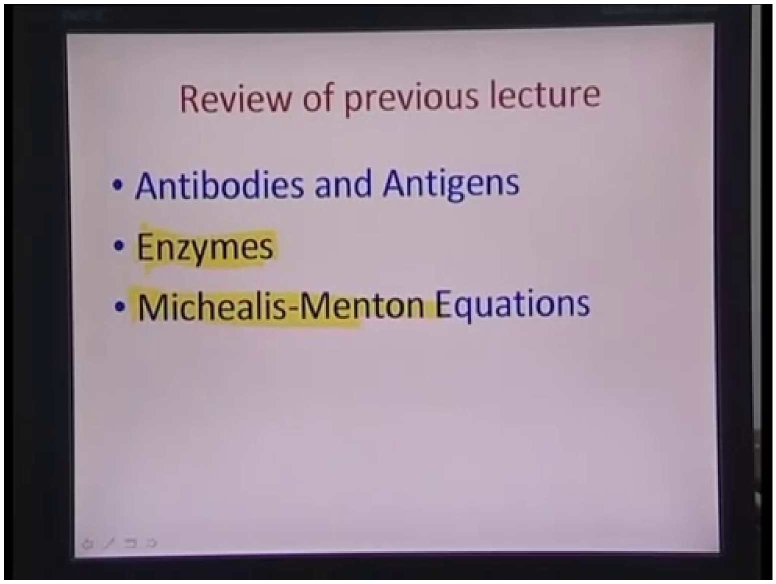
BioMEMS and Microfluidics
Prof. Dr. Shantanu Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 25

(Refer Slide Time: 00:30)



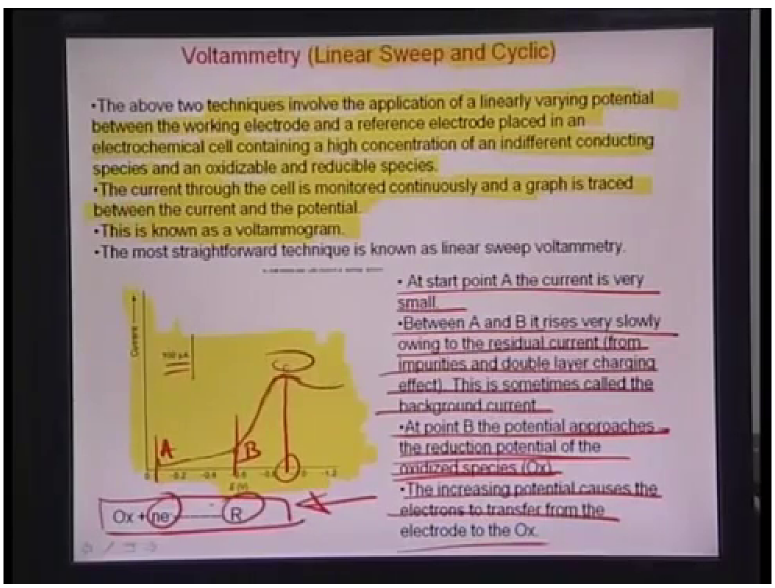
To this twenty-fifth lecture on Bio Micro Electromechanical systems. We will just quickly start with the brief review of what have been done in the last lecture. We talked about antibodies and antigens and their kinetics of binding. We also mentioned about enzymes definitions and what they would do. And then, we talked little bit about these. This is wonderful set of rate kinetic equations called Michaelis-Menten equations for studying the properties of how an enzyme binds to a substrate producing an intermediate compound and then, again breaks down into the product and the enzyme comes out as itself and essentially, how it catalyzes any process. Now, we also talked about the binding kinetics of antigens and antibodies and the way they would behave to changes in the ambient.

So, today we will try to address some issues wherein, and we also, you know, last lecture we also saw about some of the various ways and means of immobilization mechanisms of biological entities on to the sensor surfaces. Studies in details about absorption phenomena also classified it into chemical and physical absorptions. We also studied about microencapsulation, about covalent bonding or covalent bind direct, direct covalent binding of the biological moiety onto the, on to the surface of interest. We also talked about cross linking mechanisms and entrapments within gels or gel matrices.

So, all, why all these things are required is basically, because you have an entity and it causes an electrochemical change or a response we had at the very outside defined, that you know, most of the sensing mechanisms, which are commercially available as on data electrochemical. And so, therefore, with the mobilization of such biological entities on to the sensor surface, the obvious question which comes into this is what next? How to measure the electronic response? And so, therefore, a very important technique, that we had kind of touched upon earlier and I would like to detail it little bit today is voltammetry, ok.

So, we had seen in detail what Potentiometry would mean and this essentially, it is the measurement of voltage with concentration at zero current. In this case, in voltammetry, it is essentially a plot between the current and voltage and it finds out redox reactions and tries to ascertain species by measuring its reduction and oxidation potential. So, let us look at this in a little more detail, what voltammetry would involve typically.

(Refer Slide Time: 02:56)



So, there are two different kind of voltammetric techniques, linear sweep and cyclic. Essentially, these, both these techniques involve the application of the linearly varying potential between the working electrode and the reference electrode placed in an electrochemical cell in a manner, that the cell actually contains the reduction oxidation species, ok, with a high concentration of indifferent species which does not have any interfering ions with a particular electrode, ok. So, the obvious thing to assume here is that as you pump in electrons inside the cell, there is a tendency of both, the reduction and oxidation mechanisms to take place in the couple to formulate. However, if you could have an electronic response of the system by measuring the current at different voltages, what you would see is peak and the peak is more because as a species is essentially oxidized and it suddenly liberates electrons, there is a tendency of all these electrons to shoot up one all at a single potential point. And therefore, there is a huge rise in current at a certain potential and that is also known as the oxidation potential of the particular species.

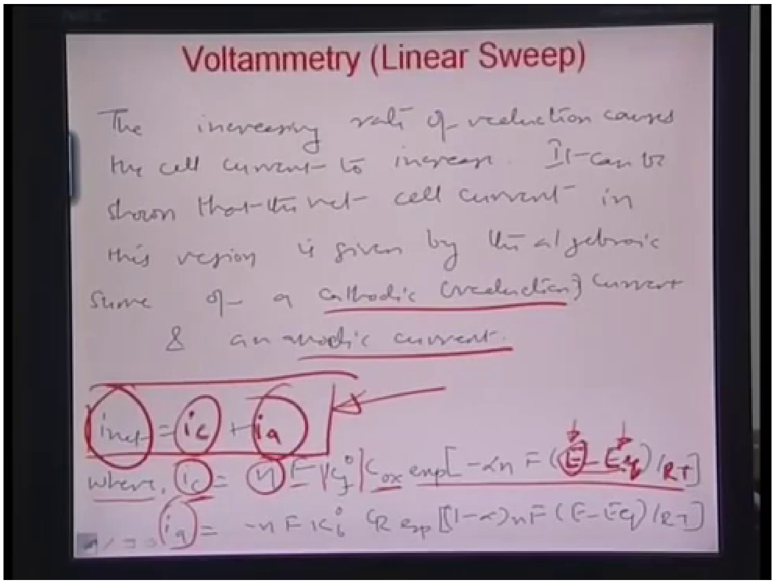
So, species can be identified by its oxidation potential. The potential of the cell at which the species get oxidized may vary as the oxidized agent or the, or the agent which is getting oxidized varies, ok, in concentration as well as in type or nature. So, therefore, in such a case, the current monitoring is very important through the cell and continuously a graph is traced between the current and the potential and we know this is a voltammogram, ok.

