BioMEMS and Microfluidics
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Lecture - 28

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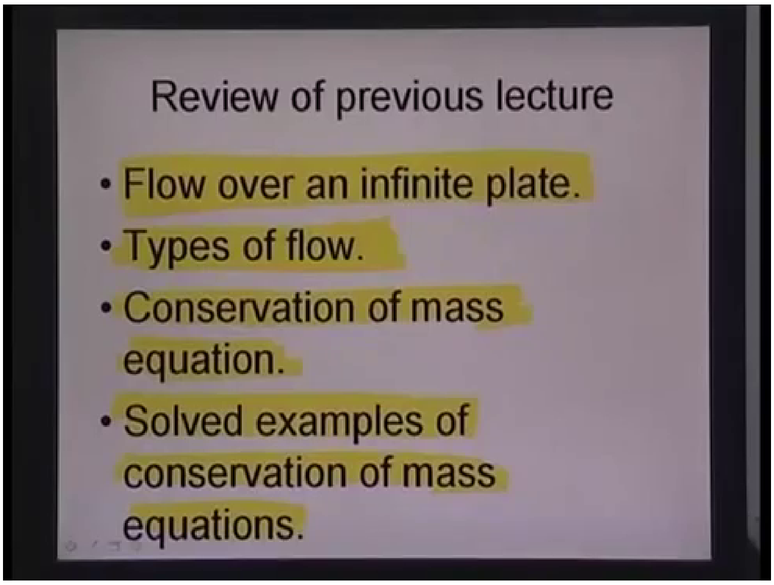

Hello and welcome back to this 28th lecture and Bio Micro Electro Mechanical Systems. Let us begin with the quick review of what have been done last time we talked about the flow over an infinite plate, the way that the boundary layer would be created as a difference between the fully developed region of the flow and the layer adjacent to the to the plate which has shear deformation. We talked about a different types of flows regarding one, two, three dimensional flows and try to calculate velocity fields. We also discussed about a control volume dx, dy, dz and try to a derived the conservation of mass equation which is also the first and Navier-Stoke equation. And then we did some solved examples for a pertaining the velocity component or the density with respect to time or space for compressible or in compressible flows. Again to retreat compressible flows of those which density change with time; incompressible where the density does not change the function of time.

So, today we will try to derive that the conservation of momentum equation which is also the second Navier-Stokes equation. And essentially all these theory is very important because when we translate this theory to micro scale the interesting part is that the mass transport becomes time independent, and that is essentially also reveals why if there are two side by side flowing streams of fluid in a micro channel, they seldom mix because going primarily because of this reason. So, if we look at the how derive the conservation of momentum, there seems to be something do with acceleration of fluid particle because, Newton's second law force is nothing but mass into acceleration. So, let us look first begin with what is the acceleration of a fluid particle in a velocity field.

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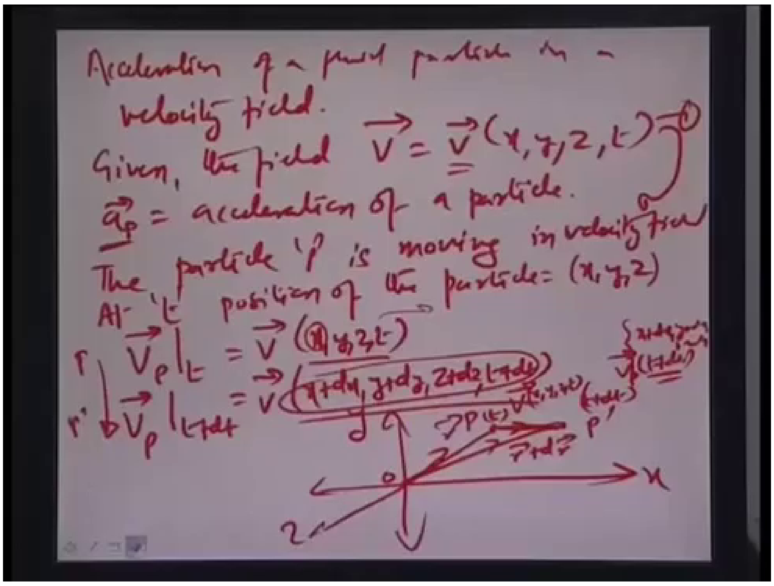

$$\vec{V}=\vec{V}(x, y, z, t)$$

So, we calculate here the acceleration of a fluid particle in velocity field. So, given the field V as a function of x, y, z in time, in space time; let us assume $\vec{a}_p$ is the acceleration vector of a field particle. So, we want to find out what $\vec{a}_p$ is in terms of velocity. Let say the particle p is moving in the velocity field as a represented here above in equation one. So, time t the particle at position x, y, z let say and this particle has a corresponding the velocity at a at point and space of time which is represented by $\vec{V}_p$ at t equals $\vec{V}(x, y, z, t)$. Let say has a $\vec{V}_p$ at t+dt which is represented as $\vec{V}(x+dx, y+dy, z+dz, t+dt)$ .. So, essentially what it really means here is that let say you have this rectangular coordinate system with x, y and z components, you have a radius vector r somewhere here at point p with respect to the origin o, and this changes to another point p dash and the new radius vector becomes r plus dr.

So, essentially this is of the position at time t, this is the position as t plus dt and the vector connecting these two are really you know the position vector that the particle is traverse from point p to p dash. And definitely it is the function of ah x plus dx alright. So, essentially this point here is x plus d x that means, the traversing of the particle in the x direction by and elemental distance is dx; the traversing of the particle the y direction while this path is being executed. So, it is three components of this path. As the particle traverses dy, and similarly the traversing of the particle in the z direction as the traverses dz from point p to p dash. They are at two different time instants t and t plus dt. So, if we consider, if we try to find out what is the change in velocity as the particle moves from p let say to p dash here.

