BioMEMS and Microfluidics
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Lecture - 26

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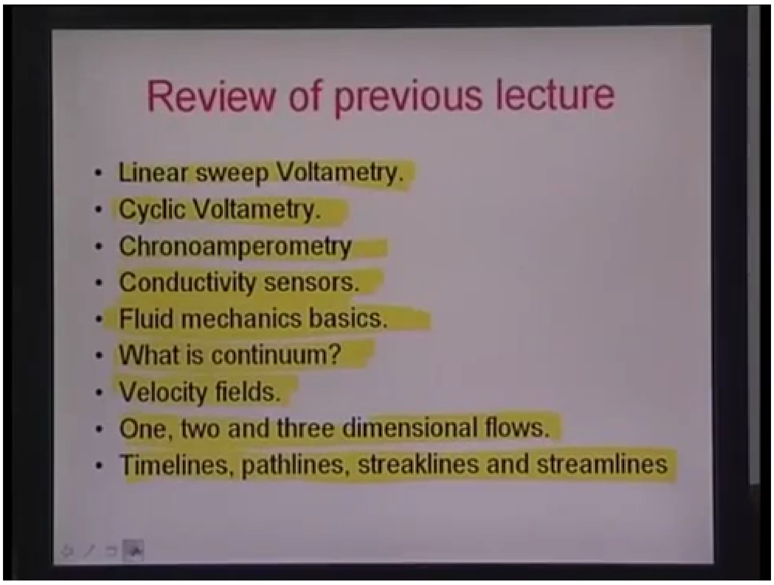

Hello and welcome back to this 26th lecture and bio micro chemical systems. Today lets us first quickly look into brief review of the previous lecture. We started with the understanding the basic voltametry mechanism, linear sweep and cyclic voltametry. Again voltametry is the technique of measurement of a reduction or oxidation potential is a various electrochemical species with rapid voltage is can the measurements made are between current and voltage, and basically get a peak which shows whether the electronics have been least or certainly absorbed or certain potential corresponding to the oxidation and reduction potential of the species, and there comparison to standards the kind of ((Refer Time: 00:56)) what the species are or in work concentration their present.

We also talked briefly about chronoamperometry where a we discussed the application of a square way if and a going to a certain peak potential which would oxidize or leaves these species and then try to understand the kinetics of d k of the current as we go temporarily. So various species is would have a different rate of oxidation or reduction or in another words various species would have a different rate of release of electrons or absorption of electrons, we should make a chronoamperometry measurements comparable, and would let us draw ((Refer Time: 01:42)) from the current versus time plot in such situations.

So we also talk about conductivity sensors, conductivity essentially means the inverse of resistivity. And the increase or decrease ions of one kind of particular species definitely lead to the increase or decrease in the conductivity. And you can use this measurement technique by assembling together what do know as Wheatstone bridge. Then we started a new area of some basics in fluid mechanics, it is very important to mention for me to mention here that because we will be studying some fundamental problems in micro fluidics, we need to understand these basics. So essentially we covered about what really a fluid is by definition. We talked about how it would deform on a shear force being applied to the system and how it compares with the solid in a similar kind of situation.

We try to understand what really a continuum amaze when it breaks down particularly at a level when in the dimensions spatial dimensions of the control volume, kind of rime with in the mean free path of the different molecules. Then we get difference is in properties like velocity, density etcetera with the time and that is where the continuum breaks down. We described in details about velocity fields; we talked about one dimensional, one, two and three dimensional flows respectively. And then we also try to categorize this very important ways of means of geometrically representing flows by mean of time lines, path lines, streak lines and stream lines. So we will kind of start from here and then go to the next agenda today which is stress filed.

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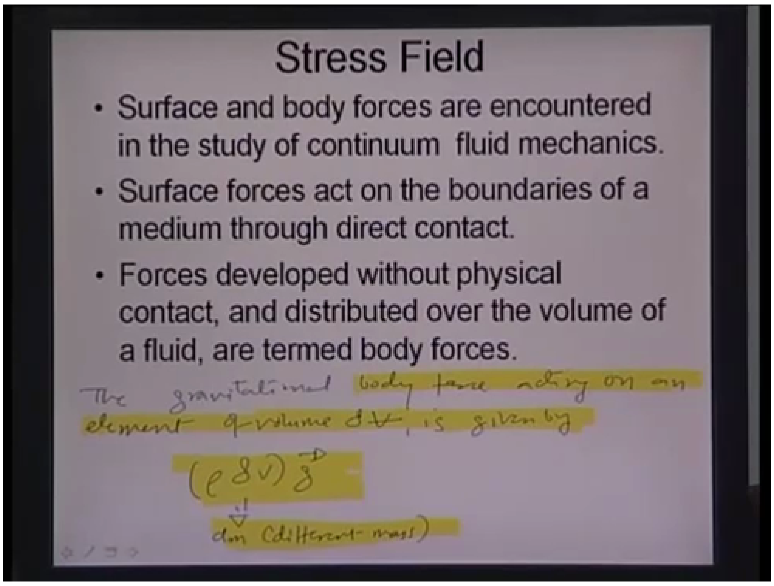

So essentially you know if you look at really what a stress, we all know that stresses a force per unit area for basic definition. So typically in a continuum fluid mechanics, we have a surface, volume body forces encounter at different points of the fluid. So if you consider control volume somewhere in the fluid because of the flow motion and because of the viscosity forces between the different layers, there is a tendency of these forces to effect the surface or the volume as such a that volume element or control volume element.

So the surface forces act on a boundaries of a medium through direct contact which means that let say consider the fluid is a ball can it is flow throw a pipe, so in the border line between the pipe and the fluid is where the surface forces are directly acting and there is an impact of these a forces through into bulk of the fluid. And also forces develop without physical contact and the distributed over the whole volume of the fluids as termed as the body force, so essentially it is a volume force that we refer to. So like for example, gravitational force acting on a fluid element is essentially fluid volume element this essentially body force, so what that is essentially rho times of v; v is the control volume times of g, rho is the density, v is the volume so that is making it equal to the mass of a certain control volume. And then you have a gravity factor g acting, so this is a body force, so this is a uniformly felt over the volume the whole volume of the control element that we are kind of figuring out. So here right here is you can see the body force acting on a on in the element of volume δ v is also given by rho δ v g, and essentially this is nothing but differential mass here rho δ v times of acceleration into gravity, it is felt within the volume.