This here, right here, as you can see, is nothing but a voltammogram essentially. So, as you can see here, this measurement indicates the potential eV on the x-axis along with the current I and the x and the y axis. And as you can see, the curve here is really something like an irregular shaped curve, starting from the point A and there is a linear rise and then, there is a sudden exponential rise where the reduction potential happens. The oxidation potential happens and there is a shoot-out or burst of electrons, which increases the current and therefore, this particular value here would also ascertain what species, number one and what concentration of the species, which is getting oxidized, number two. So, these can be obviously concluded.

So, let us look at the various parts of the curve. The start part of this curve between points A and B, as you can see here, the current essentially is very, very small because it is just an Ohmic response with the voltage. So, as the voltage increases, there is an increase in current. V is equal to IR. And from the point B, ok, where there is the potential suddenly approaches the reduction potential of the oxidized species, ok, and so therefore there is a sudden shoot-out or burst of electron, which kind of increases the current, ok. So, the increase in potential causes the electrons to transfer from the electrode to, to the oxidant Ox and so, there is an increase.

Current, of course, as you know is reverse of electrons. So, if there is an electron depletion, there is an increase in current because of that. So, here the reaction, which takes place really is oxidation. Ox happening by combining with n electrons to formulate the reduction R in the process.

(Refer Slide Time: 06:54)

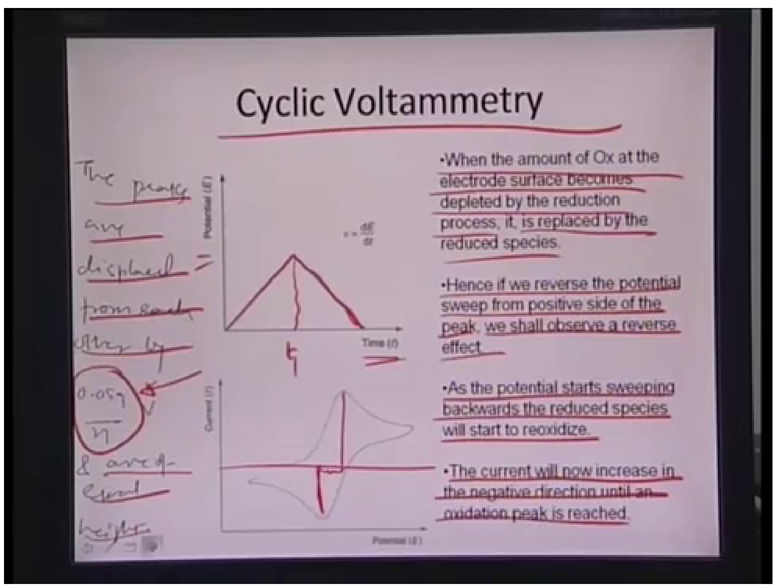


So, essentially, as you see, some of the important conclusions say, that the increase in rate of reduction causes the cell current to increase, ok. That is number one conclusion; number one. It can also be shown, that the net current in the cell in this region is given by the algebraic sum of the cathodic and anodic current basically. And let us i net here, as you can see in this particular illustration, here is given by ic plus ia, ok, where ic is the cathodic current due to the reduction and i a is the anodic current. And although the derivation of ic and ia comes beyond the scope of this course, I would like to just illustrate, that the current i c can also be measured as proportional to the number of the electrons transferred n, the Faraday constant. The rate, the equilibrium rate of the forward reaction to take place in the concentration of the oxidant essentially, and then you have this exponential term here, exponential to the power minus alpha n FE minus eq by RT. e q is the equilibrium potential and E is the potential at a particular point T, of course, as we all know is temperature in Kelvin and R is the Rydberg’s constant.

So, essentially, here the whole goal is, you know, that as the, as the E would increase, you can see, that there is automatically an increase in the cathodic current and simultaneously, there is a fall in the anodic current. Therefore, the overall e net would really depend on position of e with respect to the e equilibrium where this potential e of the current time point is located in comparison to the equilibrium potential of the particular reaction. So, essentially, that is how you can ascertain the various, the cathodic and anodic currents in a cyclic voltammogram, but the whole idea is to be able to ascertain what is the voltage at which V, at which this reduction status getting attained, the specie is oxidized species. Ox is suddenly accepting electrons and getting reduced. So, there is an increase in current. Because of that current gain is the loss of electrons, as you all know in the conventional sense, ok.

So, what would typically happen if, take the emf on the reverse side. That means, towards the oxidation side the reverse process should take place and therefore, there should always be a tendency of the new species to get reoxidized and the current go down, because there is a sudden burst or emission of electrons or availability of electrons. And so, therefore, this really is a set of two peaks and you can back and forth alter the potential into making it more reduction, reduction potential or oxidation potential and correspondingly plot a hysteresis curve because of that and that is typically what you call as cyclic voltammetry, ok. Cyclic because you have a species carrying oxidized and simultaneously species carrying reduced on the same system by just application of emf. And then, you can characterize this by measuring the current along and getting an increase as species get reduced and simultaneous decrease the species gets oxidized.

(Refer Slide Time: 09:46)

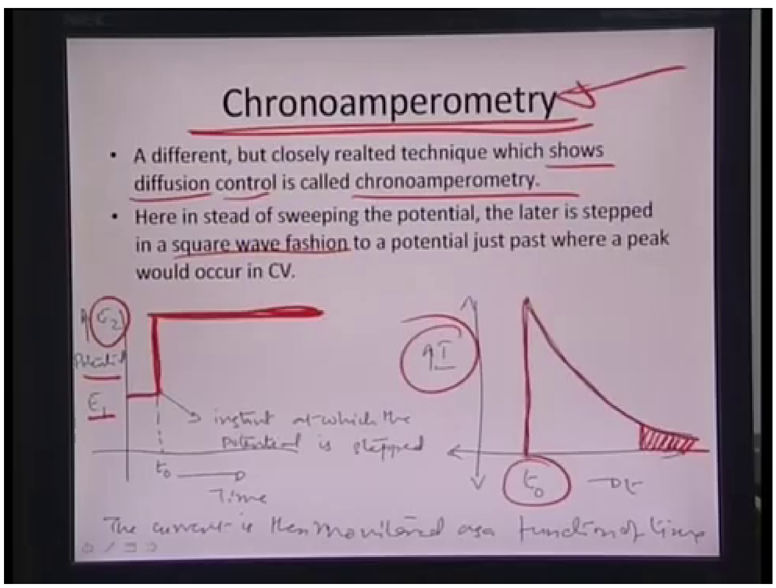


So, essentially as you can see here, this illustrates what the cyclic voltammetry process really is. So, you get a potential or give a potential with time as a triangular pulse where you have an increased potential up to the time point, let us say T 1 here and then beyond T 1 you have the reduction in the potential or the potential in the reverse direction. And as you can see here, on the forward side you have a species getting suddenly reduced and getting suddenly oxidized, sorry, the oxidized species suddenly getting reduced and losing electrons to the electrodes, therefore resulting in a, in an increased current.

And reduced species getting oxidized here with the burst of electrons, which gets reduced, correct. So, the amount of Ox at the electrode's surface becomes depleted by the reduction process and it is replaced by the reduced species in a nutshell.

And if you reverse the potential sweep from positive side to the negative of the of the peak, we shall observe an exactly reverse effect, that is, oxidation of the reduced species so as the potential starts sweeping backwards the reduced species will start to reoxidized again and the current will now increase in the negative direction until an oxidation peak is reached. What is more interesting here to observe is, that in a one electron transfer process, in n electron transfer process, these two peaks are shifted exactly by one ((Refer Time: 11:14)) slope 0.059 by n Volts and they are typically of equal heights because that indicates the equilibrium of concentration of the reduction and oxidation species inside the redox couple, the redox reaction. So, that is kind of all about cyclic voltammetry that one should know.