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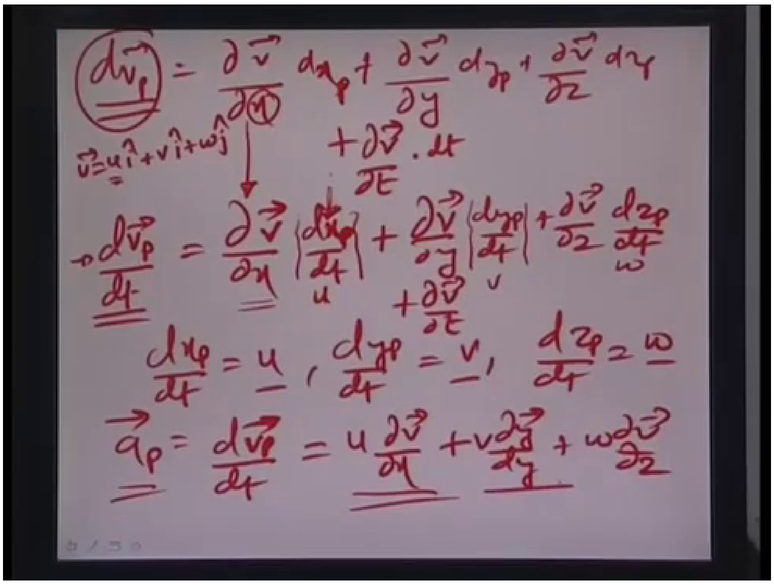

$$d\vec{V}_p = \frac{\partial \vec{v}}{\partial x} d\vec{x}_p + \frac{\partial \vec{v}}{\partial y} d\vec{y}_p + \frac{\partial \vec{v}}{\partial z} d\vec{z}_p + \frac{\partial \vec{v}}{\partial t} dt$$

$$\Rightarrow \frac{d\vec{V}_p}{dt} = \frac{\partial \vec{v}}{\partial x}\frac{d\vec{x}_p}{dt} + \frac{\partial \vec{v}}{\partial y}\frac{d\vec{y}_p}{dt} + \frac{\partial \vec{v}}{\partial z}\frac{d\vec{z}_p}{dt} + \frac{\partial \vec{v}}{\partial t}$$

$$\frac{d\vec{x}_p}{dt} = u, \frac{d\vec{y}_p}{dt} = v, \frac{d\vec{z}_p}{dt} = w$$

$$\Rightarrow a_p = \frac{d\vec{V}_p}{dt} = u\frac{\partial \vec{v}}{\partial x} + v\frac{\partial \vec{v}}{\partial y} + w\frac{\partial \vec{v}}{\partial z} + \frac{\partial \vec{v}}{\partial t}$$

The velocity $d\vec{V}_p$ can effectively written as the rate of change of the velocity vector p with respect to x times of dx plus dx p rather, because x p is the let say the position vector of the x component of the position vector at of the particle p. So, dv x dv vector by dx that is the rate of change of velocity in the x direction of the particle times of dx p plus rate of change of velocity of the particle with respect to y times of dy p plus rate of change of velocity of the particle with respect to z times of dz p. And you have a time component here let say the time is varying with respect to t, so the rate of change of velocity vector of the particle with respect t times of dt, so this is how the differential element of velocity at dv p can be written from vp. So, if you do differentiate this with respect to time, so you have dv p vector is respect to dt is essentially the rate of variation of v with respect to x times of and this is again you know spatial component. So, you would have only suppose you are v vector here is comprise of three components u, small v and small w.

Therefore, u is really differentiable only with respect to x and not in time that is how you basically represent the velocity vector; while at the position p, the vector is in variant time invariant essentially. So, del v by del x times of dx p by dt plus v del by del y times of dy p by dt plus del v by del z times of dz p by dt is plus del v by del t is essentially what you are rate of change of velocity vector of particle p which respect to t is. So, as we know here dx p by dt is nothing but the velocity of the particle at in the x direction; dy p by dt is the velocity of the particle in the y direction; and the d z p by dt is the velocity of the particle in the z direction, we can represent them by u, v and w. So, essentially the acceleration vector then which is nothing but the dv p by dt the rate of change of velocity vector of particle p with respect to t is nothing but u del v by del x plus v del y plus w del v by del z.

So, mind you again why we did not use the change rule for differentiation by taking the derivative of dv by dx, because v is essentially at a certain time point t. Again let me just reiterate this point once more, v vector is the velocity at time point t, and it changes to the velocity at time point t plus dt; later on you know so this is essentially x, y z and t and this is related to x plus dx, y plus dy and z plus dz plus times of t plus dt thus can be represented here. So, once you have differentiated this vector with respect to x, the time component is automatically gone out and therefore as for as the time is concern the differentiation of v 1 with respect to x and dependent of time. So, therefore, the first component is not differentiated with respect to time. The second component of course, represented them of a position particle xp and what we are doing here is dx p by dt which means it is a instantaneous velocity of the particle in the x direction when it is at position p.

Similarly, dy p by dt is represented by the instantaneous velocity of the particle in the y direction, when it is resting at p and so on and so forth. So these are essentially nothing but u, v and w. So, v with respect to x is already been independent of t, so it is constant as for as differentiation with respect to t is concerned, and this right here is the new term which comes out x p can of course, we differentiate with respect to t, and this gives the instantaneous velocity of the particle in the x direction at the point p. So, that is how this equation emerges of acceleration from you know the previous equation. So, acceleration of the particle therefore can be represented as u dv by dx plus v dv by dy plus w dv by dz.

