**BioMEMS and Microfluidics**
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**Lecture – 27**

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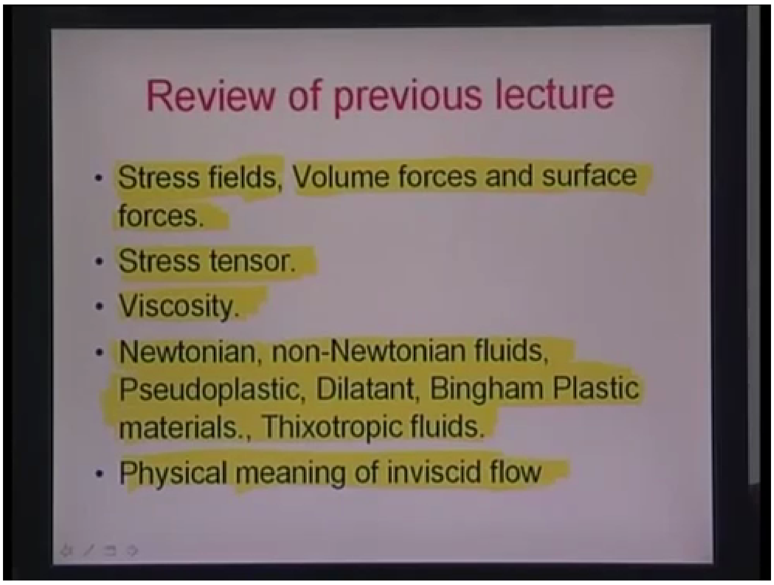


Let us do a quick review of what we did last class. So basically we were talking about different kind of fields like stress fields, we try to understands what stress fields is in terms of area vector and the force in the normal as well as tangential direction. We also discussed some basic differences between volume and surface forces and you know some examples so kind of a coin like for example, gravity is a big volume force ,it is a body force where as a forces less to viscosity are more predominantly on the surface to the surface forces. We talk that describe this stress tensor which is essentially a matrix three by three matrix where you have the principle stresses in the diagonal element in the non diagonal element here stress component. We described a basic notional classification of a how to represent the stress tau y x would mean there is a force in the x direction, and it is along an area vector pointing towards the positive y direction that is how we call it $\tau_{yx}$ stress which is shear, so area vector which this is forces as shear it is in the y direction in the force itself is in the x direction.

So we saw that basic classification, we try to derive the basic Newton's law for viscosity, Newton's law for viscous motion of fluids where in a correlation where drawn out between the shear stress $\tau_{xy}$ with the rate of deformation du/dy, and we classify different fluid as Newtonian, non-Newtonian. Newtonian where in this stress and velocity gradient are in direct proportion with each other; constant of proportionality being viscosity which later on converted into kinematic viscosity, because a better physical idea would be to compare the viscosity absolute values with density of solution on the medium. So then we talked about various different kind of non-Newtonian fluids like pseudo plastic where in the viscosity seems to go down with the deformation rate. Dilatants where the viscosity would have a reverse behaviors going up with the deformation rate, and then Bingham plastic which would essentially behave as a solid up to a certain viscous beyond which it will follow the path of the Newtonian fluid.

Then, we talked about Thixotropic fluids materials essentially where if it described about properties related to you know the variation or the temporal variation of viscosity time that means the viscosity index eta would vary typically temporarily with time, it will actually decrease with time. So then we were just about describing the difference between the viscous and inviscid flows; inviscid flow again definitional are flow where the viscosity can be treated as zero, it is an normally you know it is really an derive situation in a never exist in nature are there is no fluid in nature which exist with viscosity of a zero value, but then in micro scale flows or in macroscopic flows we can consider region which becomes inviscid, because of being away from a flat plate and we will actually a describing the situation by considering what would happen when a flow of some uniform velocity passes over a flat fixed plate. So the approximately the plate does not any more matter to create a velocity gradient, so those are inviscid region of flow.

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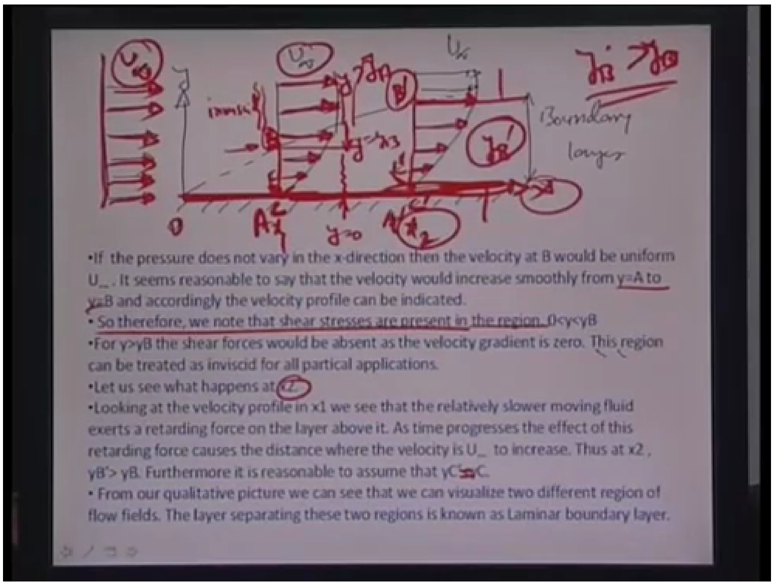


Let us go ahead and actually look at little more of what really happen is when the flow means flat plate. So we were talking about flow coming in x direction with a uniform velocity, let say $U_\infty$ as you can see here and flat plate being position in the o x direction. And then we talking about two points A and A dash, which represented as x 1 and x 2 on the x coordinate. So some conclusion about this process that is about does not vary in the x direction and the velocity at the B would be uniform $U_\infty$, so we can assume that it kind of seems reasonable to set the velocity would increase move to the y equal to A to y equal to B. So you have a case here where there is no you know gradient of pressure in the x direction here, the pressure is pretty much constant, we assume that $U_\infty$ to be constant at the ((Refer Time: 05:11)) it is approach the plate then you consider that the there is almost always zone of no slip, which comes into this layer which is close approximately of the plate which actually close all way up to infinity beyond the certain y.

And let say the point where it goes to $U_\infty$ is B, so there definitely the shear stresses in the region B C, C is the point you know at the surface here, and B is the point from which the velocity goes back to $U_\infty$ and beyond this the flow behaves as a between inviscid that is how we interpret that in the last class. So, therefore, we know that the shear stress at present in the region zero to $y_B$ in this particular region, so y equal to zero is this plane and y equal to $y_B$ this plane. And essentially for y greater than $y_B$, in this particular case, as you see the shear forces are absent, because the velocity is now all uniform and it is riming very well with the initial velocity $U_\infty$ . So there are no whatsoever the shear forces in the inviscid region. And the V viscous forces and the shear forces are only a between the y equal to $y_B$ and y equal to zero in this particular region here.