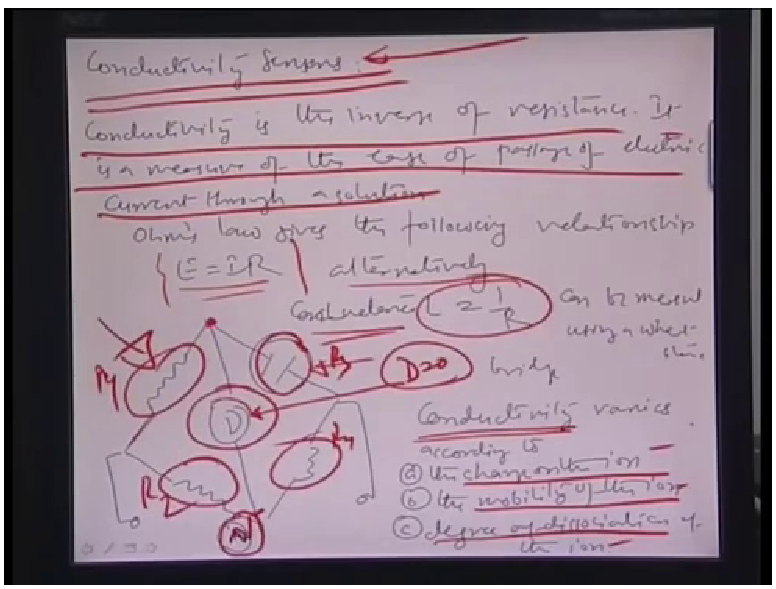
(Refer Slide Time:11:30)



Another very interesting, you know, kind of technique apart from what we have done before is potentiometry and the cyclic voltammetry is chronoamperometry. So, it is essentially, as the name suggests, chrono indicates time and amperometry is measurement of current. So, we are actually trying to measure the current by giving a sudden potential and creating an oxidation process to happen or a reduction process to happen. So, you give a square pulse here. As you are seeing here, the potential E in this particular figure at time instance t zero goes from E 1 to E 2, ok. There is a huge step or a square pulse there has been given thereby continuing the pulse at E 2 or the state to be at E 2. And what essentially chronoamperometry shows is the diffusion control of such a case, you know, where there is a square wave suddenly applied to a redox

So, if we measure the current with time, as you see here, from time t0 that is a slow decrease in the current, ok, with time; there is a slow decrease in the current. And if you might as well kind of predict the type and nature and behavior, the redox species by monitoring how fast the current will decay with respect to time at a particular square pulse from E1 to E2 is generated, ok. So, the advantage here is, that different slopes, different time aspects of decay for different species would be typically characteristic of the species type or nature and concentration and therefore, can be used as a good technique to find out what is the active concentration of the certain species, which is electrochemically active in such a redox system or a redox couple. So, we have by and large covered all the electrochemistry techniques so far starting from potentiometry to cyclic voltammetry to chronoamperometry.

(Refer Slide Time: 13:35)



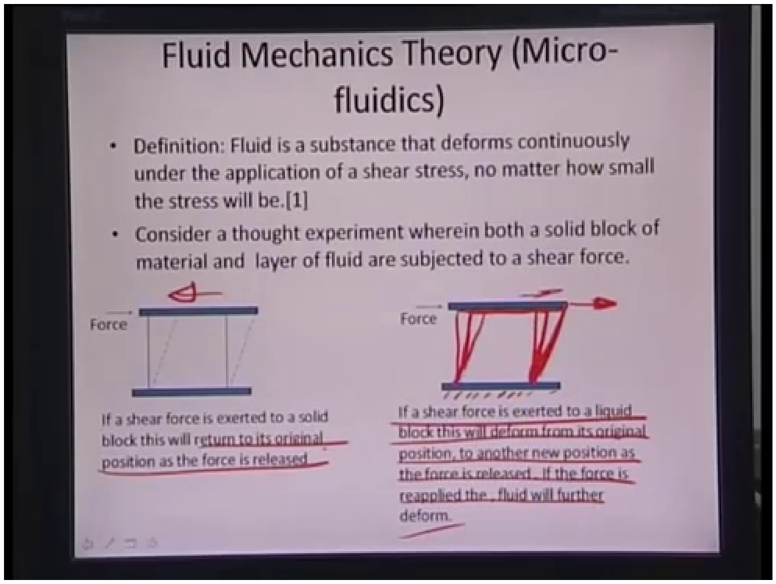
So, we now kind of shift briefly to another very interesting area, which is conductivity based sensors. So, as we know, conductivity is the inverse of resistance, ok. It is a measure of the ease of the passage of electric current through a solution. And if you see all, if you see all the solutions, they kind of obey Ohm's law. Alternatively, the conductance relation would be kind of E by L where 1 by R is equal to L essentially. So, E L is equal to I can be what the Ohm's Law can be written as in terms of conductivity.

Now, what conductivity does is, that you know, as is obvious. it would have change in behavior with change in the ion concentration of the particular medium. It would also depend on the mobility of the ion. If the ion is more mobile, the conductivity of course would be more, the flow of currents through that kind of medium would be much higher and then it would also depend on the degree of dissociation of a particular ion in especially in the redox system. So, these are some of the factors where the conductivity varies.

Typically, you can get a Wheatstone bridge kind of configuration, As you can see here, right here, you can see there are resistances, which are known in nature and then there is a meter right at the center here and this is the flow cell where the conductance typically has to be measured and the idea is that these three combinations, these four combinations can be varied in a manner, that R1 by R2 typically becomes equal to R3/R . And that is a instance whereby this principle of Wheatstone bridge, which typically the D here would show 0. There is no current here in between these two terminals at a fixed potential and therefore, you can find out the conductance of such a medium. You can also apply an AC field here as you can see here and do similar calculations of an AC Wheatstone bridge to find out what the conductivity of this cell is. But whatever it is, the conductivity senses essentially, are used for measuring the charge of a particular ion, the mobility of the ion and also the degree of dissociation of the ion and several such electrochemical information found out by these technique, very useful, is very useful for sensing or measurement.

So, we are kind of done now with this whole electrochemistry business and also seen some of the interesting behaviors of surfaces or surface potentials. Now, what I would like to explore is a little bit of fluidics as applied to the micron scale and for that we would do some basic theorizing of fluid mechanics and understand some of the basic concepts. So, what I am going to do now is to dedicate kind of the remaining part of this lecture to more towards understanding of some basic fluidics and then go into active micro fluidics where we can design ((Refer Time: 16:27)), mixers, PCR reactors, so on and so forth. Let us look at this little more closely now.

