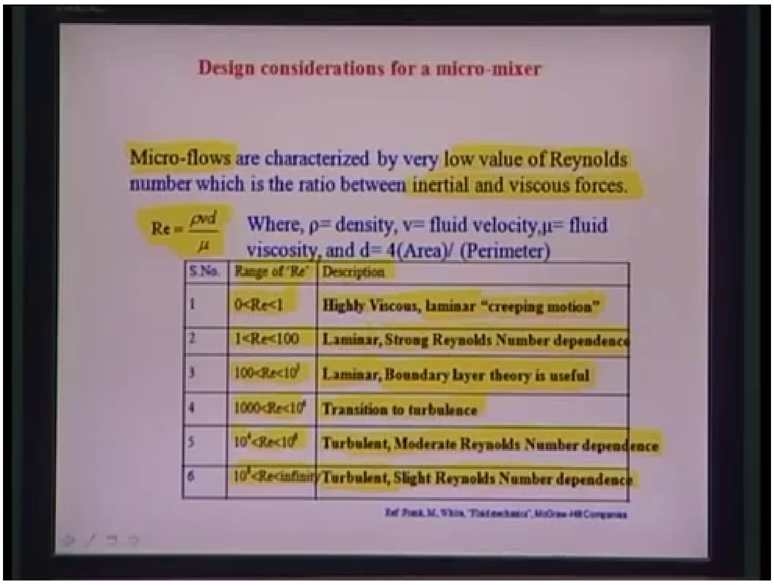

Design considerations for a micro-mixer

Micro-flows are characterized by very low value of Reynolds number which is the ratio between inertial and viscous forces.

$Re = \frac{\rho v d}{\mu}$ Where, ρ= density, v= fluid velocity,μ= fluid viscosity, and d= 4(Area)/ (Perimeter)

S.No.	Range of 'Re'	Description
1	$0<Re<1$	Highly Viscous, laminar "creeping motion"
2	$1<Re<100$	Laminar, Strong Reynolds Number dependence
3	$100<Re<10^3$	Laminar, Boundary layer theory is useful
4	$1000<Re<10^4$	Transition to turbulence
5	$10^4<Re<10^6$	Turbulent, Moderate Reynolds Number dependence
6	$10^6<Re<$infinity	Turbulent, Slight Reynolds Number dependence

Ref Frank M. White, "Fluid mechanics", McGraw-Hill Companies

So, if you look at really the characterization of a micro-flows; the first property which comes out as we have seen earlier is the low value of Reynold's number. Of course, it is the ratio between inertial and viscous forces again. And, you can mathematically define it as $e=\frac{\rho v D}{\eta}$; rho is the density; v is the velocity; d is the dimension at that particular scale; and, mu is the viscosity of the medium. And, the way you calculate d in Reynold's number is essentially four times area by parameters. And, it is also known as the hydraulic diameter of a particular section. So, based on the different ranges that this Reynold's number kind of comes into ranges of values, you could categorize flows into many types. So, this table here illustrates the corresponding range of Reynold's number with the description of the flow. So, as you are looking at R e between 0 and 1, which is mostly the case in microfluidics, the flows are highly viscous; they are highly laminar; and, they are as if they execute creeping motion as if they are moving in a very tightly packed – close-packed channel or creeping in a close-packed channel without really interfering much into each other's path. The molecules move in perfect stream lines parallel to each other.

So, for a Reynold's number of 1 to 100, the flow still remain very very laminar. And, there is of course, very strong dependency on the Reynold's number as this particular scale. So, suddenly, the flow properties may demonstrate a change in behavior with very minute change in the Reynold's number. If you go slightly above and consider the Reynold's number between 100 and 1000, the flows again are very very laminar; they have not yet changed regimes; however, here in this particular range, you could really have a very good boundary layer, which is formulated. If you may recall, boundary layer again is the layer, which is separating the fully developed flow from the flow, which is made up of these – flow which is actually having shear stress. Or, they are nominated by shear. So, therefore, this boundary between the fully developed part and the sheared part of the flow is what laminar boundary layer apparently is. And, this boundary layer formulates whenever very close to a fixed surface maybe a channel inner or some kind of flat plate or flow over a surface. So, essentially, this Reynold's number – range of 100 to 1000 would inculcate a prominent boundary layer. And, here most of the theorizing of boundary layer is very very useful.