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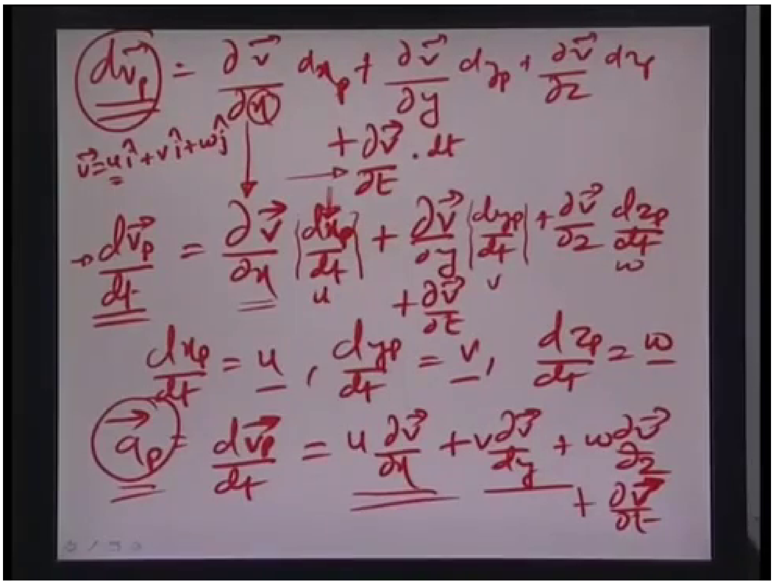

We just could rub this in again, and that is what the acceleration vector ap is, and then of course, there is a del v by del t velocity of v with respect to time t that is how you have defined the acceleration. So, the fluids maybe accelerated as it is converted into region of higher velocity, and let us look at that if suppose you have rectangular block of fluid what kind of forces that he block will feel because of stresses which are taking place on the medium with respect to that control volume. And really if you look at the type of definitions that such a small control movement would have in a shearing stream assuming the volume to present somewhere in the stream.

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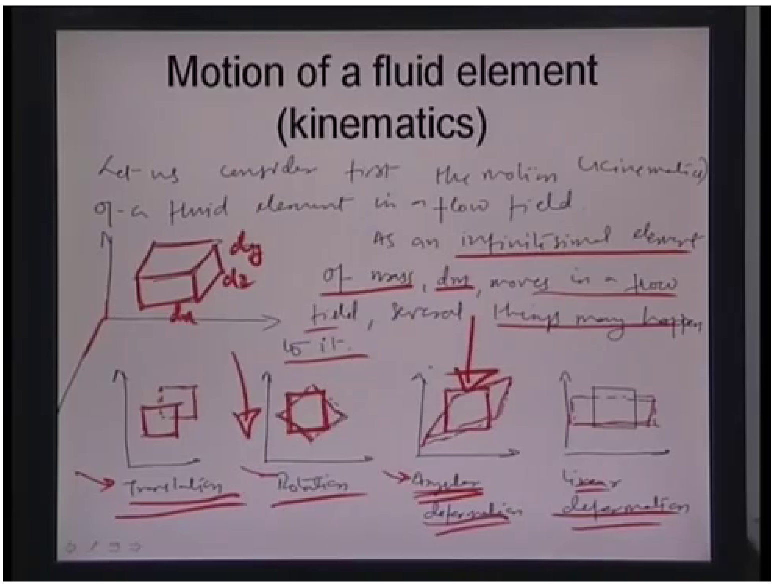

The kinematics of such mechanism can be illustrated as shown in this particular figure here.

So, this is the box at very outset present in a rectangular coordinate system the volume element has a volume of dx, dy, dz. And essentially this is an infinite decimal element of mass dm where rho is the density; we assume the density rho is constant here this is compressible flow rho, dx, dy, dz is essentially is the total mass of this small volume element let us call that dm or it knows in the flow field. So, the number of deformation that such the number of you know kinetic kinematic states that this kind of a block would have can be that the block is simply translated; that means, this was really the initial provision of this block and it change to the new position here.

So, this is simply translated motion, it may rotate which means that the block actually is essentially being integral in shape or not changing or losing its shape rotates at the same point. It may deform which means that the ball block actually starts now changing it shape from a square more like a parallelogram, something like this or it may actually deform linearly, since the linear deformation; that means, a cube will become a cuboids that means, the because of volume continuity we increase where the length of the block by pulling it together the area of cross section will definitely decrease. So, there are four different kind of kinematics states that this block can have while moving in in a fluid volume one is a translator, other is a rotary you can see here, third is the angular deformation where you are actually trying to compress the block and make it from a square into a parallelogram, and the fourth is linear deformation where you are actually trying to pull the block and trying to reduce cross section and increase length.

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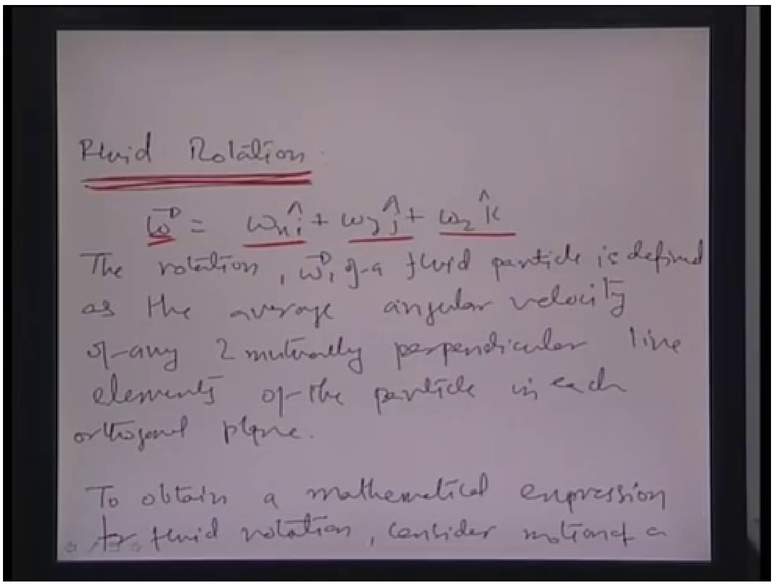

$$\vec{w}=w_x i+w_y j+w_z k$$

So, let us actually if consider the first aspect that is fluid rotation. How this small fluid element would be rotated. So, let us assume that omega to the angular velocity of rotation; if you again recall rotation is actually this particular case where how a block at one angular one position and it actually rotates about its own place, retaining the center and goes to a different location rotates to a different location. So, let us assume angular velocity here where which it is rotating is omega, so omega will again have three components omega x, i, j and k which means that this omega is a different axis in which they have independent rotations. So, really how do you define the rotation in this kind of case. If you suppose have two mutually perpendicular plane like this and you are rotating the planes you know all together retaining the perpendicularity between them that means the planes are not really deforming among it is own with respect to each other.