(Refer Slide Time: 16:36)

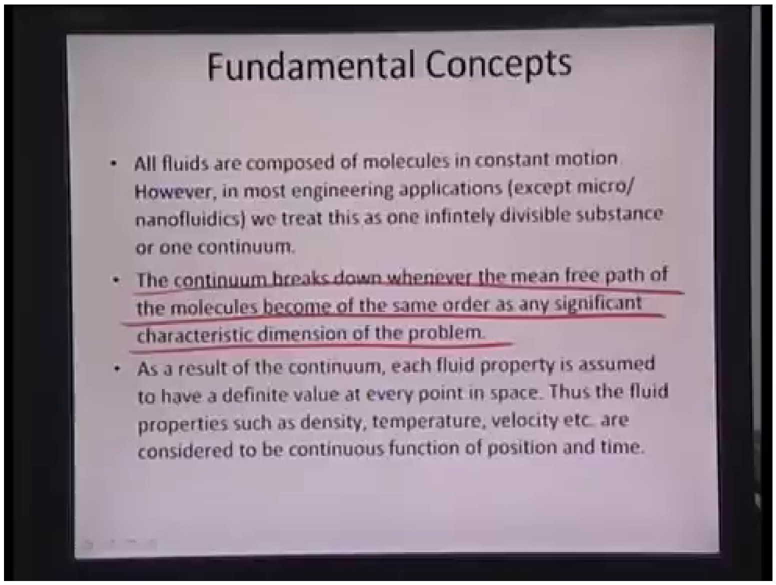


So, to start with micro fluidics, as we know, is the movement of transport of fluids of the microscopic length scale. But what really fluidics is, is first coming from the term fluid, ok. So, what is a fluid? A fluid essentially is a substance that deforms continuously under the application of shear stress. So, what happens? Suppose, I take this particular solid object and try to hold it here at the base and try to deform it from the top. So, if I deform it by holding it continuously at the base, it would kind of get sheared, right, or it would at least try to get deformed in its shape. Immediately after we release the load, it should come back automatically, elastically if it does not cross its elastic domain back into its normal shape. However, in case of fluid it is the reverse kind of effect.

So, whenever we have, let us say an experiment as we can illustrate here again, wherein you have fluid layer as you can see here, held between a mobile plate on the top and a fixed plate at the bottom, ok, and then you try to move this fluid as a plug along with this plate by moving the plate forward in the forward direction. So, as you can see here, on the fluid would kind of shear up without returning back. So, there is some molecule deformation, something where this fluid will tend to remain in the shear condition. If you apply some more force, it will sheer to a different condition here, ok. So, there is a continue shear process, which is happening because of movement of the upper plate with respect to the lower plate.

So, I can summarize it by saying, that if a shear force is exerted in this case to a liquid block, this will deform from its original position to another new position. As the force is released, this position, this new position will be retained, that is very important. The force is reapplied the fluid will further deform along the same direction as that of the force. However, in the case of a solid, it is the other way round. The moment the force is kind of released, the block would tend to come exactly in the opposite direction and try to stabilize into its own shape, return to its original position as the force is released in a particular solid. So, that is essentially what a fluid is, ok. It deforms on applying shear stress.

(Refer Slide Time: 19:07)



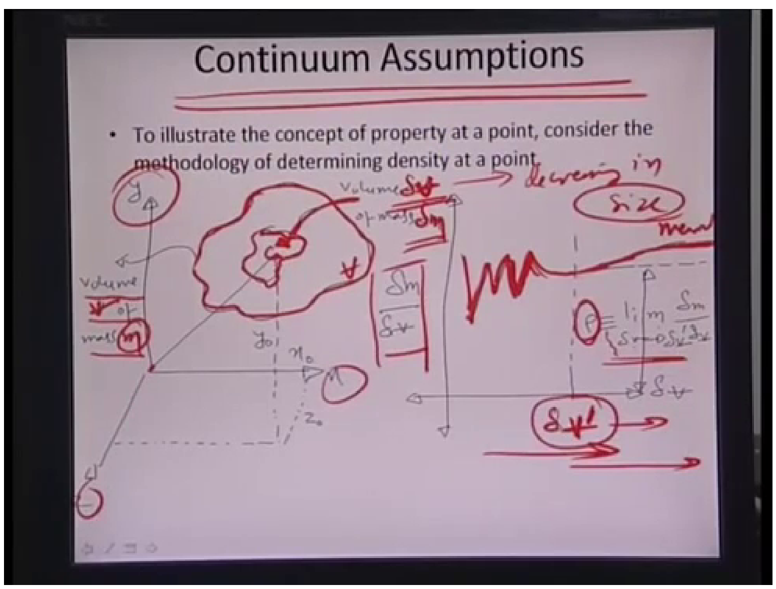
So, what is important is to know that at a more fundamental molecule level in a fluid, the molecules are not so firmly bound. They are having lessor forces and they are in constant motion with respect to one another. Then, whatever it is, you know, we still treat the fluid as one continuum body. Even if the molecules are moving around with respect to one another, we still treat it as continuum, one big continuum especially in the scales in which we can do bio sensing or diagnostics. However, if you go to a certain different domain in the nano scale, there the properties of the fluid itself vary with time because as I am going to illustrate this concept in the next slides, that what happens if you scale down to a smaller scale.

But then, essentially, in most engineering applications related to sensing and diagnostics, we still continue to believe the fluids as one big continuum, one individual substance, be it that the molecules are moving in a certain motion with respect to another, but then still they are bound by a certain domain, bigger domain and the domain keeps on changing shapes and size, but though it keeps on doing so continuously and the fluid continuity is maintained. So, the continuum breaks down whenever the mean free path of the molecules become of the same order as any significant characteristic dimension of the problem.

So, if you are considering the volume element to be too small in comparison to the mean free path of individual molecule, the probability of the continuum to break down is more in that respect. So, as a result of the continuum, each fluid property is assumed to have a definite value because you have one continuum, the fluid behave as one medium and especially, at every point of space, these fluid properties would show one fixed value, average value if it is of a certain finite size, ok, where this problem of the mean free path to be equalized to one characteristic dimension is no longer there.

The mean free path is much, much smaller in comparison to one of the characteristic dimensions. So, properties such as density, temperature, viscosity, and etcetera are considered to be continuous function of position and time in such instances where the continuum still exists. When the continuum breaks down, they change with time. They become total independent, totally independent of space as well as time. They, they keep on varying space to space, time to time, etcetera. So, let us illustrate this concept little more in detail.

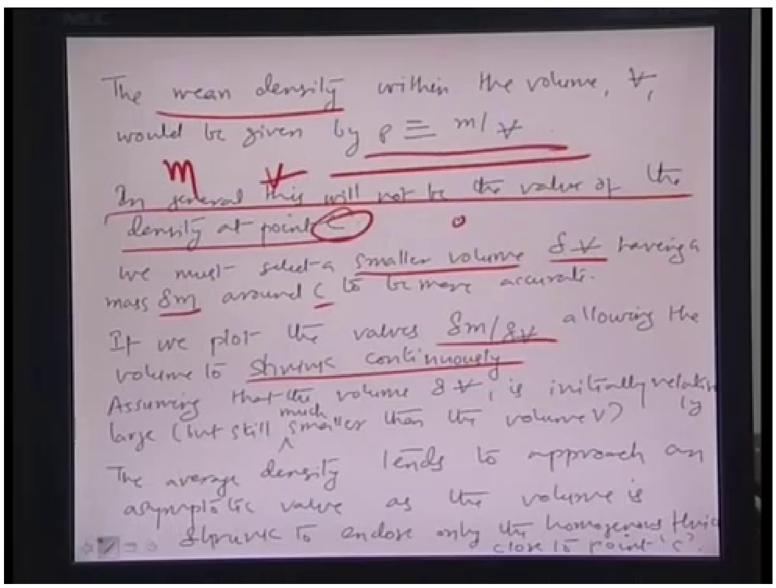
(Refer Slide Time: 21:55)



Let us suppose we are talking about plotting the volume V of a mass of fluid m, ok, as can be illustrated here within this x y z, Cartesian coordinate system, as you can see. Let us suppose there is a point C in space, which is radius vector R from the origin and you assume a one big volume around v tilde, ok. This whole thing is v, v tilde. We cross, we assume the mass of such an element m. Now, around the point C we also take much, much smaller volume, it is delta v cross ok and take the mass of this particular element delta m, alright. So, this volume here is much, much smaller around c in comparison to the overall volume, which is a v, v cross with the mass m. Now, this small volume has smaller volume, has much, much smaller volume delta v cross and much smaller mass delta m.