So, we will just see what happens in x2 and this particular point, so let us look at the velocity profile in x1, we see relatively slower moving fluid exerts retarding force on the layer above it. As time progresses the effect of this retarding force causes the distance where the velocity is $U_\infty$ to increase, thus it $x_2$ $y_B$ dash as to be further away then this point here, we just point of contact c dash. So this is kind of you know proposition rule that as the flow enters this region and let say at point c, there is a certain velocity gradient that is establish between the zero point or zero velocity and you know this b, where the velocity is now $U_\infty$ , but as that the flow propagates along plate this frictional force kind of predominate.

So, therefore, this region here where the velocity would go back to $U_\infty$ should increase, because there is a energy lose and forms of friction as you move from point c to c dash; c dash of course, is this new point here at you can see on this arrow. So if you assume this to happen then we can think that you know the fluid applying retarding force to the plates, the force an increasing as it goes long from zero two towards x 2. So, therefore, definitely the $y_B$ dash here which is essentially this distance should be greater than $y_B$, because it takes a while because retarding forces more at b dash I mean c dash b dash plane this plane. So, therefore, we know the fully develop flow here obtain y it is $y_B$ dash should certainly be greater than the value $y_B$. So we can also kind of reasonably assume that y c dash, so $y_C$ and $y_C$ dash are pretty much same as you can see here and the reason being that you know the no slip zone would always be kind of you know close to the surface. It does not go beyond into the bulk.

So from our qualitative picture, we can see that we can visualize these two different flows by a separating layer between them. One where there is a shear, which is existing at the bottom starting from the plate all the way up to where the fully developed flow as happened that is $U_\infty$ , and another which is at inviscid region, where it is starts from the U infinity I mean the fully develop flow and in goes into the bulk, so the layer which is separating these two also known as called the laminar boundary layer, it is call the laminar boundary layer.

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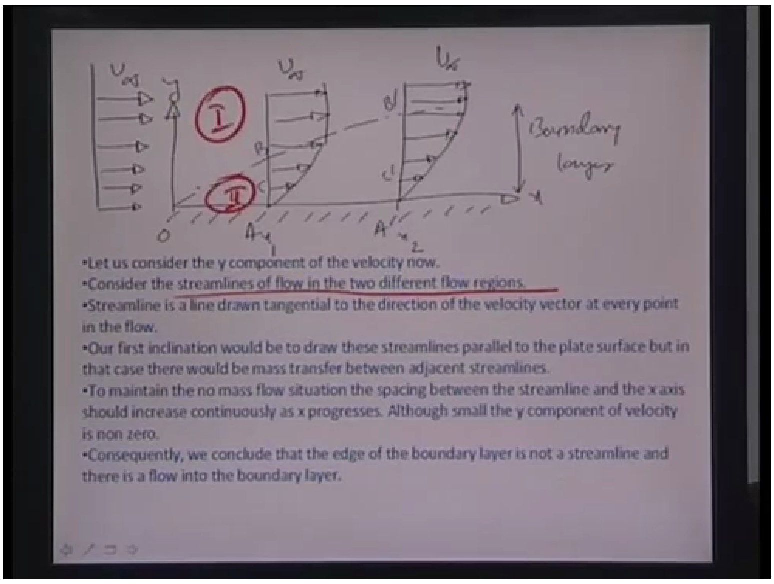


So, essentially if you consider the y component of the velocity now, there is now very interesting thing which comes out that let say you know we consider these stream lines of the flow. And as we know from a earlier definition what are the stream lines they are tangential vectors are line joining the tangential vectors to the direction of flow of particles so that is how stream lines can be categorized. So let us consider this stream lines of flow and these two different flow regions; flow region one here, which is inviscid region in flow different two rear which is the viscous domain. So what would be need to a assume to be maintain consistency, so our first information would be to draw these stream line kind of parallel to the plate assuming that you know the fluids go pass the plate parallel.

Now we interestingly if there are parallel stream lines generated parallel to the surface of the plates, and we are saying that in a one case, there is a lesser amount of velocity which increase all the way to $U_\infty$, in other case there is all $U_\infty$. They would not be much region much problem in the region one of the inviscid region, but region two definitely there is going to be mass transfer in the y direction, because of principally you know the amount of feed of the fluid, if you want the continuity to be valid or if you want to assume the fluid is the large continuum and the they cannot be any gaps in between can be one indivisible mass of a substance flowing over the plate. So in that case if there are there is a velocity gradient and there is a tendency of the lower layers you known parallel to the plate to reach the at a slower rate certain point, the upper layer should move at a higher rate, and try to free occupy that point. So there is going to be mass planes for in order to balance such a system of flow.

This situation does not really exist because as we know that they can be a velocity gradient, but the they cannot be any really any really any velocity in y direction even if there are velocity in the y direction, this continuum failure never happens within the fluid. And so what is needed to maintain the no mass flow kind of situation that this spacing between this stream lines, different stream lines go and increasing you know the distance from the surface the flat surface as the flow goes along, so the stream line are all kind of merging out for from the point, where the flow as just enter along as the flow goes along the surface, flows the stream lines gets separated by greater and greater distance, so they are not really parallel oriented, they have different directions, which go once spreading up more and more as the flow goes along the direction of the plate.

So essentially, we conclude that the edge of the boundary layer is not stream line and just because of you know stream line is something across which there cannot be typically the any mass transport, because tangential to the direction of the stream line the particles are all moving their velocity vectors are in the tangential direction to the to the stream line there is no inward radial flow from one stream line to another. So the boundary layer which is the separation layer between the inviscid flow which is in the top and the discuss flow which is the in the bottom is not a stream line, because there has to be kind of you know mass flow to maintain the balance between the lower velocity is and fully developed flow velocity U infinity across this layer. So consecutively we conclude that the edge of the boundary layer is not stream line, and there is a flow to the boundary layer assuming the difference in the velocity across it. So based on some of these the concepts, we can divided all the viscous flow regimes into laminar and turbulent.