They are just keeping in the same angel and trying to rotate, and the average velocity rotation and this case would be nothing but the velocity of plane one plus velocity of plane two which is orthogonally placed by two that is how you certain the average velocity of rotation average angular velocity of rotation of two mutually perpendicular line elements of the particle in each orthogonal planes. So, let us obtain a mathematical expression for this and then it will be a generic expression, because it can be later on translated to the case where the angle between the plane changes as the rotation goes on, which is also the case of shear deformation where the third case.

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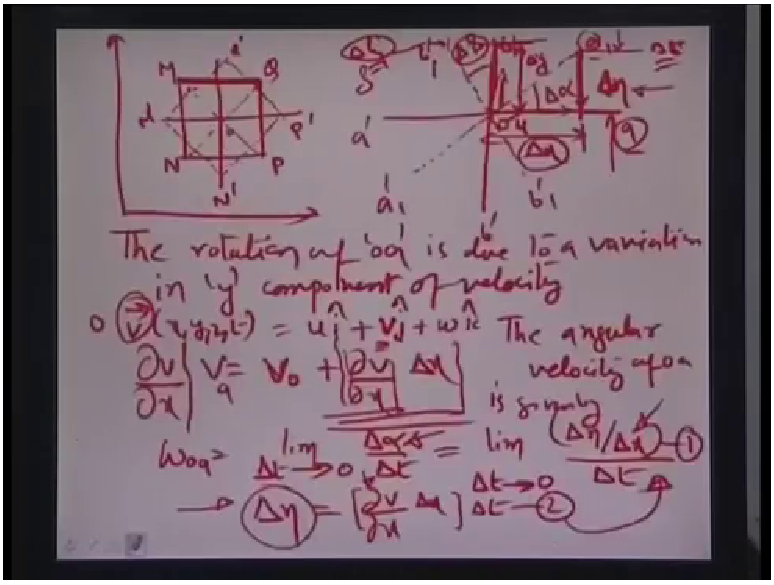

$$\vec{V}(x, y, z, t) = ui + vj + wk$$

$$\frac{\partial v}{\partial x} v_a = v_o + \left\{\frac{\partial v}{\partial x}\right\} \Delta x$$

$$\omega_{oa} = \lim_{\Delta t \to 0} \frac{\Delta \alpha}{\Delta t} = \lim_{\Delta t \to 0} \frac{\frac{\Delta \eta}{\Delta x}}{\Delta t}$$

$$\Delta \eta = \left\{\frac{\partial v}{\partial x} \Delta x\right\} \Delta t$$

So, let us actually look at that aspect. So, let us consider that in this particular case, you have a coordinate system let say x y and we are just considering rotation only two dimensional plane and you have block somewhere in position o, these are the two diagonals of the block, and the block kind of rotates now in a position where you know this basically this diagonals actually now come like this. And there is new location of the box created because of this rotation of this manner. So, basically you have the block placed at coordinate M, N, P, Q. And now the new position coordinates or become m dash, n dash, p dash, Q dash. So, it rotates about the center o. So, really this rotation can be figured out in terms of two angles, and angle which plane one this plane is able to define while it rotates on plane two; that means, this plane n q which is able to define as it rotates.

In other words, we can also consider the two diagonals which are perpendicular to this plane and they have fixed angular relationship with respect to the plane and we consider the rotation of the two diagonals in order to consider the average velocity of rotation in this particular instant. So, let us assume that and proceed, so you have a case where you have diagonals along intersecting a point o, and in the first instances after rotation these diagonals change positions to the new value, where the perpendicularity between the diagonal is maintain this case pure rotation. So, let us call these with different names, let this be o a, o a dash and this be o b and b dash, and essentially these move position a1, a dash 1, and b1, b dash 1. So, the way that rotation vector would be defined is definitely related to angle that these two planes would move in. So, let us assume this angle to be delta alpha, this to be delta beta.

Let us also assume that the amount of distance that this particular plane o a moves as in goes to a one is delta beta, and the distance here corresponding distance here is about delta x. And essentially here again the distance this position b1 is space from this position b is delta zeta. So, let me just rewrite this little more here. So, essentially as you know the diagonal has moved by delta beta, so the position here was b before this is b1 and the distance between b1 and b is given here by delta zeta. So, this is delta neta distance between a and a1, and the distance between b and b1 is given by delta zeta. So, these are some of the presumptions that we make to find out the relationship between the angles the velocities. So, one important point to be considered here that is that the rotation of vector oa, this particular line oa is due to of variation in y component of velocity.

So, let suppose this particular point o is having or define the velocity vector v, x, y, z and in time t where v in the x direction is given by u; v in the y direction is given by v and that in the z direction is given by w. So, essentially let say this small v here represents the velocity in the y direction. So, there is definitely going to be a change of this velocity from point o here in this particular region to point a, the velocity is different. So, if you assume that change to be delta v, this is small v mind you, this is not the v vector here delta v by delta x, so we can assume that if suppose the velocity at o in the y direction where v zero, the velocity at a would be equal to v zero plus delta v times that delta x times of del v by del x times of delta x. So, this is the additional velocity which is coming by the variation of the y component of the velocity as you move from o to a in this particular rotation case.

So, angular velocity of oa is where really given by as we all know the rate of change of angle. So, limit of delta t tends to zero delta alpha by delta t, we assume that these two points a and a dash are placed by a time point, placed in time by difference of delta t. So, delta alpha by delta t is really the angular velocity ω_{oa} at this particular instance of time. And therefore, this can also be represented by limit delta t tends to zero and what is delta alpha really it is equal to nothing but delta eta by delta x. We assume this to be the length of the arc, this distance here which is delta eta to the arc and we assume the radius vector to be delta x. So, really the angle delta alpha which it may moves is delta eta here divided by the radius vector delta x. So, that is essentially delta alpha divided by delta t.

So, let us now try to figure out what this quantity really comes in or becomes or what is the relationship between the rate of change of velocity the y component of the velocity with respect to x as we derived here. So, delta eta again we look at closely would be nothing, but the differential velocity change of v as it goes or the y component of the velocity as goes from while o to a which is nothing but del v by del x times of delta x divided or into a times of times of delta t that the time it takes for this point go from a to a1. So, delta eta the distance that the point a covers in order to go from a to a1 is nothing, but the differential change in the y velocity with respect to x times of delta x that is this is the change or due to which the velocity at point a here is different, then the point o here. So, this change times of delta t that exactly what delta eta would be the amount of distance that a traverses when it goes to a 1 would be. So, if I put this value of let say this is number two, and this is equation one; if I put this value of delta eta from equation two to equation one.