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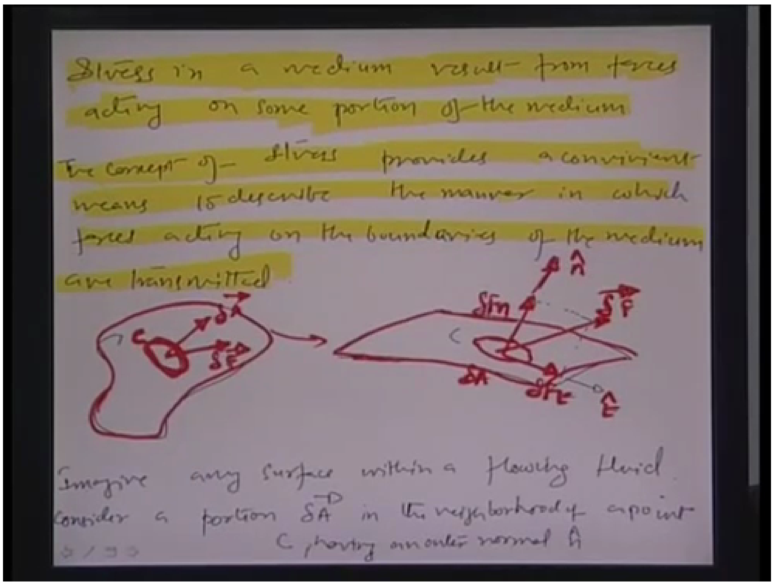
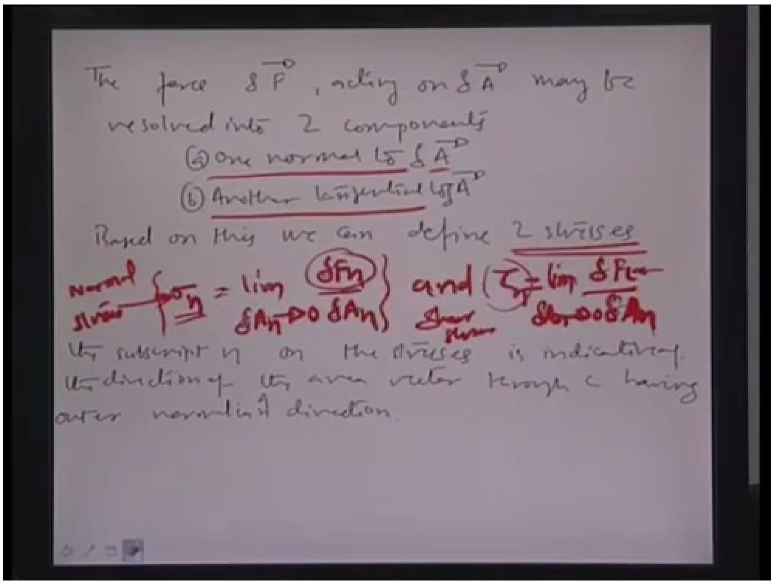


So let us actually figure out what stress really is in terms of such a control element, it is important of pertinent in fluid mechanics to understand fluid as an assembly a control volume by control volume and so on. So, therefore, if there is one represented a control volume such situations, it kind of generically represents the properties related to the flow and general like you know a velocity, acceleration, density and so on and so forth. So the stress in a medium results from forces acting on some portion of the medium, so definitely there has to be a relative force between this element in concentration and the medium which this element is for us to understand the essence of stress, there is a relative force between the control volumes an it is a surrounding medium.

The concept of stress provides a convenient means to describe to the manner in which forces acting on boundaries of the medium of the transmitted like let say for instance let us consider a control volume here as in this example, I can see this is a control volume and this control volume is essentially close to let say some points c in space. And we consider as small area delta A, which is adjacent to this point c inside this control volume is a regular control volume. Now any area as we know from vector geometry can also be represented by a direction perpendicular to the area and quotient. And if you are considering this small area element as illustrated here and let say the value of this area is delta a so we can represent this area by a normal vector a unit normal vector pointing away from this area and perpendicular to the area, and we call it delta A vector as in this particular case.

Now let suppose that there is a force delta F vector which is acting on this area vector delta A, so it is a certain angle in respect to δA delta A, but then there is a force δF vector acting on this area vector delta a. So if we imagine any surface within the flowing fluid, this surface let say is a part of this whole control volume as has been illustrated here. And we also assume that this δF which is at a certain angle with the area vector can be resolved into a normal component, which is a in the direction of this normal vector. Let say this is the normal component δF n - direction of the normal vector again, and one which is a kind of tangential to the area of interest here close to this point c and we call this F tangential δF tangential. So essentially we are kind of resolving this delta a vector into δF n that means δF into the normal direction to the area, and another component δF t delta F tangential to the direction of the area. The value here for this normal vector is also δA , and this is the tangential direction represented as t cap, this is the normal direction represented as n cap.



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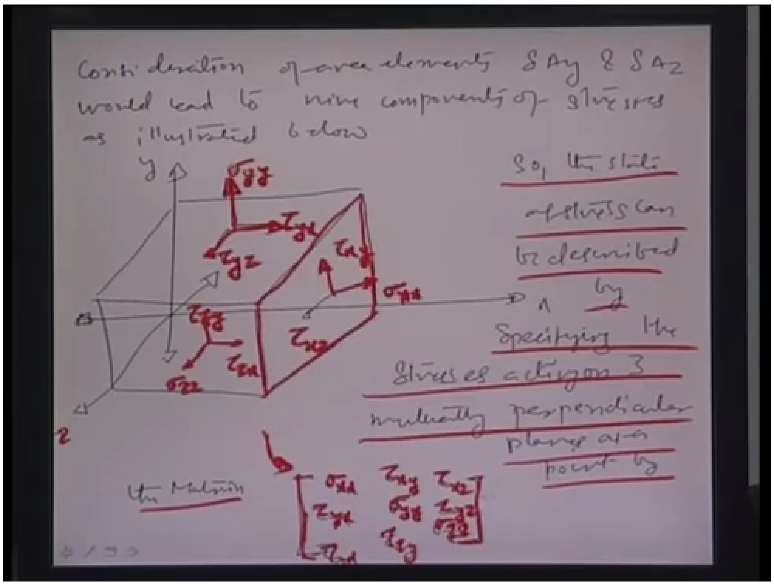
Now if you want to really see the kind of stresses, because of these two components on this area vector a, it would be represented as a normal stress and stress which is tangential, because there are only two components of the forces, the normal force and the tangential force. So based on this, we can define in the two different stresses as one in the direction of the normal vector, which is also represented as sigma m or the normal stress of the principles stress, and it can be defined is the limit of the area limit delta a and approaching zero, $\sigma_n = \lim_{\delta A_n \to 0} \frac{\delta F_n}{\delta A_n}$ and shear force, so this is known as this is called the principles stress or the normal stress. So this is the normal stress. And the other component which is parallel to the area can be represented as $\tau_n = \lim_{\delta A_n \to 0} \frac{\delta F_t}{\delta A_n}$, so the area vectors still does not is unmodified it still remains the same. So we have a stress due to the normal force parallel to the area vector and the stress due to the tangential force perpendicular to the area vector passing the normal stress and the shear stresses. So this is essentially is how you define normal and shear in certain you know the situation.