So, what I would like to do is to go ahead and plot the ratio mass per unit volume with respect to the volume change, ok. So, this is essentially the variable volume. We are assuming, that this delta v is going on decreasing in size to a certain extent, ok. So, it is kind of closing on the point C and becoming more towards the point C. That means, it is becoming a point slowly from a finite volume and so, it approaches continuously a smaller value. So, what would happen to properties like density that means, mass per unit volume. In case it achieves this, let us have a look at what that would be.

(Refer Slide Time: 24:00)

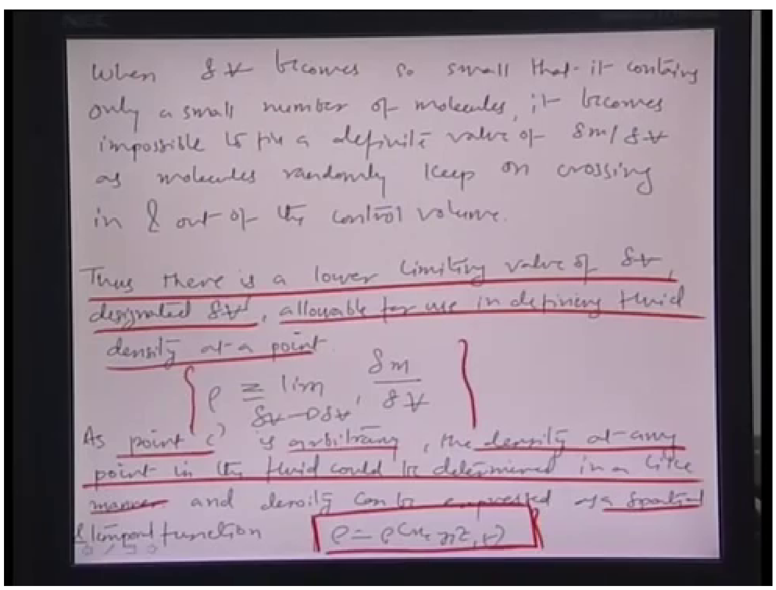


So, the mean density of this bigger volume v, where the mass is m, essentially, is given by mass per unit volume and you know, in general, this will not be the value of the density at point C though very, very close to C. If you are just having a point dimension of C, we have to select the smaller volume delta v tilde. Let us assume we have a mas delta m, I just talked to you about it just a minute back, around C and now you are plotting this, allowing the volume to shrink continuously, ok.

So, there is a certain volume here, as you can see. Let us say delta v cross dash, ok, beyond which the property density kind of rapidly increases and asymptotes or becomes asymptotic to a fixed mean volume. But below this you can assume, that the dimension is so small, that one of the characteristic dimensions is more like the dimension of individual mean free path of the molecule. So, the volume is small and the mean free path is greater. You never know there is an uncertainty of whether the molecule is within the volume or outside the volume. Thousands of this molecules, which are cross-crossing the surface of the small dimension and therefore, you were not hundred percent certain as to how much amount of mass or how much amount of volume is there within the small, infinitesimally small point volume, that you are considering. Therefore, the density should be a function of time. So, delta m by delta v dash, in that case, below v cross, below a certain v cross dash volume, delta v cross dash volume becomes very, very erratic because of this motion of the molecules or continuous sweeping of the molecules through its boundaries, ok.

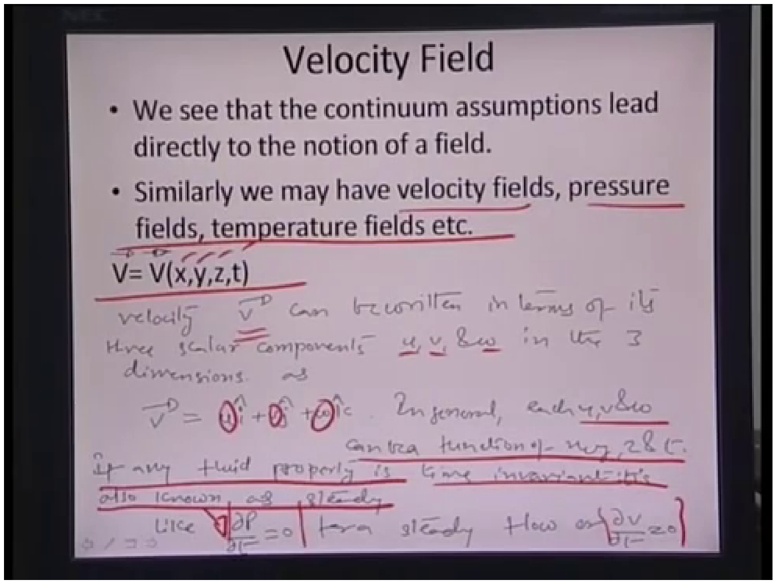
So, let us say, you behave, the density behaves like this. It is very, very erratic with the time. So, really the continuum assumption is valid beyond this delta v cross dash value beyond which you can say, that as the delta v is approaching, a limit delta v dash cross row essentially, becomes delta m by delta v cross as a continuum property and but less then v cross dash. The row is really a function of rho t, it is a function of time, so it keeps on varying. So, that is what continuum essentially means in the real sense in the real physical sense, ok.

(Refer Slide Time: 26:38)



So, in summary, there is a lower limiting value of delta v cross, designated delta v cross dash allowing for use in defining fluid density at a point. So, at point C being arbitrary, the density at any point in the fluid could not be or could be determined in a like manner and the density can be expressed essentially as a spatial temporal function, a spatiotemporal function, as a function of x, y, z and t, as you can see here in this particular equation, ok. So, rho becomes space time and function.

(Refer Slide Time: 27:15)



So, next important concept to assume are, are to, to find out is what a velocity field would be. And essentially, by the continuum assumption, we see directly there is some, some kind of a notion of a velocity field developing. That means, velocity really is a function of x y, z and time, but it is, it is a continuum property, it is an average velocity property. We should have a kind of trajectories along which these velocities have or meet a certain condition, ok, and they follow a certain space-time relationship. So, these projectories can then act to define the flow paths of the various molecules, which are transporting through this fluidic medium and they could be timeline, they could be streaklines lines, they could be streamlines, they could be path lines and all different kind of lines.

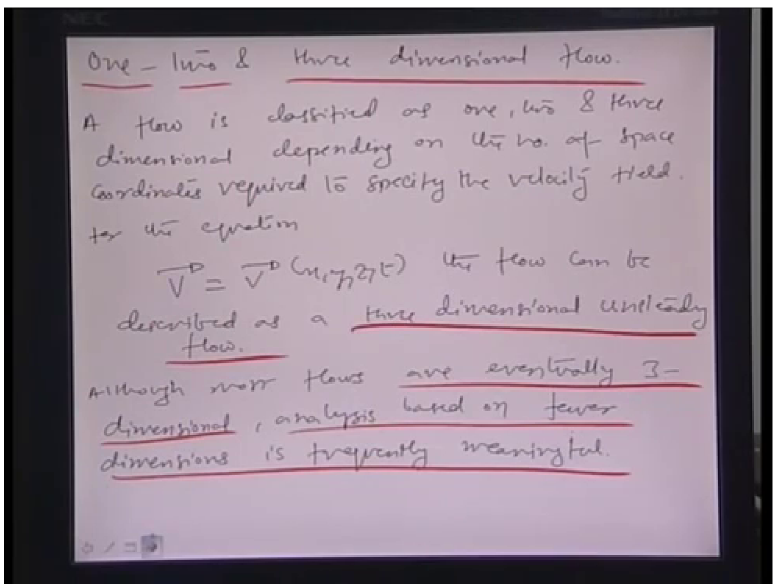
So, we will like to have some illustration as to what this velocity field is and then, go into this various aspects for various methods of defining the flow paths within the fluid. So, we may have similar equations as velocity fields, pressure fields, temperature fields, etcetera within this volume. So, let us say v, vector v is a function of x, y, z and time. $V=V(x,y,z,t)$

$$\acute{V}=ui+vj+wk$$

This typically the case, because v can be written as a summation of three components, u, v, w in the x, y, z direction as y cap plus v j cap plus w z cap. And in general, each this u v and w can be a function of x, y, z and time, ok. We can, we can assume that kind of a thing.