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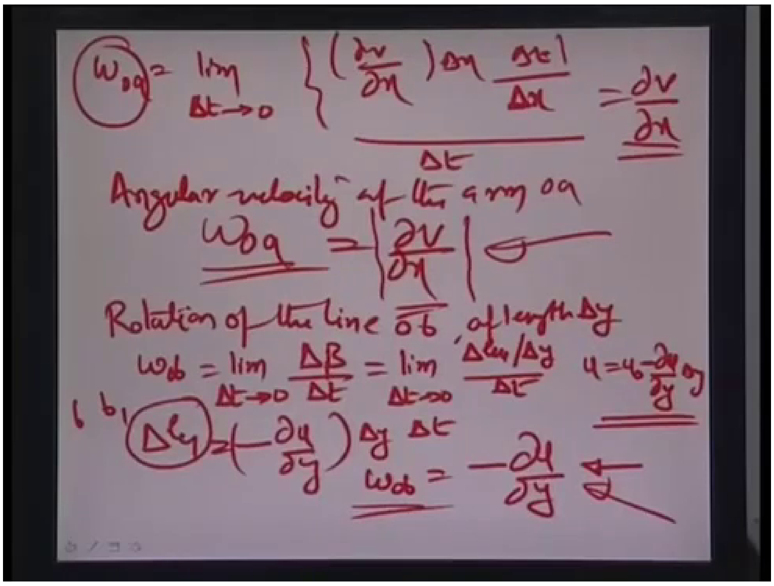

$$\omega_{oa}=\lim_{\Delta t\to 0}\frac{\left\{\frac{\partial v}{\partial x}\Delta x\frac{\Delta t}{\Delta x}\right\}}{\Delta t}=\frac{\partial v}{\partial x}$$

$$\omega_{oa}=\frac{\partial v}{\partial x}$$

$$\omega_{ob}=\frac{\Delta\beta}{\Delta t}=\lim_{\Delta t\to 0}\frac{\frac{\Delta\xi}{\Delta x}}{\Delta t}$$

$$u=u_0-\frac{\partial u}{\partial y}dy$$

$$\Delta\xi=\left(\frac{-\partial u}{\partial y}\right)\Delta y\,\Delta t$$

$$\omega_{ob}=\frac{-\partial u}{\partial y}$$

Let us see what the value of omega oa finally becomes, so omega o in that case is limit delta t tends to zero times of del v by del x times of delta x times of delta t by delta x divided by delta t which is nothing but d v by d x. So, this essentially is a d v by d x into delta into delta t is basically the delta value divided by delta x divided by t. So, this comes out to be d v by d x.

So, surprisingly enough the omega oa at the angular velocity of the r oa comes out to be del v by del x. So, the rotation a really translates into a rate of change of y velocity with respect to the x direction. Another if I do the same thing for the other part that is the component of the diagonal which was initially in the y direction a remaining to a certain what is the change in the x direction are what is what is change in the x velocity in the y direction it goes along that y what is change in the x velocity as in grows and that would essentially with omega ob.

So, let us derive that expression as well. So, the rotation of the line o b of length delta. So, essentially from to counter here, but I can say that this right here from here to here this distance delta y and you can also say that these distance is delta, delta, zeta as we have indicated in the angle move is delta beta. So, there is a similar kind of relationship should be describe omega ob and you can write that as limit of delta t comes to zero delta beta by delta t and this can further express this limit of delta t is to zero delta zeta by delta y by delta t and further we have let us say you the velocity the x velocity r to the origin here was u and here the velocity is in further in the opposite direction is a let say u zero plus d u by d y times of delta y.

So, you have a let say u is equal to u zero plus d u by d y times of delta y pretty much in the similar manner as you did it for the x variation we doing the y variation in this particular case. So, the del zeta in this case really is nothing, but minus del u by del y I am sorry this is actually minus because it will be in the reverse direction. So, essentially this would be a u minus u zero would be minus as you see here in this figure the use in this direction, but the del u by del x the differential because of this rotation is really i in the opposite direction. So, the value here would be u minus del u by del x times of del y del y del y del u by del y times of to delta y so that is what so this is opposite to the direction of the positive u that you have taken.

So, now, we are left with delta zeta equal to minus del u by del y times of delta y times of delta t right time the amount of distance that b takes to move to b prime b one and therefore, omega o b in this particular case would be nothing, but minus del u by del y now pretty much similar manner as we did before for the x component. So, therefore, we find out that the omega has really the two arms of the diagonals in the two arms are d v by d x and d u by d y with the minus sign. So, this is a very interesting that we will keep in mind of this time a, for proceeding hide with the momentum this is what the rotational component would do. So, anything related to shear with causes such a rotational component would essentially give you values in terms of du by dx and minus du by dy.

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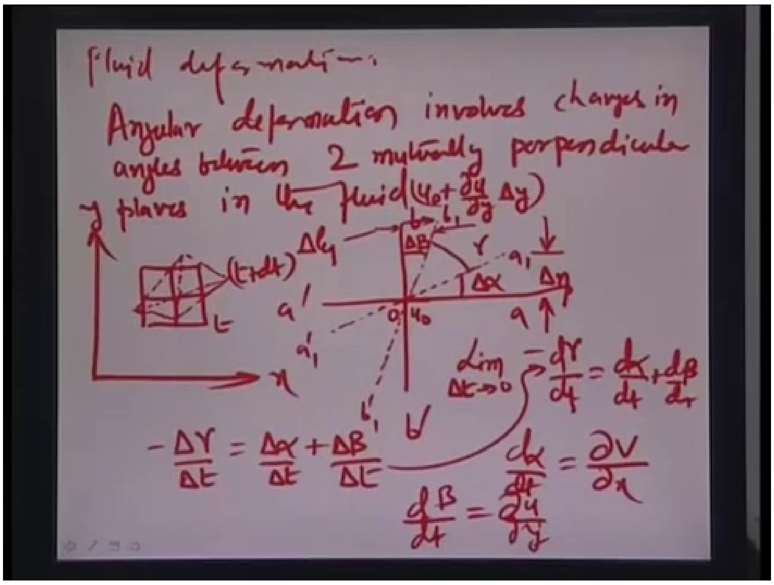