So normally it is kind of you know customary into consider the vectors are the where the component of these force vectors in orthogonal coordinates system which essentially means so in a rectangular coordinates, we may consider the stresses acting on planes as outward normal are in the x y and z direction. So essentially if this is a plane that we are talking about in the rectangular coordinate system x, y, z, we consider this stresses acting on planes which was outwardly draw normal are in the x, y, z direction. So one of the planes is essentially was outwardly normal here is drawn in the x direction, and represented in by this red line here, so let us call this as some area vector delta a x. Another would be similarly in the y direction, which is probably an element like this, which is orthogonal to this A x element, which is called delta y and similarly delta a z.

Now if you want to represent this stress vectors here, let say only on this particular plane here on this a x vector here. So let suppose this which is the plane which is draw it separately here. And so you have again a components of the force in a rectangular coordinate system, well resolved to all the three v coordinates x, y, z. And let suppose the force along the x is Fx delta along the y is a δ y along the z is δ z, so here as you are seeing, there is one principle component let say sigma xx and two shear stresses based on the resolution of the force in the y and the z direction respectively. So delta so the principles stress here $\sigma_{xx} = \lim_{\delta A_x \to 0} \frac{\delta F_x}{\delta A_x}$ and other two components who represent this as tau x y, which means the shear due to the force in the y direction the second term here applied to area vector A_x . So the second term of this is the force direction; and the first term is the area direction. So tau is the shear applied due to a force the y direction by an area vector in the x direction also represented as delta f y by delta a x; delta a x tends to zero, this is the limit of delta x.

And similarly this again is a representation where you considering τ_{xz} meaning the shear stress you to a force is the z direction on in area pointing towards the x direction the second term is the direction of the force in this subscript here and the first term is the direction of the area vector this is just purely the notational and it is needed for kind of understanding the different components the principles of the here stress when this plane change is between let



say plane point into the x direction applying point to the y direction and the plane pointing to a z direction in a rectangular coordinate system.

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So really if we look at all this together as I pointed out the four illustrated the force there are three such planes in the orthogonal coordinates as you can see here there is the plane ion the x direction right plane pointing towards the x if I really maker a control as a as I describe here all fluid mechanics is really about constructing a control volume let say we make a cube as an element which represents represented volume so in this cube you have a phase phasing this direction phasing the minus x direction and similarly a phase phasing the y and minus y and z and minus z directions respectively so along all this phases you will have shear stress and at least two components per phase and you will also have principles stresses one component.

And therefore, if you look at all the in totality number of stresses which exist so here let say in the positive x direction if the plane the normal vector of the plane points to the positive x direction you have a σ_{xx} which is essentially the Fx; that means, the delta Fx are the force in x direction divided by the area who's vector points towards the x direction that is delta a x similarly you have τ_{xy} has i define earlier and τ_{xz} if you are looking at the y phase; that means, the phase where the area vector points to the positive y direction you have again sigma y in this direction and then you have the shear stress because of the force in the z direction apply to an area vector delta a y pointing in that positive y direction and the shear because of a force in the x direction component of force of which is you have kind of resolved in the x direction divided by the area which is again of the area of the phase which is having a vector point towards the y direction so that is what y x is.

And similarly you have a similar combination on the third phase here pointing in the z direction the area vector points towards the positive z direction where you have σ_{zz} and two other shear stress components τ_{zx} and τ_{zy} is a very, very a clearly illustrated then and how you can notational express different process in such a fluid element mind you all this phases are acting together on the fluid element as fluids go all way round in pasted and there are stresses which can be shear force, there are stresses which can be in the normal direction of principles shear principle stresses, and this whole combination is what we have to evaluate dynamically to consider the behavior of such an element with time and then that also. Let us to define certain equations of motion of this fluid element goes along considering the kinematic and dynamics, which we also known as the Navier Stokes equation.

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

So in probably the next lecture I would also the trying to derive some of this equations there are principle three such equation, equation of a conservation of mass because conservation of a mentor and conservation of energy. So here if you really put all these stresses together in a matrix form you can really define in a matrix which is also known as the stress matrix where you have the diagonal elements which are principle stresses sigma in the in the x y and z directions respectively, $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ and the non-diagonal elements here really represent the different shear stresses as has been illustrated before. How this just this combine with the certain notation incase of this is τ_{zy} this is τ_{yz} and so on and so forth. So the straight of stress can really within described by specifying in the stresses acting on the three mutually perpendicular planes of a rectangular coordinate system in the a orthogonal system at any particular point by this is stress matrix this stress matrix is also known as the stress of this particular fluid element. So, now, I would like to kind of a go ahead and evaluate the very first and very important property of a fluid before this viscosity so what really is viscosity so let see how we can understand this concept of viscosity.

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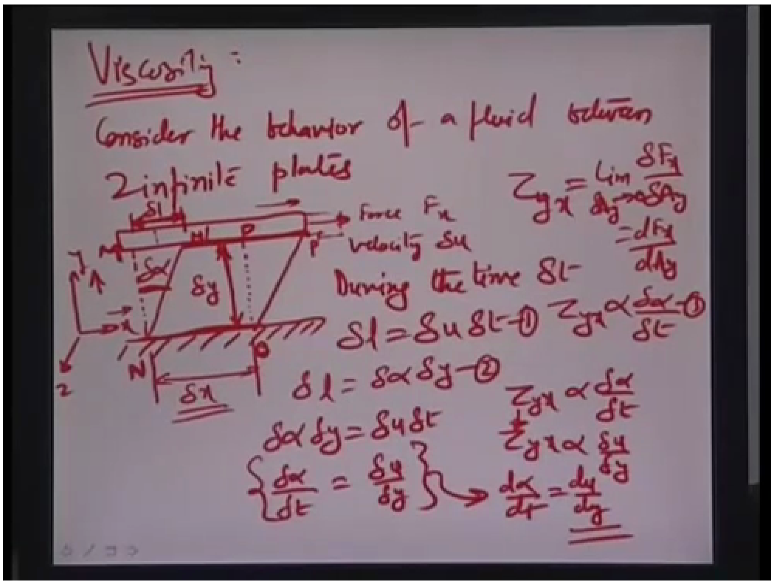
So, let say we consider the behavior of a fluid between two infinite plates. let suppose we have a an infinite this plates are essentially infinite in the z direction there in the towards into



this particular dream in so there is plate here and then there is a fixed supported the bottom and if you recall we have done this back in lectures related to finding out the parabolic velocity profile how you know moving plate influence a fluid column by shearing as with the respect to a fixed boundary so here let say a time instants e we have a fluid which was static and having a boundary like this and let say we have apply a force on this particular upper plate to an extend Fx because which the plate moves with the velocity delta u so I will just delta u illustrate here.