If you change the Reynold's number a little bit from 1000 plus all the way to about 10,000; there is a slow transition, which takes place; and, the flow goes from laminar to turbulence. But, then this is really a transition; you have instances where the flow is slightly laminar or instances where the flow is turbulent and there is slowly changing the behavior after a certain value let us say 10 to the power 4 or so up to about 10 to the power 6. The flow becomes fully fully turbulent. And of course, in this particular range, there is some Reynold's number dependence, which happens particularly from 10 to the power 4 to 10 to the power 5 Reynold's number; beyond which up to all the way up to infinity, not only flow this turbulent, but it is having any very less dependency on the Reynold's number. And essentially, that is probably because the flow really gets defined, but the local eddies and vertices more than the overall bulk flow. And, that is essentially what these different categories of flows are with respect to different values of Reynold's numbers. So, what I would like to reiterate here is that, as I told you before in the 1 to 100 case or even less than 1 case as is mostly the case in case of… It is mostly the case in all the microfluidic devices. The mixing behavior or the mass transport between the mixing inter layers typically is not really dependent on the momentum of the flows. There are no eddies or no vertices which would cause mass transport between the two mixing inter layers just by virtue of motion. You need something else to drive the flow in a perpendicular direction. And, what could be more appropriate done, a concentration gradient. And so, therefore, the diffusion approximation comes into picture. Most of the micro-scale flows mixing takes place primarily by diffusion.

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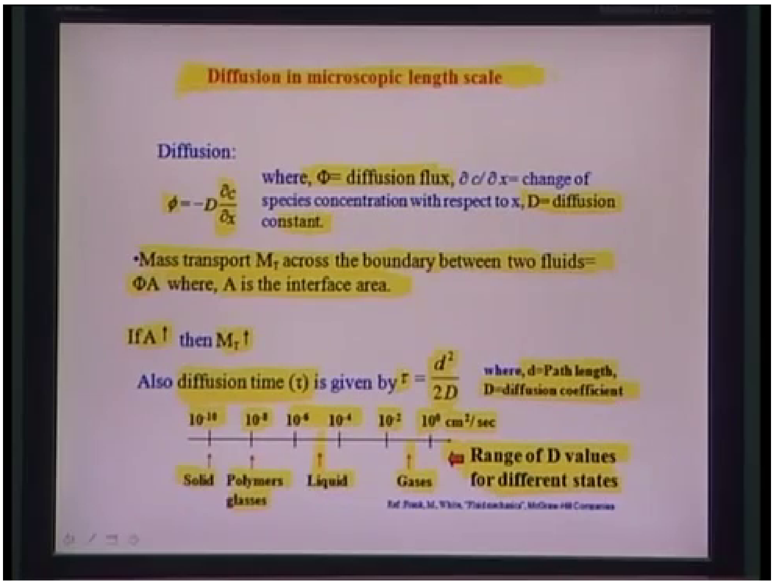

Diffusion in microscopic length scale

Diffusion:

$\phi = -D\frac{\partial c}{\partial x}$ where, Φ= diffusion flux, $\partial c/\partial x$= change of species concentration with respect to x, D= diffusion constant.

•Mass transport M_T across the boundary between two fluids= ΦA where, A is the interface area.

If A ↑ then M_T ↑

Also diffusion time (τ) is given by $\tau = \frac{d^2}{2D}$ where, d=Path length, D=diffusion coefficient

Range of D values for different states

If you look at the diffusion equation really, the diffusion flux, because of the existence of this concentration gradient here dc/dx, they are kind of proportional to each other. So, diffusion flux is proportional to the negative gradient of concentration with respect to the perpendicular distance. And, the proportionally constant really is d here; the diffusion constant, which can be varied for different states of the matter. For example, diffusion constant is very very low if you consider solid. You see this scale below here – diffusion constant is in the range of about 10^{-10} in case of solid. In case of polymers and glasses, this constant increases a little bit; goes to about 10^{-8}. But, still is very very low; most liquids have a diffusion constant in the range of 10^{-6} to 10^{-4} in this particular region. And so, therefore, it is of some significance; the diffusion is of some significance as far as liquid is concerned. But, in case of gases, as this constant is very high, it is about 10^{-2} to 10^{-1} centimeter square per second. And so, therefore, you have to be very very careful about diffusion as you go to gas flows.