If the fluid property is time invariant, which is also known as study state flow, ok. So, essentially the $\frac{\partial \rho}{\partial t}=0$ or as a matter of fact, any other property like dv by dt or dp by dt, p being the pressure, they are all equal to 0. There is no time variation of the particular flow. However, in flows, which are heated up, this may not be the case because the densities keep on changing from the cold surface towards more, towards the hot surface in touch with the fluid in question. Therefore, as we will see later on, especially in micro scale fluidics, we hardly need this component of heating and we can do away with the energy equation of Navier Stokes just because of that reason. And so, in micro fluidic situations mostly we use time invariant flows, ok. So, dv by dt or dp by dt or d rho by dt or any other property with respect to time does not really vary at all within such domains.

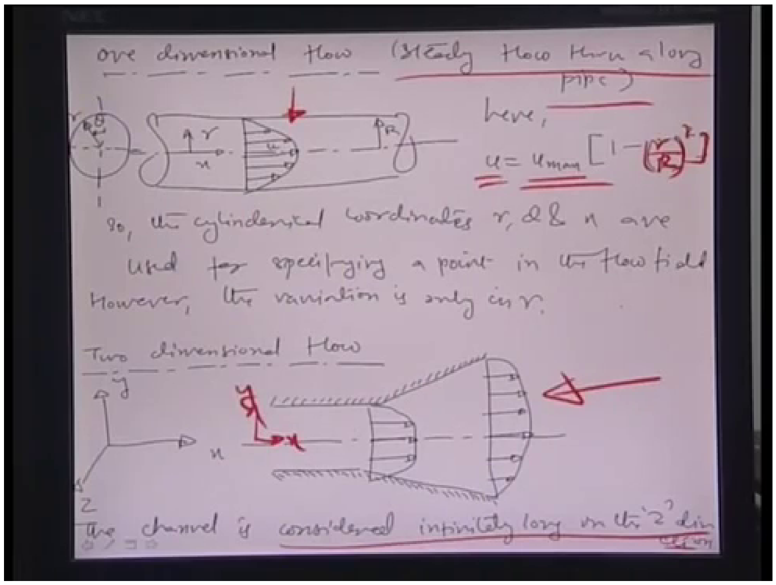
(Refer Slide Time: 30:10)



So, flows can further be described as one-, two- and three-dimensional. As the name suggests, it really depends on the number of, number of space time coordinates that is needed to specify the velocity field for a certain equation. So, essentially, if it is just dependent on x and y and z or time, that is called a one-dimensional velocity field and which is steady state and if it is varying on time, it can be also known as unsteady one-dimensional. So, therefore, it varies with all the three coordinates. You can call the three dimensional unsteady flow system ok.

Although the flows are eventually three-dimensional analysis almost is always helpful. So, if you can convert it somehow into fewer dimensions, two or one in most of the cases. Depending on the symmetry of the situation or the geometry through which the fluids are flowing, you could easily convert it into more one dimensional, then two or three. Although actual in the real life, it is more towards three-dimensional. So, this is just for calculation simplicity sake, that you are eventually assuming all three dimensional flows or are predicting all three dimensional flows on basis of analyses on one or more dimensions, but fewer than three, ok.

(Refer Slide Time: 31:31)



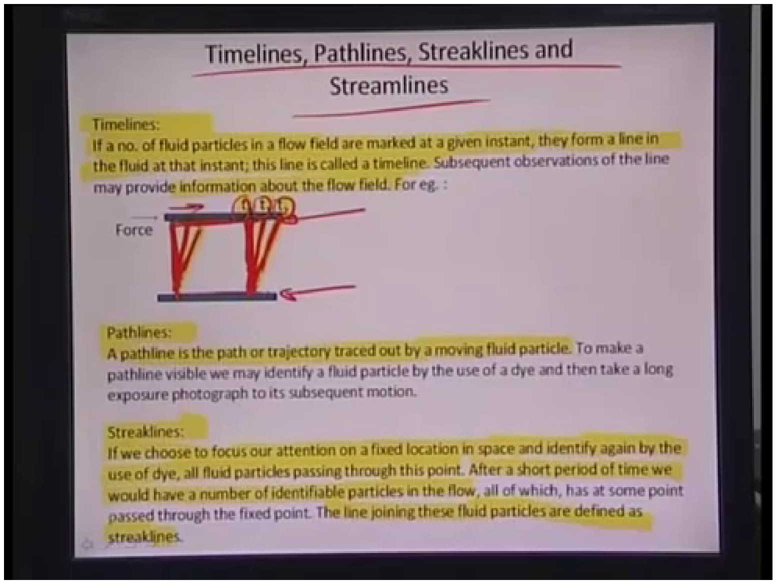
Let us discuss some examples of one dimensional flows, especially steady flows through long types here. You can see here, this really is a variation of velocity profile with respect to the radius r, small r. So, u here is actually equal to u max with somewhere in the center of the pipe,

$$u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

which is 1 minus small r by big radius R square. So, it is varying only on the radius vector, it is symmetric along the angle phi, describe the cylinder it is only varying along the radial direction. So, typically this is the case in cylindrical coordinates, ok, or same is true for the z direction. z is too long in comparison to the width of the pipe, you know, the radius of the pipe.

So, if you are talking about micro flows where the dimension of the pipe itself is very small in comparison to the length in which it is kind of laid out, you typically can consider it to be one-dimensional flow problem, which is radially symmetric, ok. If you assume, that the continuum assumptions are approximations whole true, two-dimensional flows are represented here. In this particular figure, as you can see, the channel is considered infinitely long in the z direction, that means, in the plane of this particular illustration or in the board. So, therefore, the variation though is mostly in the x and y. If you can see x and y are these dimensions here and the flow velocity would vary in only two dimensions in this particular case, illustration.

(Refer Slide Time: 33:03)



So, let us talk a little bit about these concepts of timelines, path lines, streak lines and streamlines. These are important for understanding of later on topics of microfluidics or micro flows. So, if you talk about timelines here, the timelines are these, ok, various instances, t0, t1 and t2. What they are typically is, that you know, if a number of fluid particles in a flow field are marked by a given or marked at a given instant of time, let us say at a certain point in space and then, let us say they form a certain line, you know, it can be a straight line, this line is known as the timeline, ok.