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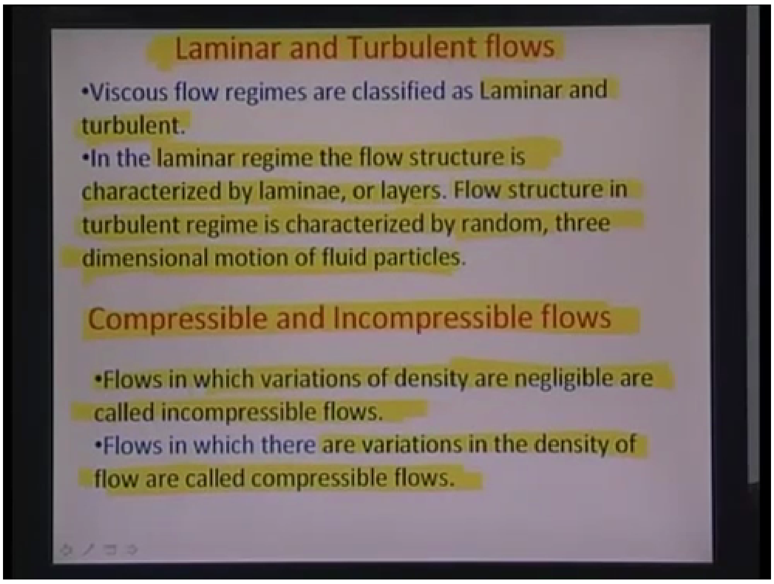


In a laminar regime, the flow structure is characterized by laminae, or layers, this is the regime where micro flows are kind of packed up. And in the flow structure is turbulent regime this is mostly macro scale version of flow is by random three-dimensional motion of fluid particles. We can also categories, so we have already classified fluid as viscous and inviscid, we have categorized fluid in a laminar in turbulent. We can also categories fluid into compressible and in compressible. And essentially the main differences that in compressible flows, there are variations of density along the fluid medium; whereas incompressible flow assume the density to be just constant across the whole continuum of the fluids. So the flows in which variations of density are negligible are incompressible, and there where the variations density of are substantial they are called compressible flows that is how you divided flows into compressible and incompressible apart from laminar and turbulent, and viscous and inviscid flows.

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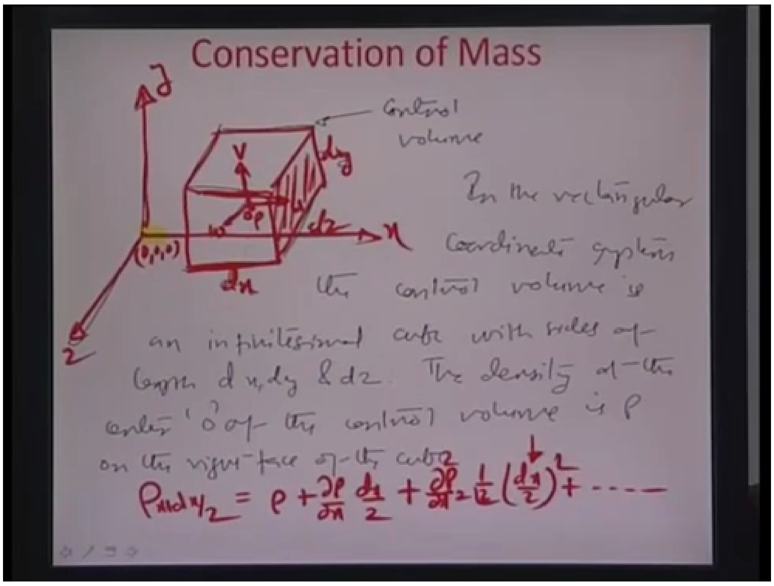


So let us now try to go ahead and derive the first equation of conservation of mass or what we call the first Navier-Stokes equation, so for that a need to assume again the small control volume. Let us say we are trying to see the amount of mass flow into this control volume, and the amount of mass flow outside this control volume. And it is center around point o, in this further like a cube around this point o with dimensions dx are the rectangle around this point flow around this point o with the dimension dx, dy and d z and the x, y and z direction. And what we would be looking at that if you assume that there is no creation of mass with in this control volume, so whatever is in flowing in to the mass is exactly the control value is exactly equal to the mass that is out flowing of control volume, so this is also known as continuity equation or the conservation of mass equation.

Let us try to mathematically or geometrically derive this particular equation. So let say we have x, y, z direction here. It is a rectangular coordinate system, and we assume a control volume of a rectangular shape cubical with the values dx, dy and dz dimensions, so further assume this point over all which this control volume is equally spaced, symmetrically spaced. And we have three components of the velocity u, v and w, it is a three dimensional flow this is the origin 0, 0, 0. And the we are trying to investigate what happens in this point o. So the very first thing that we would like to investigate is let say the density given we have a density rho here at a point what would be the density at let say $\rho_{x+dx/2}$ , which is this particular face here. So this can be expressed again as a you know Taylor approximation as $\rho_{x+dx/2}=\rho+\frac{\partial\rho}{\partial x}\frac{dx}{2}+\frac{\partial\rho}{\partial x}\frac{1}{2!}\left(\frac{dx}{2}\right)^2\pm--$ .

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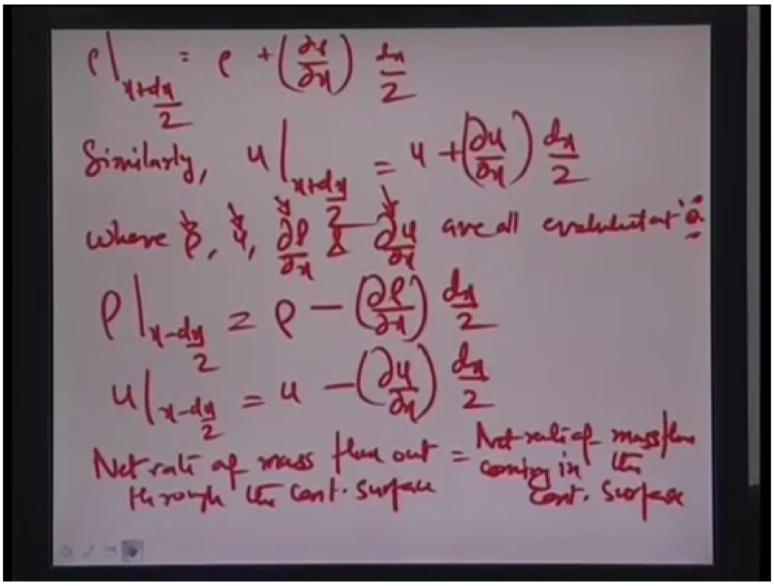


So if you assume these dx to be infinite decimally small element and neglect all the higher order terms here the d that the rho x plus dx by 2 really comes out to be y we are taking the this by 2 is because we assume this whole length to be dx and this at the center, so therefore, this face s this shaded face s here is at distance of exactly dx/2 from o that is why in the dx by 2 term. So, therefore, $\rho_{x+dx/2}=\rho+\frac{\partial\rho}{\partial x}\frac{dx}{2}$ . Similarly $u_{x+dx/2}=u+\frac{\partial u}{\partial x}\frac{dx}{2}$ , where rho u del rho by del x and del u by del x are all evaluated at o, you have to keep this in mind, because there essentially evaluating what is happening at one of the edges based on the properties of the point o, and all these values that means, including the change of a density with respect to x change of velocity with respect to a velocity and the density must necessarily be at the point o.