And let us actually see that if we try to move this with the force of x at rate tell you what happens of t plus delta t. So of course, this plate here would move forward right and let say the new position of this plate is formulated somewhere here because of that moving so that it moves in total are in totality by some finite distance here. So what will you expect would happen to the fluid column a fluid column would actually try to get shear like this so if as if know that is how fluid is define that if you apply a force in this kind of is situation the fluid will just simply go a deform plastically and not come back as it happens in solids normally. So in fluid it would just go plastically and still there and if you apply a little more force of again force it will again bend and keep on shearing as you proceed along.

So here essentially a let us assume that we have able to successfully move this fluid layer by a total amount of distance delta l. So here one of the elements here is we can see let us mark it as M N O P and this moves to new position M’ M OPdash so as you maybe already a where this particular layer here of the bottom is a static cause the lower plate is fixed in nature. So there is a zone of step which is formulated and as you go ahead in the y direction you have a velocity gradient which comes because of this there are is layers which are kind of shearing of are sliding over each other as the fluid e forms from the position M N O P, m dash n o p dash m n o p here this is m n o p m dash m o p p dash.

So let us assume that the distance between the two plates they are parallel in the distance between the plates is delta y and essentially the total amount of length that this fluid element processes at the very outside is delta x so that is an change much although the shape changes from rectangular in little more like parallel, because of the shear that the fluid layer would have respect to the zone of no slip close to the surface n o so during the time delta t the amount of distance has been moved delta l can also presented as delta u times delta t right and essentially the shear stress here t at y x; that means, the stress due to the force along the x direction on the area vector pointing towards the positive y direction. Let suppose we have a right hand rectangular coordinate system x, y, z are the different direction. So the area vector pointing to the y direction is really in this particular direction here and the forces in the x direction so that is what would come along this particular plane m p or m dash p dash whatever you may call so therefore, τ_{yx} is defined as $\tau_{yx} = \lim_{\delta A_y \to 0} \frac{\delta F_x}{\delta A_y} = \frac{d F_x}{d A_y}$ whatever you may call and essentially as we know that form the young's law from the loops law τ_{xy} is also the portion to the rate of a angular the formation.

So let assume that this angle change here because of the component moving from of the fluid element moving from MNOP m dash n o p dash is $\delta \alpha$ and this delta alpha happens in delta t time so $\tau_{yx} \propto \frac{\delta\alpha}{\delta t}$ so the rate of change of angle that is what huge law defines expressed. So if you consider all these factors together we are left with another very interesting observation that δl which is actually this particular element change in the length are this is the displacement by this the layer n p moves to the new position n dash p dash as the plate use ahead is also given by $\delta l = \delta \alpha \delta y$, because this s essentially can be consider in very, very small situation is same as the length of the r that this radius δy would execute as it moves position m to m dash.

So essentially what we are talking about here is the length of the r delta *l* by virtual the fluid element moving from position m p m dash p dash the elements moves by an angle delta alpha so delta alpha times radius delta y here you define what this delta *l* is so let called this equation one this equation two and this equation three if you if you actually correlate equation one and two you have a situation where $\delta \alpha \delta y$ becomes equal to $\delta u \delta t$ why and therefore, $\frac{\delta\alpha}{\delta t}=\frac{\delta u}{\delta y}$. So, now, as we know that the sheer force τ_{xy} really τ_{yx} is really proportional to delta for the delta t so we can easily say that $\tau_{yx} \propto \frac{\delta\alpha}{\delta t}$ also is proportional to $\frac{\delta u}{\delta y}$ y taking limits here we can get a situations where $\frac{d\alpha}{dt}$ is equal to $\frac{du}{dy}$, and this is what the velocity gradient is. So thus the fluid element when subjected to a shear stress τ_{yx} experiences rate of deformation given by really $\frac{du}{dy}$ as can be seen in this illustration here.

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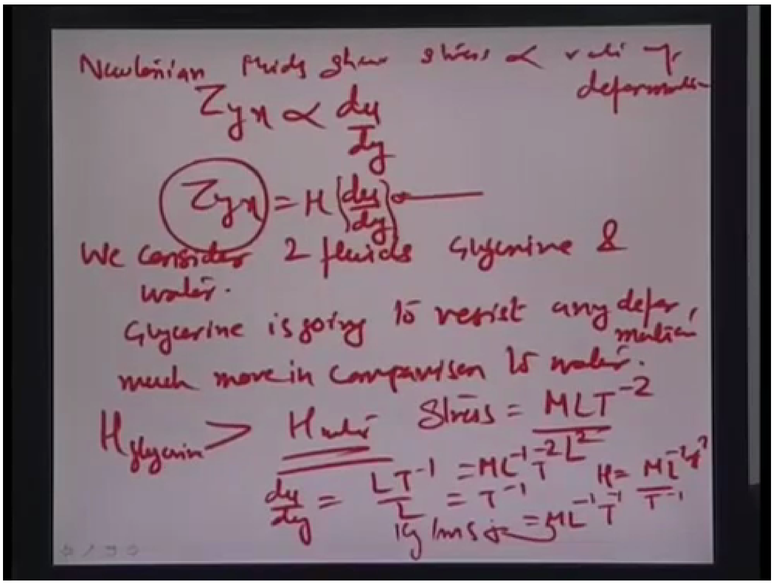

So, at least fluids in which this proportionality between shear stress and rate of deformation exists are known as Newtonian fluids as we all know. So let us define this again here that in Newtonian fluids and we will in just about minutes see what happens in the nonNewtonian case how that is different this and this particular illustration. So Newtonian fluids the shear stress is also directly proportional to the rate of deformation. As we have illustrated here before, I just what mention that this proportionality only holds valid for Newtonian fluids that is how the fluids are defined. So, therefore, in such a situation, we have τ_{yx} is proportional to $\frac{du}{dy}$, and the constant of proportionality in this case is also known as viscosity of the medium mu. So what really viscosity physically means is that let say if you consider two different fluids glycerin and water, so we consider two fluids glycerin and water. So definitely glycerin is going to resist as we all know by natural experience, the glycerin is going to resist any deformation much more in comparison to water. So this definitely because glycerin is much more viscous or in other words the mu for glycerin is much, much higher than mu for water, which means that amount of here which is needed for a certain velocity gradient to be crated that means, you talking about movement of inter layers there are two layers which are moving with respect to each other. So is gradient du by dy for a certain finite gradient to be created, we need much more shear stress or much more force or effort in glycerin because mu value is a higher in comparison to water, so that is the essence is what viscosity is all about.