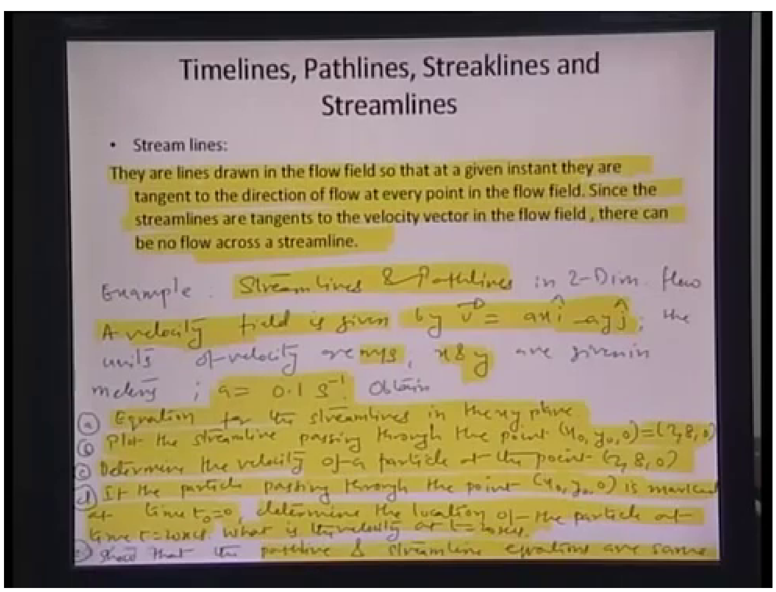
And the variation of this line would provide information about the flow field, like for example in this particular case, as you can see, the fluids at the very outset were held in a very symmetrical manner between the mobile plate here in the top and the fixed plate. Now, this plates, the mobile plates start moving and it being fluid in the property dictates to be or take the shape of the relative displacement between the upper plate and the lower plate, which kind of varies with time, kind of increase with time, as you see here from time t0 to t1 to t2. You have all these different, you know, lines, which are getting formulated. So, this was the time t0, ok. All the particles, which were here in this particular edge of the fluid were defined from the time line at t0 and it moves to t1. The time line also changes to t1 and t2. So, the flow field here would be defined by tracing these timelines here. So, that is what timeline means.

What are path lines? Path line is again path of trajectory traced out by moving quick particle. So, we know, we can use identification techniques to make these timelines kind of or these pathlines kind of visible, you know, things like we can introduce a small dye or an ink in water and we can see how this ink kind of diffuses through or a cross. So, the ink particle moves in a path of trajectory traced out by moving the fluid around it or the fluid itself, the ink itself with time, ok. So, that is the path line. So, we can take a long exposure photograph of such a flow situation and see where this dye is kind of slowly going into or diffusing into, that is, on the path line of the dye would be in, let us say, water.

The other important classification is streak line, ok, and this is little bit indirect way of describing flow fields, ok. So, if you choose to focus our attention on a fixed location and space and identify again by the use of dye, all fluid particles passing though this point and after the short period of time we would have number of identical, identifiable particle, which flow though that point in space in the flow field, ok. And all of these particles, of course, flow through that same fixed point as indicated before. Now, the lines, that would typically join these fluid particle once they are out of that small point and they are somewhere along the fluid after that points the lines, which would join all those particle, which came from this one special location is known as a streak line, ok.

So, we have already seen what a timeline, path line or streak line would be and they all have different connotations. You could use these to explain fluid flow behavior or motion of molecules, you know, in in a, in a continue fluid mechanics very easily.

(Refer Slide Time: 37:27)



The other very interesting factor is streamline, ok. So, what are stream lines? Definitionally, their lines drawn in the flow field so that at a given instance they are tangent or they are, they are tangential to the direction of flow at every point of the flow field. So, you know, by drawing, so let us say, you have a particle, which moves along a certain trajectory and this trajectory is like curvilinear, like this, you know, and there is directional variation in the velocity, this variation in the velocity by virtue of the direction. That is the reason why the particle describe the trajectory. Combining all these tangents together would give you what you call this streamline, ok, of the flow.

Very interesting, very interestingly, the streamlines do at a micro scale define whether, at all possible scales define whether the flows are laminar or turbulent. If they are very, very streamlined and they are very, very laminar in nature, that kind of indicates the less mix ability between two, three different flows, which would have similar lines parallel to each other as they are going along in a small channel side by side, ok.

So, since the streamlines are tangent to the velocity with the flow field, there can be no flow across a streamline. So, what I would go ahead is to give you an example problem to calculate how we can really estimate an equation between one, two dimensional flow between the x and y coordinate estimate, what we call a streamline, ok. So, this particular example, a velocity field is given by the expression v is equal to $\acute{v} = ax\acute{i} - ay\acute{j}$

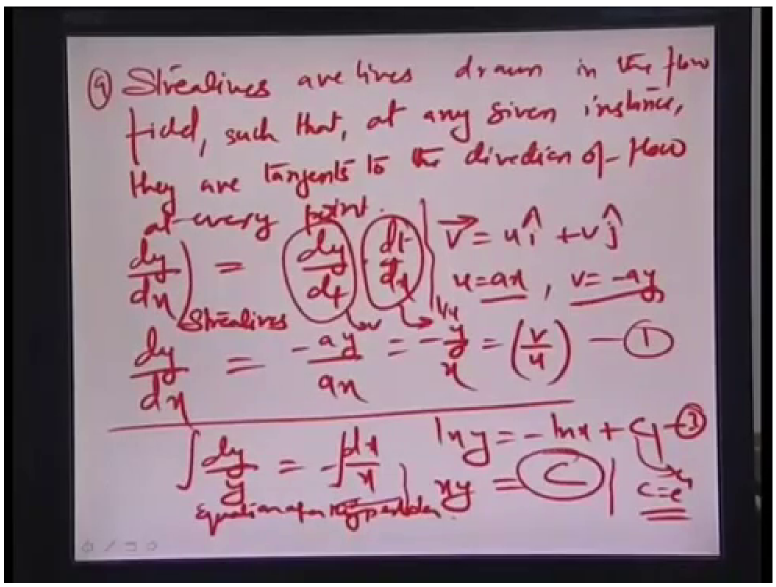
x i cap minus a y j cap. x and y are the position coordinates and a is some constant and $\acute{i}$, $\acute{j}$ essentially, as you know, are unit vectors in the x and y direction, a and minus y direction and the units of velocity here are meters per second. a of course, is given by the expression 0.1 second inverse, just to be consistent on this equation and the dimensional aspect. So, we want to obtain the equation for the streamlines, in this particular case in x-y plain, ok.

So, basically as in this example the velocity field is given by this equation, v vector is equal to a x i minus a y j. i cap and j cap are the unit vectors in the x and y direction and a basically is having the units of second inverse, ok. So, a is given as point one, 0.1 second inverse, x and y are the x-y coordinates. Basically, the velocity v is given in meters per second in this particular expression.

So, the first problem here is to find the equation for streamlines as we have talked about this little bit in the x-y plane. We also want to plot the streamlines passing through the point (x0, y0, 0) where x 0 and y 0 are 2 and 8 respectively. And then, we essentially, so essentially these two things are something like a plot that we would be able to define in terms of relationship geometrical relationship between x and y. We also, we would also like to determine the velocity of the particle at a point (2, 8, 0) and also, that if the particle passing through the point x y (x0, y0, 0) is marked at time t0 is equal to 0. We have to determine the location of the particle at time t equal to 20 second.