So, therefore, we can write similar equation for the other face that is the face on this a negative side, this particular face. And here we can say $\rho_{x-dx/2}=\rho-\frac{\partial\rho}{\partial x}\frac{dx}{2}$ and $u_{x-dx/2}=u-\frac{\partial u}{\partial x}\frac{dx}{2}$ . Now for concentration of mass, we say in the one direction of the one dimension, we have to necessarily assume that the net rate of mass flux out through in the control surface is essentially equal to the net rate of mass flux coming in the controls surface. Since the assumption suppose that we have to have necessarily made here. So, therefore, we have to really see that there not only the x faces, but also y faces and z faces and also faces along the minus y and minus z direction. And so the whole equation can be thought of as you know problem with there all these different faces are in flows and out flows, and we are trying to see how the fluid masses concern in this particular case.

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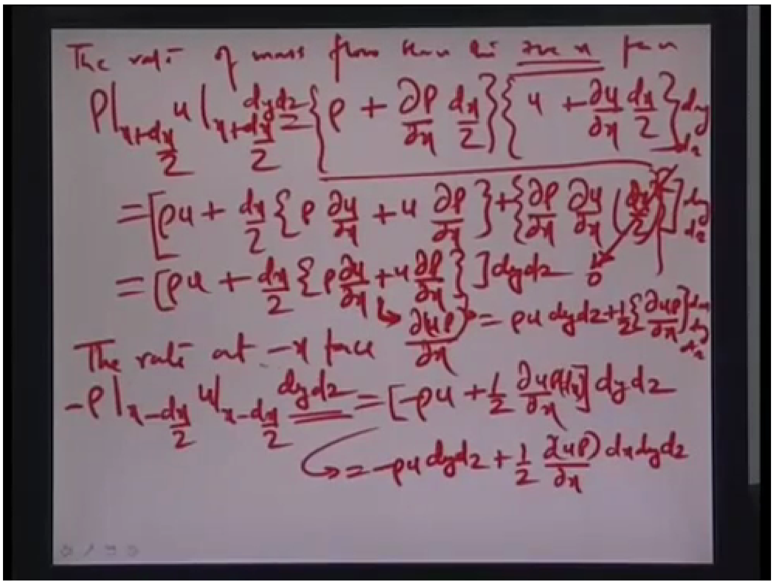
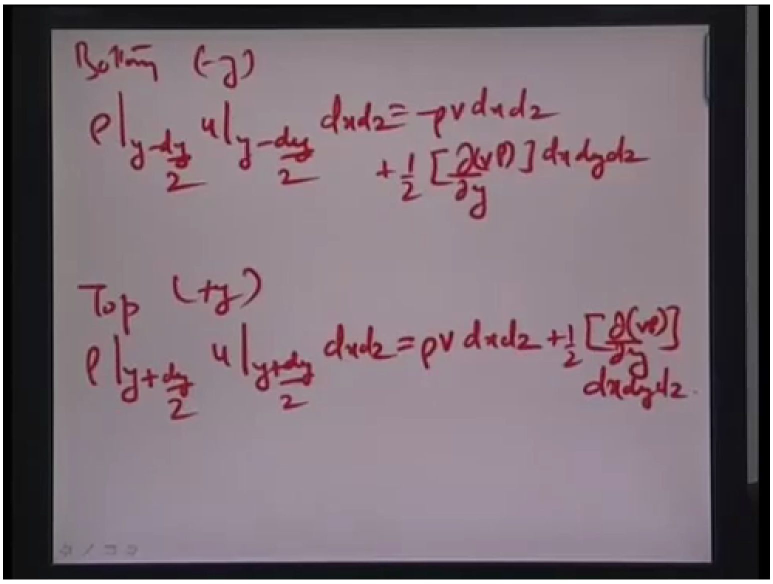



So what we do here is that let say evaluate the rates of mass flow in flow and outflow at all the different faces. So the rate of mass flow through the positive x face is one to begin the control volumes. So we know the density times of velocity is times of area is really the mass per second, so density times of velocity times the area of the face is mass flow per second. So here in this case, we can write the $\rho_{x+dx/2} \times u_{x+dx/2} dydz = \left\{\rho + \frac{\partial \rho}{\partial x}\frac{dx}{2}\right\}\left\{u + \frac{\partial u}{\partial x}\frac{dx}{2}\right\} dydz$ .

So if you solve this particular equation, you have the resolve value the as $\left[\rho u + \frac{dx}{2}\left\{\rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x}\right\} + \left\{\frac{\partial \rho}{\partial x}\frac{\partial u}{\partial x}\left(\frac{dx}{2}\right)^2\right\}\right] dydz$ . Now we assume that the component dx being very, very small into dy d z - the area the component dx by 2 very, very small this




actually can be approximated zero which eliminates together this particular term here. So we are left with the equation

$$\left[\rho u+\frac{dx}{2}\left\{\rho\frac{\partial u}{\partial x}+u\frac{\partial \rho}{\partial x}\right\}\right]dydz=\rho udydz+\frac{1}{2}\left\{\frac{\partial u\rho}{\partial x}\right\}dxdydz$$

. This is the basically just comes from you know the differentiation of the product formula so that is what the mass flow rate is really towards the positive x face.

Now, let see what would rate would be at the negative x face, and the only difference this case would be the rho and the U both are evaluated at x minus dx by 2 face, the area vector always remains same in the magnitude dy/dx so this typically of course, you have to have so the area vector is all though it is same in magnitude by it is actually negative in direction. It is exactly opposite direction points to minus x side in this case you have to have a minus sign representing the direction of the area vector dy dz. So in this case, the expression can be simplified as

$$-\rho_{x-dx/2}\times u_{x-dx/2}dydz=\left[-\rho u+\frac{dx}{2}\left\{\rho\frac{\partial u}{\partial x}+u\frac{\partial \rho}{\partial x}\right\}\right]dydz=-\rho udydz+\frac{1}{2}\left\{\frac{\partial u\rho}{\partial x}\right\}dxdydz$$

so that is how the rate of negative x face.