So dimensionally again if you investigate what viscosity is really you know that stress essentially is force per unit area, so we can represent that as $\frac{MLT^{-2}}{L^2}$ so that is $ML^{-1}T^{-2}$. And du/dy, if you look at really has LT minus 1 by L which has dimensions of T minus 1 and. So, therefore, mu would have unit ML minus 1 T minus 2 by T minus 1 or ML minus 1 T minus 1, so ML minus 1 T minus 1. So, therefore, the unit of viscosity is kg per meter second that is what viscosity is defined as. So in fluid mechanics seldom use these units of viscosity we rather express viscosity as a ratio between the absolute value of the viscosity the density we also know that better kinematic viscosity.

So, therefore, we can also write here that kinematic viscosity the new term, which is normally used very often in fluid mechanics, and it is very obvious because you know there may be substances where density is higher, and same is the viscosity. So what really matters and if there is a substance which is very, very diluted in nature, it would normally I mean by intuition we can say that it would have a lower viscosity value. So what is important to consider in a physical sense really the ratio between the viscosity and the density that gives you a better prospective of the full medium that you are investigating. So kinematic viscosity here, therefore is equal to the absolute value of the viscosity divided by density of the medium, so I would like to go ahead and do an example problem.

As you can illustrate here that there is an infinite plate is I will just showed and moved over a second plate, which is fixed and one layer of liquid. So this is essentially is the plate, this semi finite that it means semi finite in the z direction plate and it is moved over this fixed plate here. And for a small gap of width t, which is equal to 0.3 mm as you can see here, we assume a linear velocity distribution. So, therefore, the velocity varies from zero here point of no slip to all the way up to about v equal to zero point three meters second the maximum velocity of the plate. The fluid here adjacent to this plate would move at the same velocity because there is another zone of no slip here, and so that is the relative velocity between the point at the top here and the point at the fixed plate surface of the bottom here.

So the liquid viscosity which is used here is in this case is 0.65×10^{-3} kg per meter second, and the specific gravity is a 0.88. So specific gravity as we all know basically how many times density of water is the density of a particular fluid, so it is the comparison, the ratio comparison between the density of fluid to density of water at standard conditions. So you have to calculate, this case the kinematic viscosity of the liquid, you also find out what is the shear stress, which is generated in this process, give me a minute here. So we had to find out the shear stress particularly on the lower plate and you have to indicate the direction of each of these shear stresses. So let us solve this problem to understand about the viscosity.

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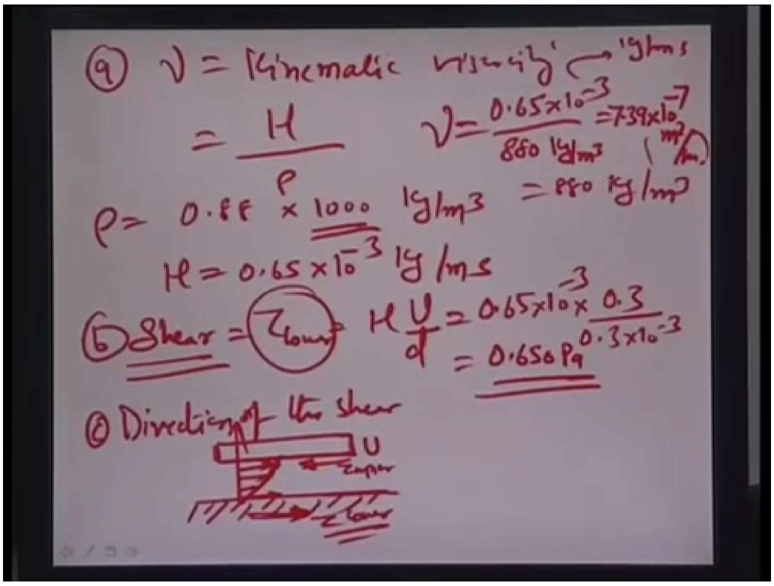

So the first question is what really is the kinematic viscosity here, as you know kinematic viscosity we call it or we represented by the symbol nu. This is really the absolute value of viscosity per unit density. Density, in this case, we know it is a 0.88 times of thousand kg per meter cube which is the specific it density of water at standard condition, so this 880 kg per meter cube. And viscosity from our earlier these things question is given to be 0.65×10^{-3} kg per meter second. So nu here therefore, would be $\frac{0.65 \times 10^{-3}}{880}$ which is equal to 7.39×10^{-7} and the units in this case is 10 to the power of minus 7. Let me just write this little more properly here, give me a minute, so 7.39×10^{-7} and the units in this case as you can see this unit here as kg per meter second, this unit being kg per meter cube, we are left with meter square per second that is what the units of kinematic viscosity is.

So second part of the equations says what is a shear stress in lower plate, so shear stress here can be represented as tau again on the lower is mu viscosity times of U by d; u is essentially 0.3 meter per second and d has dimensions 0.3 mm, so here the total stress would be the viscosity $0.65 \times 10^{-3} \times 0.3 \times 0.3 \times 10^{-3}$, so it essentially comes out to be 0.650 Pa or Newton per meters square that is how you define the shear force on the lower plate. About the direction of this shear force, if you look at really the plate combination, you have this is upper plate, this is the moving fluid, and this is the fixed plate in the bottom site, you have this velocity vector here going from some finite value u to all the way to zero. So you can consider that if this element is moving along with the upper plate, it would exert of force which is in the reverse direction, it is a reaction force that it would exert on the on this plate.

As if it tries to get the plate back into it is normal position so that is what the upper direction would be. Simultaneously, you are trying to deform the fluid element, so it is giving the pressure to this fluid in the other direction here I mean more towards in movement direction here on the lower plate, because it would have been better this plate would have been able to carry this through along with it, but since it is not carrying it, therefore, the force that is being felt on this due to this resistant layer at the at the junction here is actually towards the direction motion of the upper plate.