$$\frac{-\Delta\gamma}{\Delta t}=\frac{\Delta\alpha}{\Delta t}+\frac{\Delta\beta}{\Delta t}$$

$$\frac{-d\gamma}{dt}=\frac{d\alpha}{dt}+\frac{d\beta}{dt}$$

$$\frac{d\alpha}{dt}=\frac{\partial v}{\partial x}$$

$$\frac{d\beta}{dt}=\frac{\partial u}{\partial y}$$

Now, let us look at what the fluid deformation would be the case two is regarding fluid deformation. Let us just refer back to that particular diagram here, fluid deformation really is this second case angular deformation as you can see. So, the element actually changes its shape from a regular square into a more like you know it is a quadrilateral or parallelogram basically. So, essentially what rumbas? So, essentially there is a relative angular change between the two perpendicular planes; they did not remain at ninety degrees or such as the fluid moves. So let us actually calculate using mathematics, what this angular are deformation would be. So, angular deformation involves changes angles between two mutually perpendicular planes in the fluid. So, you have a x and y coordinates here, you have a square with two diagonals you angularly deforming this manner, so that the new shape that assumes has the diagonal moved away from perpendicular to this new angular shape.

So, essentially you are doing a deformation in the particular shape, now as you do that what essentially you doing is that there is a diagonal let say in the again the same o a, b, a dash, b dash direction and this diagonals are actually now changing in the a1, b1, b dash 1, a dash 1 direction. So, here let us assume that we have two angles the similar to the supposition before made before in the last case is delta alpha and delta beta. And let us also assume that you know the distance between a and a1 becomes delta eta, the distance between b and b1 as just is a last case becomes delta zeta. We also assume another angle here gamma which is in between which essentially see your delta gamma alright. So, here what really delta gamma is it is the let say this is gamma this is not delta gamma, this angle is gamma. Any change in this angle is delta gamma. So, change in this angle delta gamma is nothing but this change mind you is negative the angle is decreasing as you go along with time. So, minus delta gamma is nothing but delta alpha plus delta beta.

Essentially you know if you just divide the whole by delta t assuming that this shape was at time instants t and this shape came at d plus t. So, that is a time interval of del t between b t in between. So, you get minus gamma delta gamma by delta t is equal to delta alpha delta t plus delta beta by delta t. So, take limits on both sides as delta t a approached zero this whole expression becomes minus d gamma by d t equal to d alpha by d t plus the beta by d t so that is what angular deformation would really mean as we already know from you know earlier derivations that delta alpha by delta t a d alpha d t is nothing but the rate of change that the y velocity in the y direction. And similarly delta beta by delta t d B by d t is nothing but del of u by del y are the change x velocity in the y direction. We have just done these two proof if you may reiterate in the last slide, and when we were trying to see the rotation of a component without deformation this case there is a rotation and deformation simultaneously happen, actually the rotation is happening because of the deformation of the two sides.

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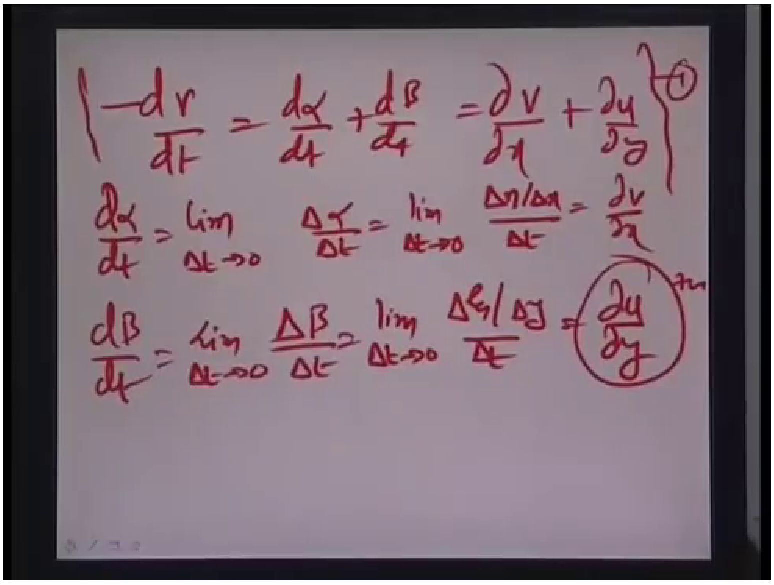

$$\frac{-d\gamma}{dt} = \frac{d\alpha}{dt} + \frac{d\beta}{dt} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

$$\frac{d\alpha}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\alpha}{\Delta t} = \lim_{\Delta t \to 0} \frac{\frac{\Delta\eta}{\Delta x}}{\Delta t} = \frac{\partial v}{\partial x}$$

$$\frac{d\beta}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\beta}{\Delta t} = \lim_{\Delta t \to 0} \frac{\frac{\Delta\xi}{\Delta y}}{\Delta t} = \frac{\partial u}{\partial y}$$

So, essentially then this minus d gamma by dt which is also d alpha by d beta by dt would become same as del v by del x let us del u by del y may retreat that d alpha by dt is nothing but limit of delta t times to zero delta alpha by delta t limit of delta t times to zero delta eta by delta x divided by delta t which is a del v by del x and delta beta by delta t is nothing but the limit of delta t times to zero delta beta by delta t which is equal to again the limit of delta times to zero delta zeta by delta y by delta t which is nothing but del u by del y. So, essentially here mind you du by dy is positive, because if you look at deformation really the deformation here if in the same direction as the u here. So, u was u zero here and here it is u zero plus del u by del y times of del y it is in the same direction as u, so therefore, this is a positive addition.