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Let us do the same for the positive as well the as the negative y face, so for the bottom pointing towards the negative y direction, we can represent this as $-\rho_{y-dy/2}\times u_{y-dy/2}$ times of this case if you look at the area vector in the negative y direction it is dx times of dz. So, therefore, this can be represented as $-\rho_{y-dy/2}\times u_{y-dy/2}dxdz$ , and this comes out to be again further simplified in a simplified manner comes out to be

$-\rho_{y-dy/2} \times u_{y-dy/2} dxdz = -\rho v dxdz + \frac{1}{2}\left\{\frac{\partial v\rho}{\partial y}\right\} dxdydz$ . So for the top surface pointing towards the plus y direction, this would come out to be $\rho_{y+dy/2} \times u_{y+dy/2} dxdz = \rho v dxdz + \frac{1}{2}\left\{\frac{\partial v\rho}{\partial y}\right\} dxdydz$ .

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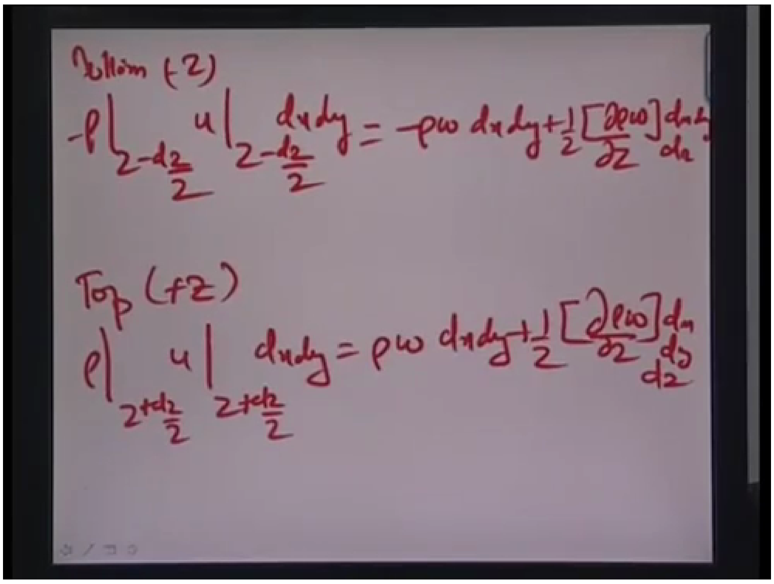


Similarly, we do the same for the face pointing towards the minus z direction. And here, we can write the velocity vector to be $-\rho_{z-dz/2} \times u_{z-dz/2} dxdy = -\rho w dxdy + \frac{1}{2}\left\{\frac{\partial w\rho}{\partial z}\right\} dxdydz$ . And similarly for the top pointing towards the plus z direction, we have $\rho_{z+dz/2} \times u_{z+dz/2} dxdy = \rho w dxdy + \frac{1}{2}\left\{\frac{\partial w\rho}{\partial z}\right\} dxdydz$ .

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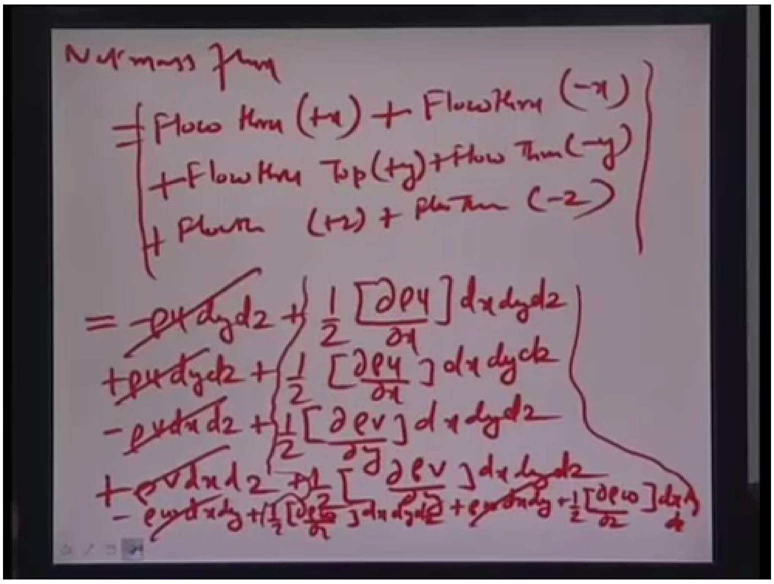

So essentially the net mass flux then as I already talked about should be equal to the flow through plus x direction plus the flow through minus x face plus the flow through the top plus y plus four through minus y plus four through plus z plus four through minus z direction. And we also assume here that if there were also make it very generic in nature. First of all let us find out what is the summation of all these different force so that comes out to be equal to minus rho u and I am just borrowing this from the earlier cases that we have derive minus rho u dy dz plus half of del rho u by del x times of dx dy dz plus rho u times of dy dz plus half of del rho u by del x times of dx dy dz plus we have similar terms for you know for the plus y minus y and plus z minus z direction let us write them down.

$$\rho u dydz + \frac{1}{2}\left\{\frac{\partial u\rho}{\partial x}\right\} dxdydz - \rho u dydz + \frac{1}{2}\left\{\frac{\partial u\rho}{\partial x}\right\} dxdydz - \rho v dxdz + \frac{1}{2}\left\{\frac{\partial v\rho}{\partial y}\right\} dxdydz + \rho v dxdz + \frac{1}{2}\left\{\frac{\partial v\rho}{\partial y}\right\} dxdydz + \rho$$

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And what you are left with are the second terms here right here which if we you know kind of sum up together.