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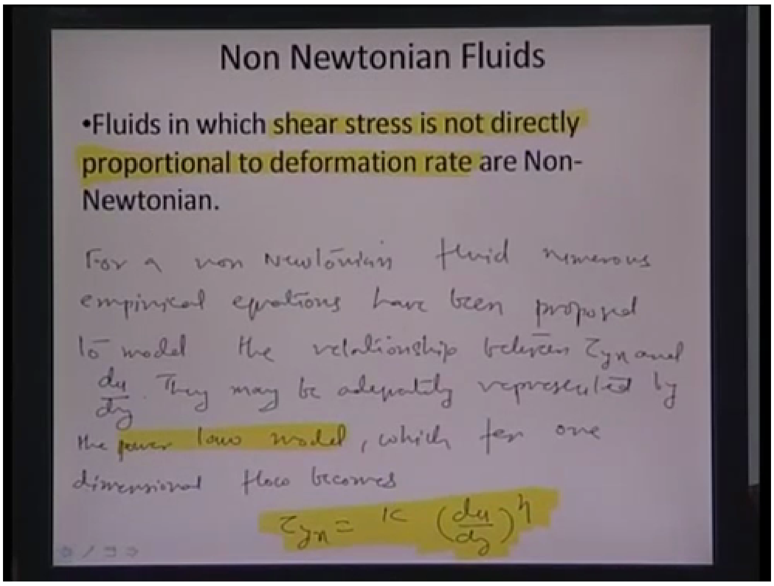

So, now, once we have done Newtonian fluid let us actually look in to the next very interesting topic of what really non Newtonian fluids are. So essentially it is again based on the relationship between shear stress and the velocity gradient. In a non Newtonian fluids just contrary toward the Newtonian fluids would the show, the shear stress is really not directly proportional to the deformation rate. So essentially for such fluids, you know there numerous empirical equations which have been proposed model, one of them being the of the power law model for describing such fluids. And here if you see the shear stress $\tau_{yx} = k\left(\frac{du}{dy}\right)^n$, where n can be either more than one or less than one. You know and depending on what it the fluid would very inert property or you know physical properties etcetera. So there different aspects like this different cases for different values of n, for which this equation with signify at the different property all together for such a fluid. So let us look them look at them case by case.

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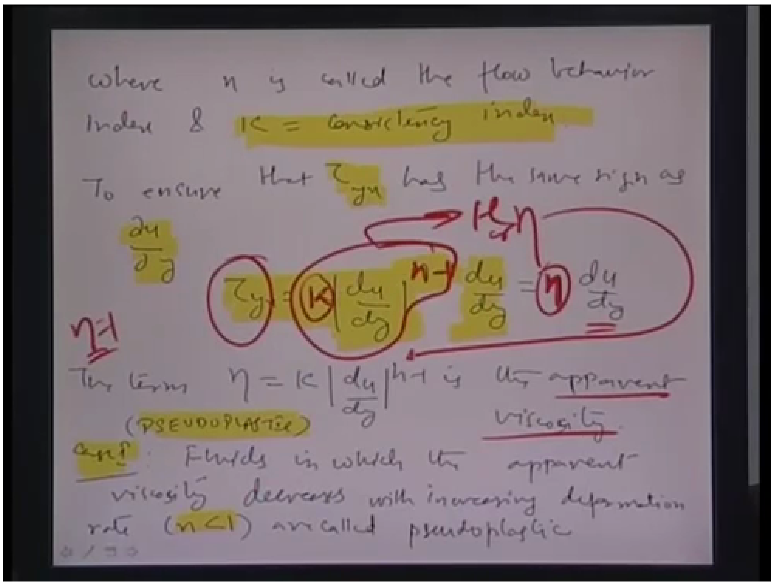

So essentially what term your k here in this particular equation is also this is also known as the consistency index and it can re modify this equations slightly to make it $\tau_{yx}=k\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}$; this ensures that the tau has the same sign as du/dx. And essentially this $k\left|\frac{du}{dy}\right|^{n-1}$, this can be represented as the viscosity η whatever you may call, so essentially in this case, $\tau_{yx}=\eta\frac{du}{dy}$. So the eta here in this particular expression is also known as the apparent viscosity, this is really not the real viscosity.

So if you are n is one really in that case the eta comes out to be constant in which is the case of Newtonian fluids with time. And if it is a more or less than one, there would be different properties associated with that fluids. So just look at the case where in this rate is of this n value is less than one, so such fluids also known as pseudo plastic material. Here the apparent viscosity, because n is less than would be decrease with increasing the deformation. Look at this particular equation here, n being less than one means that this $\left|\frac{du}{dy}\right|^{n-1}$ would be essentially negative quantity, the exponential here would be or the power here would be or the index here would be negative in nature. Therefore, the increase du by dy would essentially mean at decrease in this viscosity value.

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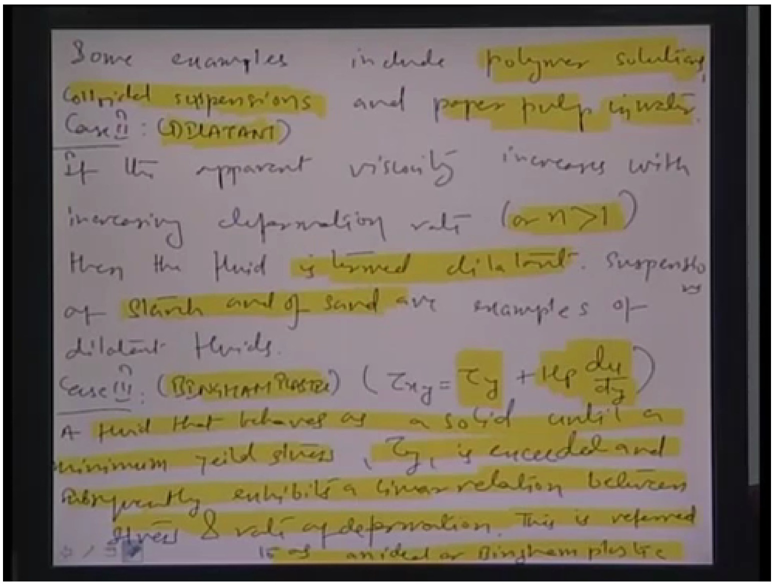

So similarly n is more than one in that case that the fluids would be categorized dilatants. Essentially what definitionally that means is that the apparent viscosity with increase with increasing deformation rate. So if n is more than one then the coefficient n minus one of du by dy mod which we just saw in this slide back would be positive, and because of that index being positive within the increasing du/ dx or du/dt the shear stress τ_{yx} would increase because of that. So viscosity mu would increase because of that; viscosity being $k\left|\frac{du}{dy}\right|^{n-1}$, so such a fluids are known as dilatants. Some example so in case of, the first earlier case of pseudo plastics can be polymer solution, which means that with an increase in the velocity gradient that means, two make or stir the polymer more and more, the viscosity value kind of decreases because of this stirring action.