However, most microfluidics being single phase and mostly liquid dominated, the diffusion constants that we will be considering in all our calculations are in the range of about 10^{-6} to 10^{-4}. So, the mass transport M t across the boundary between two fluids is also equal to the flux times of the area of interface between the two mixing fluids. So, you have let us say two fluids running parallel and their interfacial area is a and flux is phi. So, the amount of mass transport Mt really is equal to phi times a; phi being the mass transport per unit area or the flux of mass per unit area. So, therefore, interfacial area in such cases becomes highly prominent especially when mixing is concerned. So, if you have somehow a mechanism wherein you can accommodate two flows over a longer amount of or over larger amount of area, automatically, their mass transport would be more. Sometimes it becomes architecturally promising to design micro-mixers, where this area of interface between the two flows is increased resulting in some mass transport. So, area is more than the mass transport M t is more.

Also, of special significances, an equation, which I am not going to prove here though; but, then I have just borrowed it from normal diffusion kinetics. And, here it talks about the time of diffusion. And so, the time of diffusion essentially equal to d square by 2D; d being the path length over which the two flows are going for which they are diffusing at the end or throughout as they are going with respect to each other; and, big D here is the diffusion coefficient. So, if you have a length l of a channel over which these two flows are mixing and going as they move along; the length l is nothing but the path l – the path d – a path length d. So, if the length is more, then in that case, the time of diffusion tau is automatically more.

So, this length is essentially the cross-sectional length when we are talking about two flows, which are going in a particular channel of the cross-sectional length l. So, it almost makes sense to assume here that, the time of diffusion will reduce if the d value, that is, the cross section length value of the two flows is reduced. So, if I am able to somehow create shorter laminase of these flows and thus the cross-sectional length of the flow reduces by division of d into let us say n; where, n is the number of laminase; then, automatically, the time of diffusion should also reduce as the square of the reduction in the d. And, it is a very important understanding that one must have in order to design what you call passive micro-mixers, where the whole idea is how the diffusion length can be split up into smaller values, so that time of diffusion would reduce. And, the whole essence of microfluidic structures or microfluidic architectures is how the time where these two flows have been introduced from one side and the side within the chip is going to be more than the time that the fluids take to diffuse into each other. So, by the end of the day, at the end of the day when it emerges out from the other side of the chip, they are fully mixed.

So, range of d values for different states are indicated here in this particular illustration. And essentially, that is a little bit of fundamentals of how mixing would take place at the microscopic length scale.

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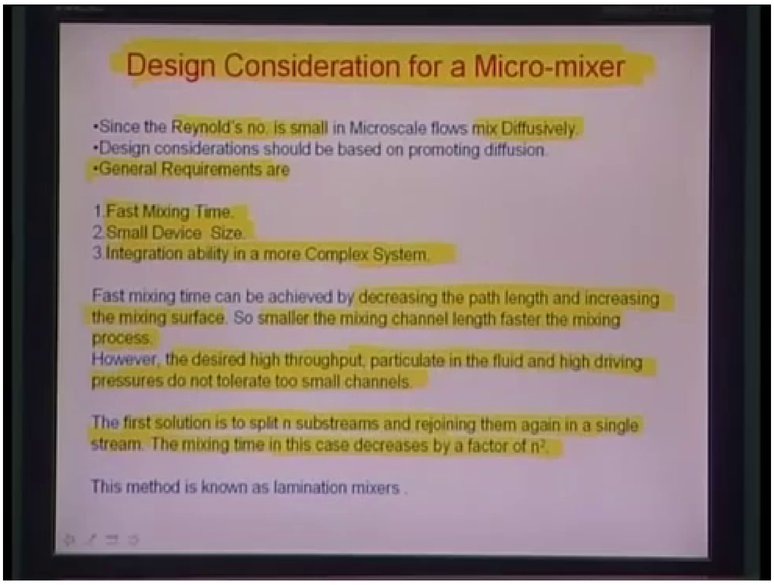

Design Consideration for a Micro-mixer

- Since the Reynold's no. is small in Microscale flows mix Diffusively.
- Design considerations should be based on promoting diffusion.
- General Requirements are

1. Fast Mixing Time.
2. Small Device Size.
3. Integration ability in a more Complex System.

Fast mixing time can be achieved by decreasing the path length and increasing the mixing surface. So smaller the mixing channel length faster the mixing process.
However, the desired high throughput, particulate in the fluid and high driving pressures do not tolerate too small channels.