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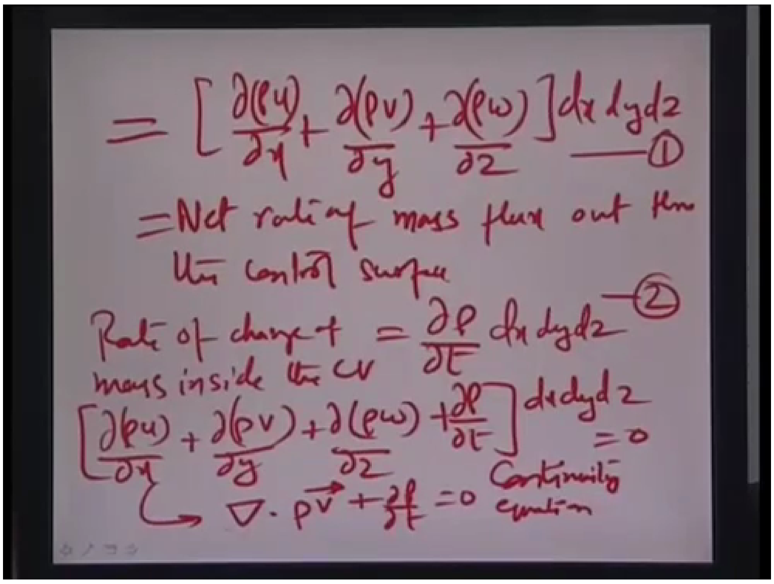


We would be getting in equation with $\frac{\partial u\rho}{\partial x} + \frac{\partial u\rho}{\partial x} + \frac{\partial u\rho}{\partial x} dxdydz$ . So essentially, this is really the net rate of mass flux out through the control volume surface, so I can write this as the net rate of mass flux out through the control surface. What is interesting here is to point out is that if the rate of change of mass inside the control volume is a function of you know time that there is the mass which is generated of created inside them to control volume. In that case we can always write down that the rate of change of mass inside the controls volumes - CV is equal to $\frac{\partial \rho}{\partial t} dxdydz$ , the rate of change of density and this can be a case of compressive flows where there is a rate of change of density with time. Incompressible force of flows of course, this dp by dt does not make any times d rho by dt rho s it is zero, we assume that the density these constant temporarily is the variation of density with time of the control volume del x del y del z. So in a more generic manner, the equations one and two here, which have been derived; if added together should give you a situation whether it is for compressive flow or incompressible flows.

And what you can do is that you know the total amount of mass in this manner which either inflows and outflows are gets generated should be equal it to give zero because of the conservation of the mass so mass cannot be crated or destroyed. If you are assuming continue assumption inside the such a control volume particular fluid, so in that case the ¿ $\frac{\partial u\rho}{\partial x}$ ¿ +

$\frac{\partial v\rho}{\partial y}+\frac{\partial w\rho}{\partial z}dxdydz=0$ . And it actually can be in a more rigid manner written down as $\nabla \rho \acute{v}+\frac{\partial \rho}{\partial t}=0$ , so this is what the first of the Navier-Stokes equations are about continuity or conservation of mass.

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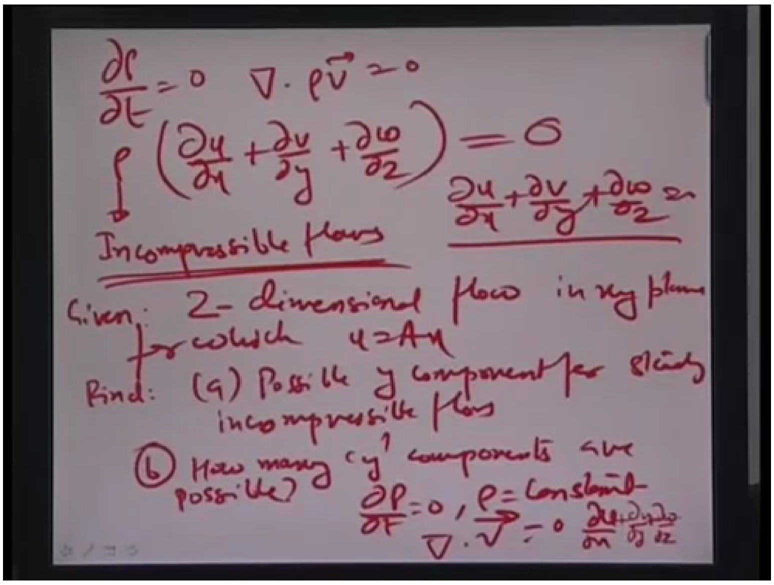


So typically for an incompressible flow though what you would be left with is a just this part of the term, so what we will left with just $\nabla \rho \acute{v}$ . So incompressible flow cases, when particularly $\frac{\partial \rho}{\partial t}=0$ , the continuity equation really reduces to $\nabla \rho \acute{v}=0$ so that means the $\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0$ . Remember in incompressible flows this rho does not vary time more space so there is absolute no variation in the density, the density either in time or space both remains same and constant, so this is the situation of incompressible flows.

So, therefore, this becomes equal to zero, and other words $\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)=0$ is new form of the continuity equation particularly for incompressible flows. So this kind of try to understand this for an example let say we have given there exists two-dimensional flow in x y plane, for which u becomes equal to A x. So you have to find in the possible y component for steady flows, steady incompressible flows using the continuity equation, and also how many y components, how many such y components be possible. So as we know here $\frac{\partial \rho}{\partial t}=0$ or rho is an constant and therefore the whole continuity equation changes to del cross v vector equals zero del u by del x plus sorry del u by del x plus give me a minute here plus del v by del y plus del w by z equal to 0.

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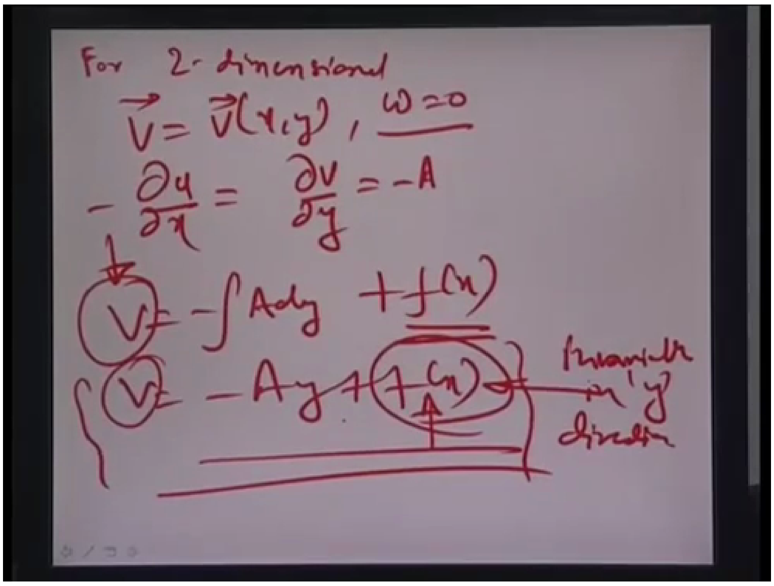


So, essentially this means that and also we although already know that the flow is two-dimensional, so flow such a two dimensional flow v essentially should be a function of x y right velocity v vector should only be a function of x and y. So, therefore, there is no third component which exists or w equal to zero. In many case, so that compressible equation on the continuity equation changes to $\frac{\partial u}{\partial x}=\frac{-\partial v}{\partial y}=-A$ . So, therefore, we can safely kind of try to integrate and find out the value for the velocity v, so as you do that we can get v is essentially $v=-\int Ady+f(x)$ or $-Ay+C$ . We can assume of function of x to be constant along the y direction. So the V essentially becomes minus a y plus function of x so this is essentially in variable in the y direction, it is a pure function of x.