Some other suspensions could be colloidal suspensions or paper pulp actually mixed in water; where if you move it more and stir it more, the viscosity decreases because of that stirring action. On the other hand, there may be the dilatants fluid like starch solution or sand probably where more and more stirring action would ensure that there is a greater of packing between the different grains, which would cause the viscosity to go up, so the du/dx is more in this case, and n being greater than one, then the viscosity mu would go up because of increase du/dy, so that is what a dilation would be.

There is another case however who is related to really the way that shear stress would vary and how or where up to where which point it would be a solid and then change state. So it is essentially kind of material where there is up till certain shear stress, the property is more like the solid that above cut of shear stress, the fluid will behave in an Newtonian manner, so such fluids are also known as Bingham plastic. Here the basic equation to represent tau x y be in terms of some you know some kind of intercept value tau y up to which the fluid behaves as a just normal solid; beyond which, it would also have this new p du by dy component, which is a related how to of fluid looks like. This fluid behaves as a solid and tells a minimum yield stress attained let say tau y; and then after it is exceeded, it is starts subsequently executing a linear relationship between stress and rate of deformation which is same as the Newtonian fluid. So this is referred to as an ideal or Bingham plastic.

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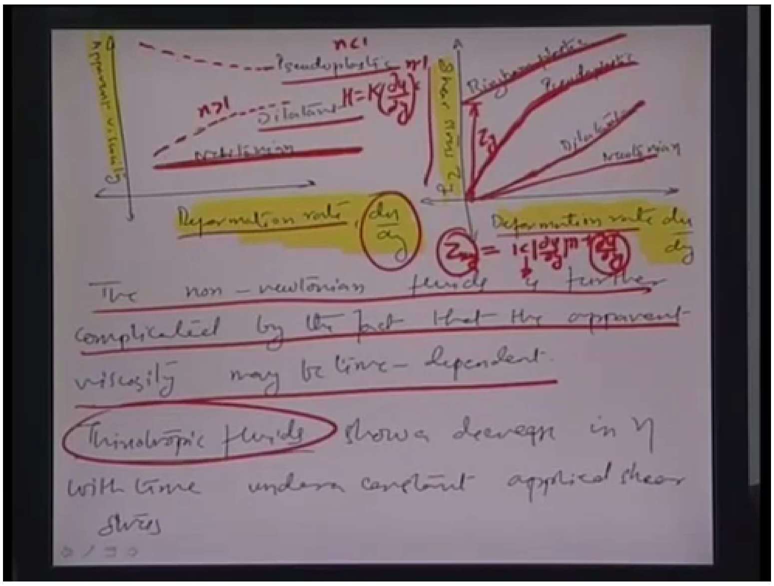

So, let us actually c what these some of these would look like on a scale of shear stress versus viscosity or shear stress versus and deformation rate du/dy. So if you really try to draw you know pseudo plastic dilatants Newtonian kind of fluid on a scale of apparent viscosity versus deformation rate du/dy this can be seen here apparently. The Newtonian fluid is one where this would be a constant parallel to the x axis, which indicates that there is a constant apparent viscosity irrespective of whatever the du/dy is or whatever the velocity gradient is. And the case of pseudo plastic as you know it is material where if the du/dy increases, because n being the less than one, the apparent viscosity come down because of that index being negative if you may remember. So this is essentially whatever pseudo plastic would behave like.

So if deformation increases, apparent viscosity comes down. And for a dilatants, it is the opposite behavior, so as the deformation increases in that case, the apparent viscosity goes up so that is what dilatants essentially would mean. So this is the pseudo plastic with the viscosity apparent viscosity falls down, the deformation rate; dilatants where it goes up with the deformation rate, and Newtonian fluid with the viscosity actually is constant with the increase in deformation rate. So if you have similar kind of materials or element plotted on a scale of shear stress tau y versus deformation rate, the Bingham plastic thing can be accommodated here. As you see here the Bingham plastic really definitionally something which would be acting like a solid up to a certain yield stress tau y, so this is the yield stress intercept tau y; after which it would start behaving as if Newtonian fluid. So here in this range the deformation rate is really proportional to this shear stress, after this intercept stress the yield stress has been crossed over.

So pseudo plastic material with increase in the deformation rate of course, because as you see here of the apparent viscosity kind of goes down, you know with increase in deformation rate initially there is an increase in the shear stress up to a point, after which it kind of you know again starts becoming you know kind of asymptotic to a certain value. For a dilatants as you see the behavior is this stop is that way; that means, you know it kind of increasingly goes on adding up the shear stress, and one of the reasons why this pseudo plastic and dilatants behave in this manner that if you may remember, for a pseudo plastic, the mu viscosity is really equal to the consistency index times of $k\left|\frac{du}{dy}\right|^{n-1}$, times of du and so, where the pseudo plastic as you know n is less than one; and for dilatants, it is more than one.

So in one case as you have seen viscosity is going up, and continuously, another case the operand is called viscosity is coming down, but as you plot the shear stress tau x y really would be proportional or it would be equal to this $k\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}$, which means that if there is an increasing shear stress τ_{xy} because of an increasing du by dx in both the cases, but as the du by dx increases in case one that means in case of pseudo plastic, the viscosity comes down with time. So, therefore, and there is instance, or there is a cut off of deformation rate beyond which the viscosity factor starts outweighing really, so the viscosity it is kind of out ways the increase in the du/dx. And so therefore, it kind of stabilizes to the certain value and these falls down beyond it.