So, therefore, this is positive and this case and thus the relationship one whole valid minus d gamma by d t two is del x plus del u by del y. So, all certain done, we now have found out relationships of a what would happen in case of a fluid element rotating by itself and a fluid element deforming by itself. So, we all this knowledge in mind and also the way we did the acceleration of a point vector in a fluid space, we combine all these things together to find out what to dynamically alter the cube as it moves along in such a flow filed, there would be stresses, there would be principle stresses, there would be shear stresses which take place and some of them we have illustrated in our last class when we are talking about the conservation of mass equation, where you will have sigma xx, tau xy, tau yx, tau zx as a three stress components because of a the force and the x direction; and similarly three stress components force in the y and z directions. So, we had a stress matrix or stress tensor that we define in this manner.

So, let us see that if all these things are individually implemented and the we came some have find out the force by multiplying with the area element of the difference stresses can we equate that to the mass of this q in to acceleration and find out what is the equation which is emerging from that. So, these essentially the conservation of momentum right a Newton's second law a rate of change of momentum is nothing, but the force and takes place in the direction of the force. So, so apply the second law here that way in try to see what the final states of this relationship are between the different stresses shear ah principles stress and so on and so forth acceleration the mass and the area elements.

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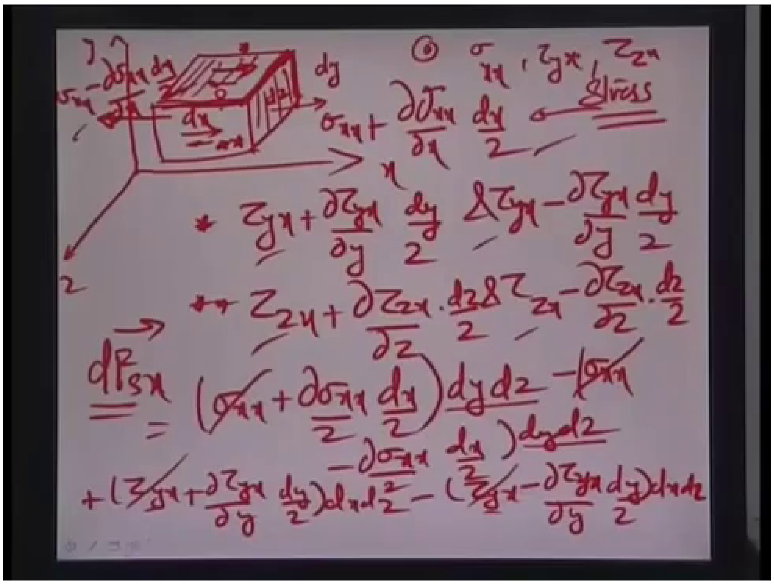

$$\sigma_{xx}+\frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}$$

$$\tau_{yx}+\frac{\partial \tau_{yx}}{\partial y}\frac{dy}{2}$$

$$\tau_{yx} - \frac{\partial \tau_{yx}}{\partial y}\frac{dy}{2}$$

$$\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\frac{dz}{2}$$

$$\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{dz}{2}$$

$$dF_{sx} = \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz - \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x}\frac{dx}{2}\right)dydz + \left(\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y}\frac{dy}{2}\right)dxdz - \left(\tau_{yx} - \frac{\partial \tau_{yx}}{\partial y}\frac{dy}{2}\right)dxdz + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z}\frac{dz}{2}\right)c$$

So, let us a look at our cube again on the control volume again you have a small cube which is of a volume dx. So, this is x y and z direction. So, dx is the distance of this particular dimension of the cube the z is this particular dimension of the cube and d y this things of this particular dimension of the cube and. So, you have a again different forces which are acting in the x direction let say you have a force, let us just a for the time being delete this arrows in the interest of space. So, we delete this by and large we know what this dx, dy, dz are. So, here we try to again this stress components, we have this component of stress we assume that this cube is centered about a point o, where you have a sigma xx, tau y x and tau give me a minute here…

So, we have sigma xx, tau y x which means due to a force x in the y direction in the area vector is in y direction and tau zx. So, at this particular edge of the cube or this particular phase your stress components would differ this would be sigma xx plus let say we have a variation in the x direction. So, del sigma xx by del x times of dx by 2 would be the stress component in this phase. The stress component similarly in the other phase here would be sigma xx minus del sigma xx by del x times of d x by 2 similarly the here components that would be here. And also in the opposite direction in the lower phase would be let put this is a star the star would be tau yx tau yx by del y times of d y by two and tau y x minus del tau yx by del times of del y by dy dy by 2. Similarly we will have this z components which are essentially in this phase in the phase in the back side these components would again. So, this double stars tell tau z x plus tau del tau z x by del z times of d z by two and tau z x minus delta was zx by del z times of z by two are the different components, so with all done d effects.

Therefore, which is essentially the total amount of force in the x direction is that this a result of all these different components component one here, two, and these three, four, and five six all put together and these are mind you, all stress terms. So, you need certain area of cross section across which these stresses are applied in order to find out the net force in the particular cube of interest. So the net force in this case would be equal to sigma xx plus del sigma xx by two times of a dx by 2 and area vector here is dy dz dy times of dz as I had pointed out before. Similarly, you have stress in the opposite direction which is same as sigma xx and where you have a of course, a minus del sigma xx by 2 times of dx by 2 times of dy by dz. And similarly you have components related to the shear. And I will just try to illustrate this here as plus tau yx plus del tau yx by del y times of del dy by two times of dx dz minus tau yx minus delta yx by del y times of d y by 2 times of dx dz. So, these are the various components in the x direction in mind you is tau yx again has a across section area dx times of dz that means, this times of this.