So, therefore, maybe treated as a constant in this particular case it could also be a normal constant from that so essentially a possible y component for the study incompressible flow can be expressed as minus ay plus function of x. So you can also say that because there is a function of x involved then many such solutions of the y component of velocity that is possible using the continuity equation. So let us also do a little bit of different kind of example related to an operating piston and certain cylinder pressure to understand the continuity equation little better.

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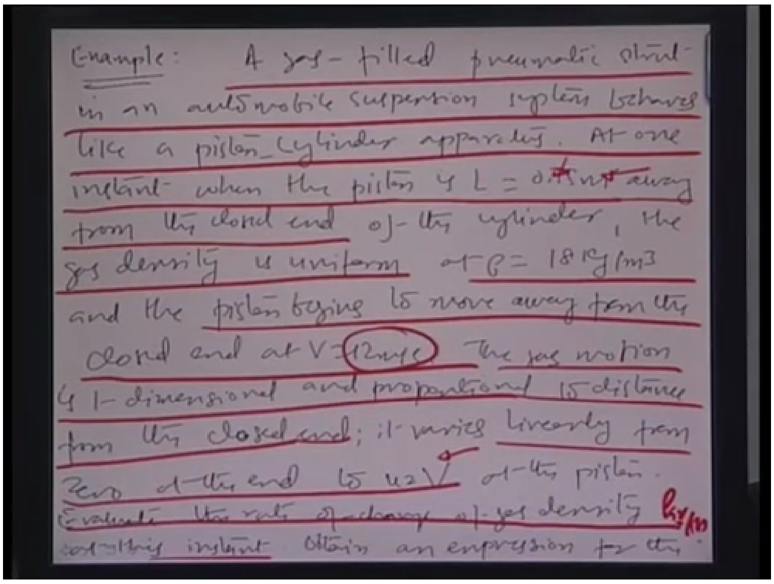


So in this particular example, as you see that there is a gas filled pneumatic struts in an automobile suspension system. And it behaves like a piston cylinder operators. The boundary conditions that are given is that one end or a one instance when the piston is at say the total length l equal to zero 0.15 meters away from the closed end of the cylinder. The glass density is uniform rho is equal to 18 kg per meter cube. And the piston begins to move away from the closed end that velocity equal to roughly about 12 meters per second. And the gas motion in one-dimensional really one-dimensional in this case and proportional to the distance from the closed end, so it varies linearly from zero to velocity v, which means that at two ends of the close n in the present velocity is zero and when you goes to the length l equal to 0.15 meter, which is at one end from that means this is away from closed end, for that is for the ((Refer Time: 46.44)) at that the velocity U becomes V. So we have to evaluate the rate of change of the gas density at this particular instance, particularly when the piston is at 0.15 meter from the closed end and we also need to obtain an expression for the average density is the function of time. So we need to find what rho average is in terms of rho t here.

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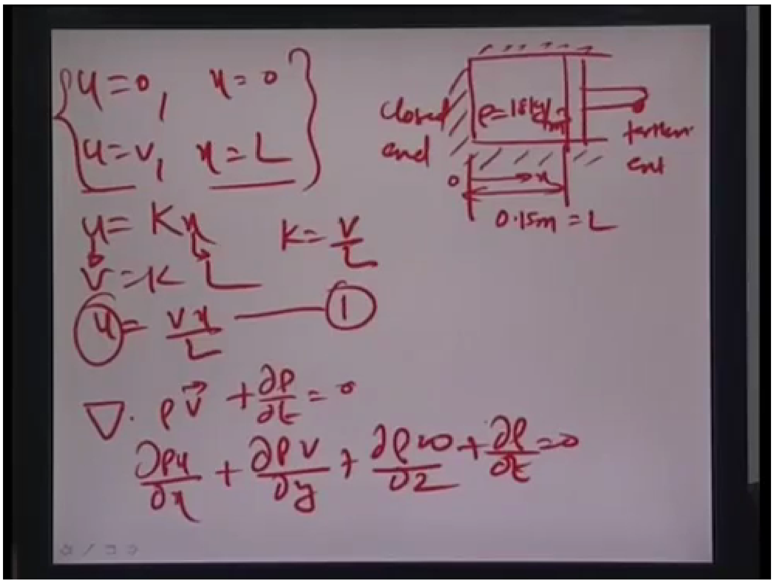


So, let us actually try to solve this using continuity equation we have a cylinder here in the example, three fix ends and movable piston is say here. And essentially what has been indicated here is that the gas density within this volume is 18 kg per meter cube. And this is the closed end and this is for the textometry that the piston can travel and here in the question the textometry has been given as 0.15 meter, so essentially distance here is a 0.15 meters. So we also further know that the velocity zero, let say we are talking about the x direction is starting from zero here all the way up to l, so velocity is zero when x is equal to zero, and velocity really is v when x equal to this l value.

So, therefore, and also we further know that as it is given here that the gas motion is one-dimensional and proportional to the distance from the closed end. It varies linearly from zero to u x, because it is a linear variation. We can assume that the u is really equal to some constant k times of x; v becomes equal to kl because u is v as a x is l essentially or k becomes v by l. So u essentially as a again vx yl that is how u is x equal to 0, u 0 or x equal to l, u is equal to v. So all set in this is what velocity equation is in terms of x. So, now apply the continuity equation here, we know that by the continuity equation, we have del dot rho v vector; v is the velocity vector is essentially plus del rho by del t essentially equal to zero. So

$$\frac{\partial u\rho}{\partial x} + \frac{\partial v\rho}{\partial y}+\frac{\partial w\rho}{\partial z}+\frac{\partial \rho}{\partial t}=0 \quad .$$