And in the case of a dilatants, it is opposite effects that there is add on and so therefore, the du by dy to the power n minus one component kind of starts dominating after awhile and it further increases the shear stress value. In case of Newtonian fluid though as the viscosity is constant would expect linear behavior between the shear stress tau y and deformation rate du by dy. So in non-Newtonian fluid situation is further complicated by the fact that the apparent viscosity may the time dependent. Some of these fluids are all also known as thixotropic fluids, where we showed typically a decrease in the viscosity value with time under constant applied shear stress. So essentially thixotropic fluids may pose a situation where with the time you may feel that just temporally the viscosity changes I mean decreases after some maybe with or without formation. Sometimes if it is with deformation, the viscosity is changing it may be classified as a rather a pseudo plastic fluid. But if suppose you just keeps something like let say glass and beyond certain things you see beyond certain time you see it kind of deforms and shears out and you know slowly the viscosity decreases with time so that can be categorized as the thixotropic fluid.

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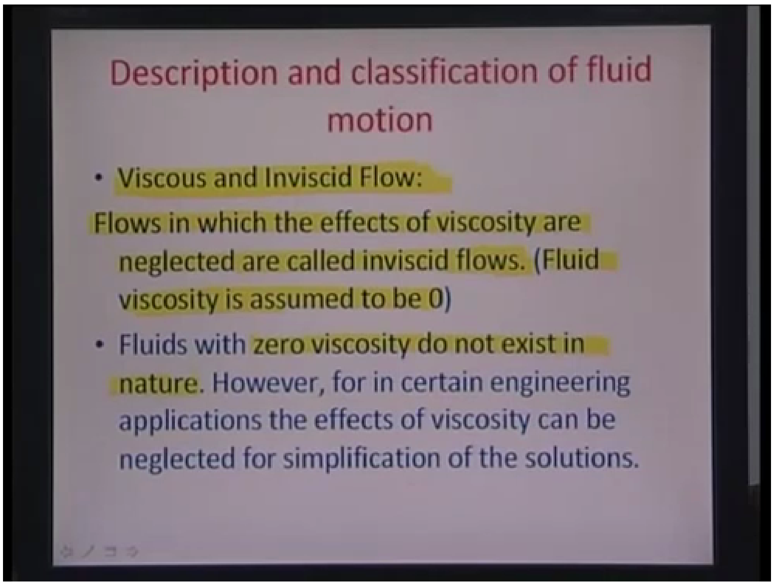

So, basically in a nutshell you know you can describe a fluid flow to be either viscous or inviscid these concepts are very, very important at this stage as I again would like to reiterate that because in case of micro scale flows typically all fluids. So basically the whole idea is that you know fluid flow can be really divided into viscous and inviscid domains. Again I would like to reiterate that these concepts are very, very important for particularly micro scale flows, because essentially all micro scale flows have very prominent viscous forces and effects, which makes this flow behave flows we have totally differently than the macro scale counter parts.

So intuitively whatever you think about would normally have to be set of fluids in macro scale cannot be really translated to the micron size scale or micron scale transport. So effectively you can categorize viscous and inviscid flows essentially as flows in which effects the viscosity or either felt or negative once is which is neglected is known as inviscid. So viscosity is assumed to be typically zero, this is really not a real world situation, but as I illustrate just in little bit how the viscosity can be taken zero specially in macro scale, whenever there is let say a fluid layer which is approaching a fixed plate you might have a zone or domain where we can treat the viscosity safely as you know zero. So it is more in approximation then an ideal situation, more in an approximation ideal situation. So normally although that would not exist in nature, I mean with zero viscous particularly; however, certain engineering application the viscosity can be small enough to make neglected.

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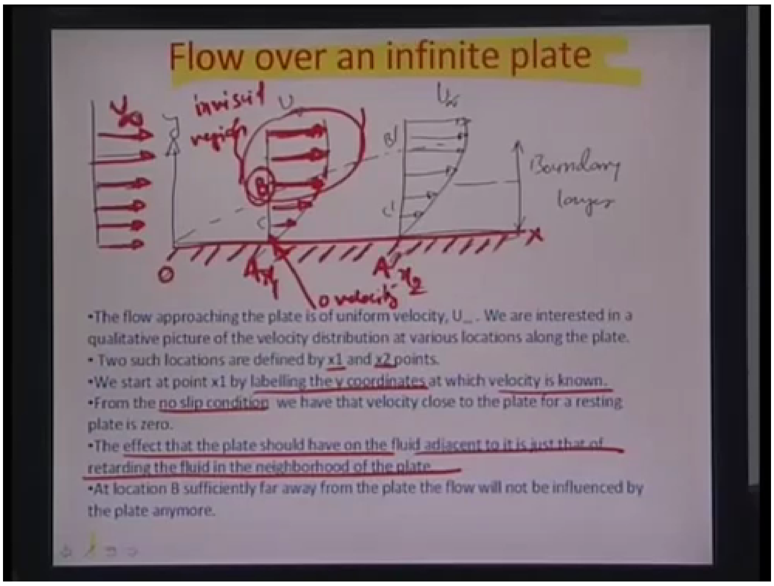

One such application is flow over in an infinite plane, as you can look at in this particular situation so let us suppose you have this fixed plane here which is represented by this surface ox, and this is also in the x direction of ox. So here as you see you know the flow approaching the plate is of uniform velocity let say u infinity, so there is a certain flow which is approaching which has velocity U infinity. So the flow when it approaches, we are first probably interested in getting a two picture qualitative picture of what would happen to the flow, when it is starts just about entering the zone where that is fixed plated the bottom. So let say we have two locations along this plate x 1 and x 2 at points a and a dash respectively, where we are trying to investigate what kind of behavior will be expected, so we have x 1, x 2.

We start let say at point x one here by enabling the y quarantines at the velocity these known and then ultimately plotting the velocity as a function of or in the y direction, your plotting the x velocity magnetite as it moves from x one all the way to let say the point b here. So as we know that very close to the plate, we have no slip condition or no slip zone where typically the velocity is zero as indicated in this particular region; whereas the case of no or zero velocity or no slip in this particular region. And what really would be the effect of fluids which are close to this particular point, so there the effect that the plate should have on fluid adjacent to it just that of a retarding the fluid in the never would of the plate so it has viscous forces.

Now, at a location b which is sufficiently away for away from the plate, the flow will never been influenced by this particular no slip layer because the velocity is already attained a certain value U infinity beyond that. So this particular region, we can actually kind of approximation as an inviscid region, where the viscous effects are some not fell. So velocity here respect of the fact that plate is close by already has attained the u infinity magnitude in all say so that is what inviscid flow would typically look like in a physical situation. So I would like to continue little more of discussion in probably the next lecture, you have kind of closing onto the time here. So next topic that I would illustrate would define these things in a little better manner, and try to understand a physical understanding as to how the flow develops or what is the layer which separates from the fully develop flow from the developing flow. So I will do that analysis in the next lecture.

Thank you.