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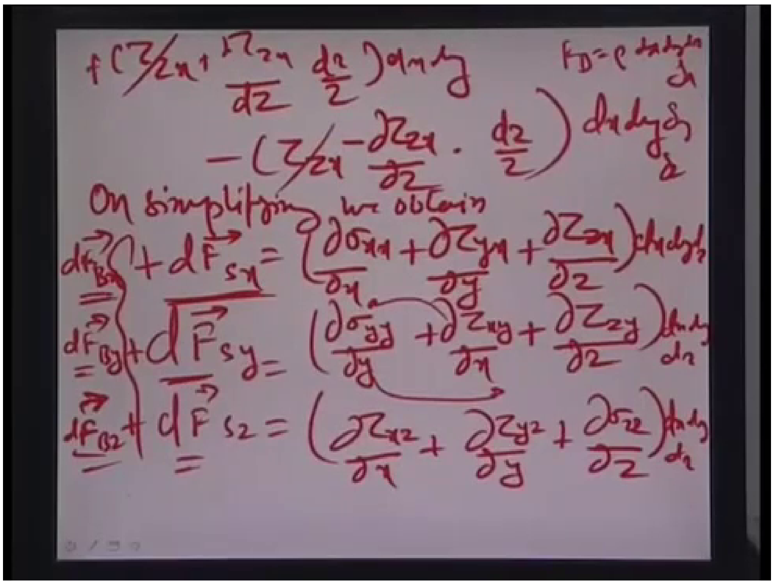

$$d\overrightarrow{F_{Bx}}+dF_{sx}=\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}\right)dx\,dydz$$

$$d\overrightarrow{F_{By}}+dF_{sy}=\left(\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{zy}}{\partial z}\right)dx\,dydz$$

$$d\overrightarrow{F_{Bz}}+dF_{sz}=\left(\frac{\partial\sigma_{zz}}{\partial z}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\tau_{xz}}{\partial x}\right)dx\,dydz$$

So, here you also have a third component of course, as a part of the whole deal, where is tau zx plus delta zx by del z times of dz by two times dx dy minus tau zx minus del tau zx by del z plus dz by 2, I am sorry into dz by 2 are multiplied by dz by 2 and times of dx dy which is the cross sectional area. So, if I take a simplified form of all this, as you know here from this expression, you can easily find out that the sigma x is actually really cancel because dy dz are similar on both similarly same goes true for the tau x y and only differential components here are what is retain really. And similarly here tau xz and minus tau zx, so what are the and if you assemble together all these different terms when on simplifying we obtain df sx is nothing but del sigma xx by del x plus del bar yx by del y plus delta zx by del z times of dx dy dz as the total amount of the force the x direction.

So, therefore, you know on a more simpler note, similar kind of expression can be generated for the different force components in the y and the z directions as well. Let say write this down here. So, in the y directions should be sigma yy times of del y plus del times of tau xy by del x plus del times of tau z by the z times of again dy dx dz. And similarly you have the other components here is really like a stress matrix, so this comes here and this goes actually here. So, in the other component, you have the del sigma zz with respect to the del z, and you have the component del tau of a now you have the y component. So, this is actually zy i am sorry this is zy, and so you have delta tau yz by delta y plus delta tau, you know it is kind of xz by delta x as into times of dx dy dz as a three different components of the force.

So, if you involve the body force that this stage and try to figure out what the overall would really look like will have this equation slightly modified because we them have to add the body forces along the x direction, the y direction and let us call that df xb b for body df By and df Bz as the different forces. So, therefore, if we have all these difference forces together, we will have to have and we can say that this is actually you know the body force f B can be represented as the rho times of the volume which is dx dy dz times of g right and g has again three components again gx gy and gz. So, let us try to all these equations together in the next page.

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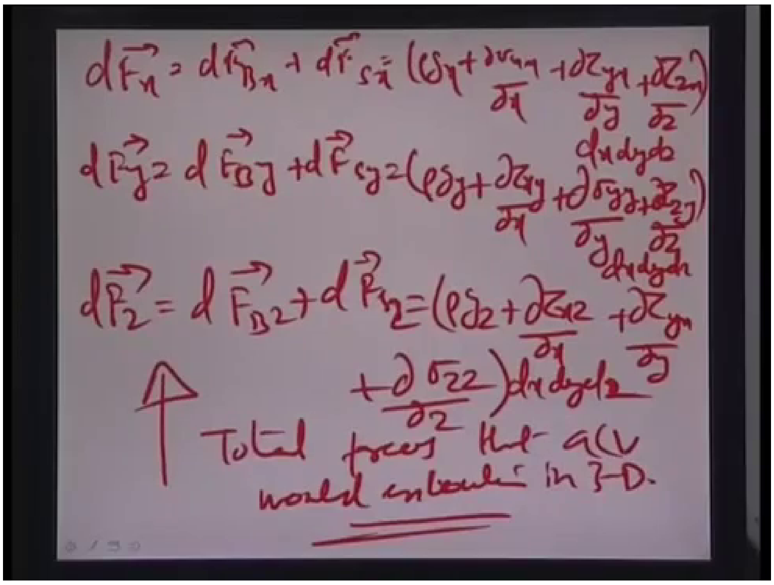

$$d\overrightarrow{F}_x = d\overrightarrow{F}_{Bx} + dF_{sx} = \left(\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) dx\, dydz$$

$$d\overrightarrow{F}_y = d\overrightarrow{F}_{By} + dF_{sy} = \left(\rho g_y + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{zy}}{\partial z}\right) dx\, dydz$$

$$d\overrightarrow{F}_z = d\overrightarrow{F}_{Bz} + dF_{sz} = \left(\rho g_z + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{xz}}{\partial x}\right) dx\, dydz$$

So, the total force in the x direction because of the body force f Bx and the stress force the force due to the stress vector can be represented as $d\vec{F}_x = d\vec{F}_{Bx} + dF_{sx} = \left(\rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) dx\,dy\,dz$. Similarly d f y is again having a body force in the y direction plus a stress force in the y direction. It is can be rho j y plus del x y with respect to a plus del sigma yy with respect to y is del tau z y with respect to z times of dx dy dz and dfz is equal to del body force in the z direction the stress force in z direction which can be rho gz plus del tau xz by del x plus tau beta xy by dz dy plus del tau and del sigma zz because it is not a shear for this principle stress sigma zz by del z times of dx dy dz. So, these are really the equations for the total forces that the control volumes CV would encounter due to stresses as well as shown weight in all the three directions. So, this kind of brings us to end of this particular lecture.

So, the next lecture what I would like to illustrate is that we move ahead with this force equation and try to kind of compare it with mass times of acceleration and see what the final outcome of equation would be.

Thank you.