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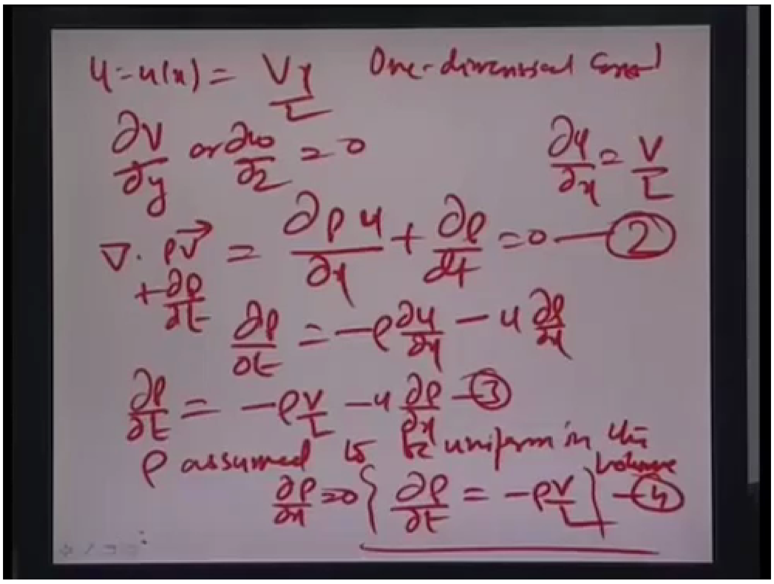


Essentially if we just put the value of u equal to $u=u(x)=\frac{vx}{L}$ value and with respect to this is the only velocity mind you this is only one dimensional case has been indicated, piston cylinder arrangement. So, therefore, $\frac{\partial v}{\partial y} \vee \frac{\partial w}{\partial z}=0$ , so only the other value which comes out of this whole del cross rho v is essentially

$$\nabla \rho \acute{v}+\frac{\partial \rho}{\partial t}=\frac{\partial u\rho}{\partial x}+\frac{\partial \rho}{\partial t}=0$$

$$\frac{\partial \rho}{\partial t}=-\rho\frac{\partial u}{\partial x}-u\frac{\partial \rho}{\partial x}=0$$

rho del u by del x or del rho u by del x. And we know that do you know from because it is actually a compressible flow in this particular illustration, we have this plus del t is essentially equal to 0. So if we try to figure out what this value would be del rho by del t becomes equal to minus rho del u by del x minus u del o by del x and as we know that du by dx tau u by tau x is essentially constant v by l. And therefore, we have also the value $\frac{\partial \rho}{\partial t} = -\rho \frac{v}{L}$ , so this minus rho v by L minus u del rho by del x.

Now if you look at this, let say this is equation three, so if you really look at the question and the problem statement, rho has been assumed to be uniform in the volume are not with the time t here; the rho is varying with time t is temporarily varying, it is not varying spatially really. So, therefore, del rho by del x, because it is uniform within the volume is supposedly equal to the zero and we are left with no other choice, but del rho by del t one side equal to minus rho v by l and that is equation four. So let us try to figure out what on integration this quantity would result in what would really with the density function in terms of the velocity, length etcetera.

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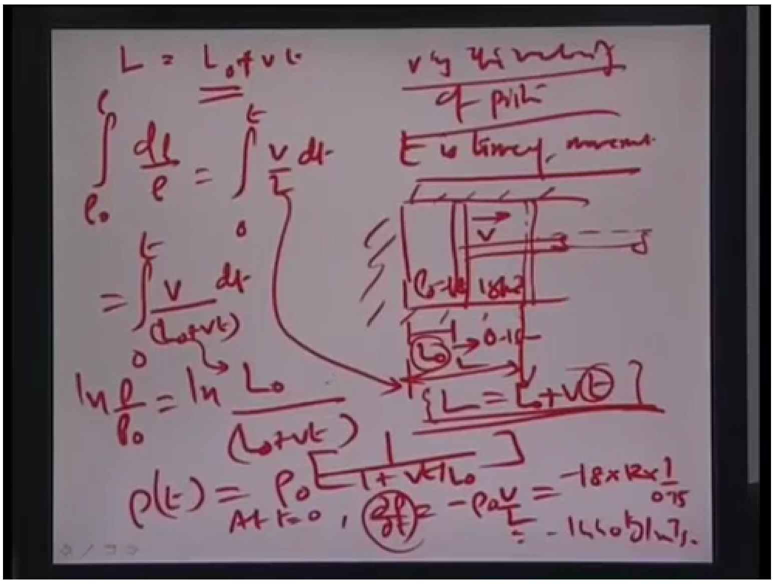

So here, we would like to illustrate that the length L really is we can assume this to be equal to let say the initial length $L_0$ that the piston is at plus v times of t, where v is the velocity of the piston and t is the time of movement. So there is some L 0 value, let say the piston is somewhere, if you see in this particular figure here, the piston is at some value when let say moves at a certain velocity v, this value is may be zero. And we want to consider any length L which is equal to L 0 plus v times t, after time t it would be here, and this is really the new length L so that is how define this whole length of traverse of the piston in side this cylinder. So if you assume this to be the final length at time t assuming this to be the length in have initial or the process started, we have integral d rho by rho where rho varies from let say some quantity rho 0 to some value of density rho t equals integral minus zero to t v by l dt. And essentially as you know that this is L the actually comes from this L 0 plus v t, so will have this as zero to t thing of zero to t v by l zero plus vt dt.

$$L = L_0 + vt$$

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_0^t \frac{V}{L} dt = \int_0^t \frac{V}{L_0 + vt} dt$$

$$\ln \frac{\rho}{\rho_0} = \ln \frac{L_0}{L_0 + vt}$$

$$\rho(t) = \rho_0 \left[ \frac{1}{1 + \frac{vt}{L_0}} \right]$$

So, therefore, ln rho by rho naught really becomes equal to ln lL naught by L naught plus vt. If you just solve this integral it is essentially and put limits zero and t comes out to be L naught by L zero plus vt. In other words, density is a function of time really is equal to the density time t equal to zero plus 1 by 1 plus v t by L 0. So at time t equal to zero therefore, as the second part of the question is assumes del rho by del t the change of density rate of change of density would be rho 0 v by l. And rho 0 being of already given equal to 18 kg per meter cube and this is at length l 0 0.15meter this particular length and it is moving with the velocity 12 that the whole density variation with respect to time becomes equal to minus 1440 kg per meter cube second.

So as you found out here that the continuity equation can be very easily used for this kind of compressible flow problems is well density varies which time as well as earlier problem as you saw was that of incompressible flows. So in micro scale though, if you consider the flow mechanics mostly flows are treated to be incompressible and strictly laminar in nature, because although there are twin faces flow problems at the micro scale, but the modeling becomes extremely complicated and difficult, so limiting ourselves mostly to the single phase flow problem in such a situation. So this kind of brings as to an end of this particular lecture, we will try to cover up the second Navier-Stoke equation that is the conservation of the momentum in the next class.

Thank you.