The first solution is to split n substreams and rejoining them again in a single stream. The mixing time in this case decreases by a factor of n^2.

This method is known as lamination mixers .

So, what really would be considered when we talk about design of micro-mixers? So, the number 1 problem that we are left with this is the small value of the Reynold's number; and therefore, the diffuse of mixing. So, design consideration should be definitely based on somehow being able to promote diffusion between the two layers. And essentially, which means faster mixing time in general requirements, small device size; and also, it should have kind of integration ability particularly in complex system with the overall setup. So, fast mixing time again can be achieved by as I told you before, decreasing the path length. The path length means the cross-sectional path – the path that the diffusion happens along, is perpendicular to the direction of the actual flow of these fluids. And, you can also have more accurate or better mixing and faster mixing by increasing the interfacial area between the two or more mixing streams. So, smaller the mixing channel length, faster would be the mixing process. However, the desired high throughput particulate in field and high driving pressures do not tolerate too small channels; that is unfortunate a part.

Most of these flows in microfluidics as you know are pressure driven. And therefore, too small a channel would have too high a resistance. And therefore, the yield at the end of the channel of the flow would be so small that, it is very negligible. So, you have to design something where in you can still be able to get substantial amount of throughput, but at the same time the mixing can be faster. And so, therefore, lamination is something that comes into picture automatically; that means you split up let us say two streams of two different dyes or two different colors, which you want to mix into multiple streams stacking with each other. So, if you have n such splits, what would happen is that, you have one stream with a diffusion length d – a cross-sectional length or two streams going into a cross-sectional length l or d, which is also the diffusion length. And, in another instance, after n splitting, the cross-sectional length is simply reduced by d by m.

We are going to investigate this a little bit further as we do mixer designing. So, therefore, the first solution really is to split n sub-streams and rejoining them again in a single stream, so that the mixing time reduces by a factor of n^2. So, this is basically the principle of lamination mixers. So, this kind of brings us to an end of this particular lecture. We have a take-home message that, mixing essentially is diffusional, this is dependent on the interfacial area, it is dependent on the diffusion length; time of diffusion essentially has to be kind of lesser than the time of residence for the flow to be effectively mixing with each other. And, overall requirements of a micro-mixer should be it has a faster mixing time, small device size and integration ability particularly in a higher order complex system. So, we will consider these aspects in a little more detail in the next lecture. Thank you.

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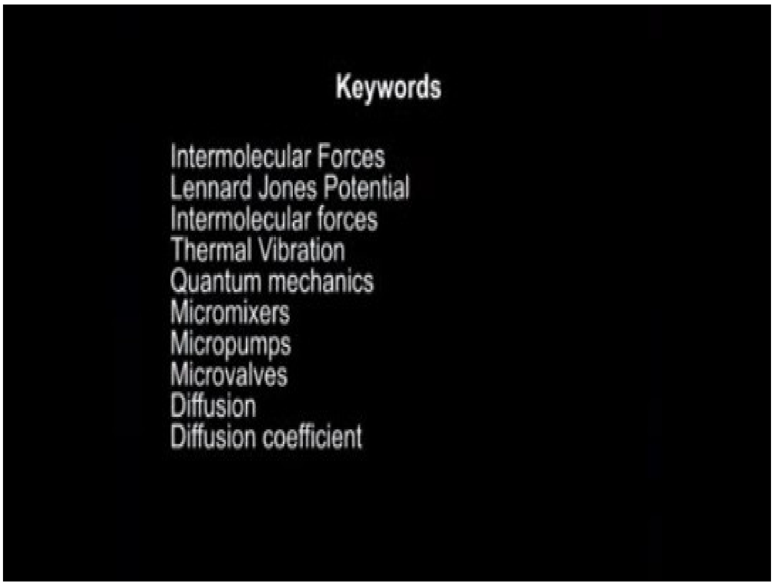

Keywords

Intermolecular Forces
Lennard Jones Potential
Intermolecular forces
Thermal Vibration
Quantum mechanics
Micromixers
Micropumps
Microvalves
Diffusion
Diffusion coefficient

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