So, typically it is almost everything related to streamlines and the path lines, you know, how to plot them and also we need to ascertain the velocity at t equal to 20 seconds. So one thing is to determine the location and other at that particular new location we have to also find out what the velocity of the particle at t equal to 20 seconds. And eventually, yes we will see and actually that is what we have to prove also, that the path and streamline equations are one and the same, same in this particular case, ok. So, let us actually start this problem

(Refer Slide Time: 41:31)



So, essentially what are really streamlines? If you just go back to different streamline. So, streamlines essentially are lines drawn in the flow field and such that at a given instant they are tangent to the direction at every point of the flow field, ok. So, in a flow field such that at any given instance they are tangents to the direction of flow, ok, at every point. So, essentially here, you know, we have to find out dy by dx, which is the tangent for the streamlines and this can also be represented as dy by dt times of dt by dx by using the chain rule, ok.

$$\frac{du}{dx} = \frac{du}{dt}\frac{dt}{dx}$$

$$u = ax$$

$$v = -ay$$

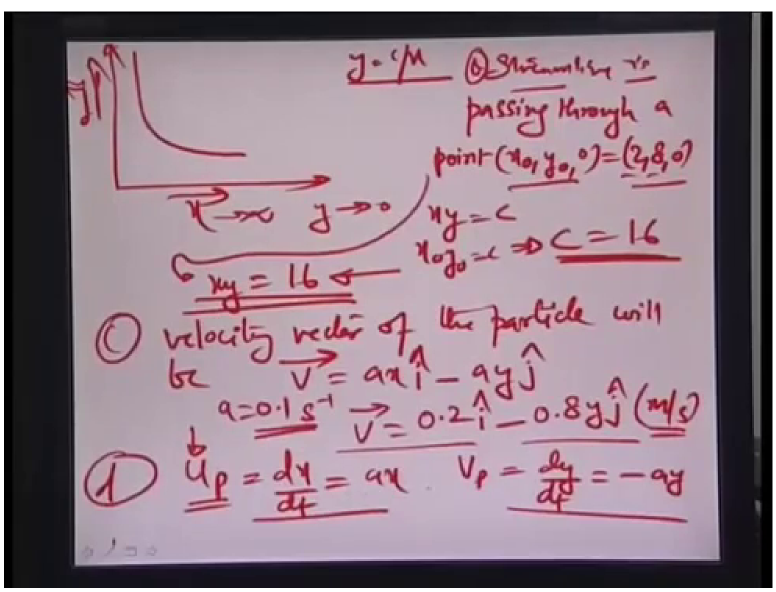
$$\frac{du}{dx} = \frac{-ay}{ax} = \frac{v}{u}$$

As can be seen here, in this equation, dt by dx, alright. And what do you really know, the velocity relationship here has been given as u i plus v j where u is given as ax and v is given as minus ay, right. That is how the velocity is defined really in the problem statement. So, therefore, dy by dx for the streamline really is nothing but minus ay by ax, ok. This is ax, so minus y by x, typically it is v by u, that is exactly what this equation here says, dy by dt is v, right. The rate of variation of y dimension in terms of time and dt by dx, it is 1 by dx by dt that is 1 by u, ok; rate of variation of x with respect to time. So, therefore, if we just solve this equation number 1 here, we are left with dy by y equals essentially dx by x with q minus sign and we can integrate this to obtain l n y on one side is equal to minus l n x plus c, on the verge c 1 may be, ok.

$$\ln y = -\ln x + C1$$

And so, in other words, we have from this particular equation number 2, we can find out x, y is equal to a constant, may be c. c is essentially e to the power of c1, ok. c1 being a constant, e to the power c1 is also a constant. So, this in fact, presents the equation of the hyperbola and it is very convenient to plot all the x, y points in a manner, whereas x tends to 0. The y value tends to infinity and vice versa as y tends to 0. Then, again the same the x value y tends to, tends to infinity. So, you can actually plot this equation as normally as a set of streamlines.

(Refer Slide Time: 45:41)



Let us say, this is y, this is x and these equations would be typically something like this, you know, order of a parabola where if x goes to infinity, let us say, y would assume 0 value, x being, y being equal to c by x, ok, and vice versa. If y goes to infinity, x would assume a 0 value. They are asymptotic to both the x and y axis, as can be seen here. So, for a streamline, so this is essentially the equation of a streamline, ok.

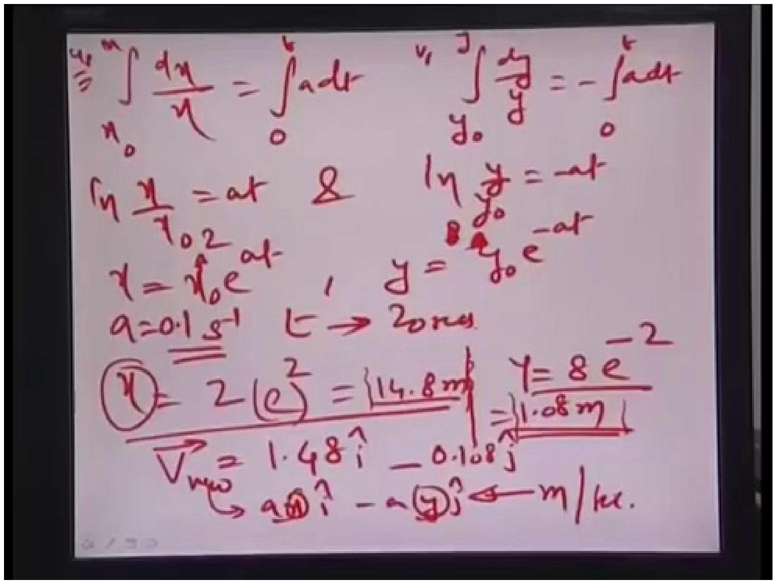
So, the part B of this problem statement says, that for a streamline passing though the point (x0, y0, 0) where x 0 and y 0 are 2 and 8 respectively. What would be the equation of streamline, ok? So, it is an interesting question. So, let us say, in this particular case, the streamline is passing through a point (x0, y0, 0) equal to 2, 8, 0 respectively. So, assuming, that streamlines follows in equation xy equal to c, therefore x0, y0 should be also equal to c and c becomes 16, in this particular case, ok. And therefore, the equation of such a particular point here would be typically xy, as you can see here, sorry, xy equal to c equal to 16. So, this is the equation of the streamline passing through the point in (x0, y0, 0) (2, 8, 0).

So, the third part of the question says or asks or determine, you know, or tries to investigate the velocity of a particle, which passes this, through this point. So, you have to determine what the velocity vector of the particle will be when it is on the particular streamline xy equal to 16. So, therefore, in this particular case you can see, velocity v was given by $\acute{v} = ax\acute{i} - ay\acute{j}$, right, where a is 0.1 second inverse and velocity was given in meters per second. So, therefore, velocity here would be represented as 0.2 i cap minus 0.8 y j cap in meters per second. Assuming, that this 2 and 8 units have or has the units of distance in meters, ok. So, that is what essentially third part of the question can be addressed as.

Now, the next part kind of asks, that if the particle is passing through the point (x0, y0, 0) and it is marked at time t 0 equal to 0, then we have to determine the location of the particle at time t equal to 20 seconds. And also, we have to further determine what is the velocity at time t equal to 20 seconds. So, let us say or let us suppose, that you are actually kind of trying to, you know, move from the point, this xy equal to 16 to new trajectory. The velocity, as we know here, from the earlier equation is given by this 0.2 i minus 0.8 y, ok. And essentially, we have to see at the new position, we have to first calculate what the new position really would be if we assume this velocity. So, as we know, U p or you know, which is actually equal to the dx by dt, ok, equals to ax, right. So, this is U at a particular position or velocity, the x component of the velocity at a particular point p. Similarly, we also know, that v of the particular particle U will be dy by dt equal to minus ay.

From these two equations what we really need to ascertain is what is the relationship between xy and time t because we need to find out what will happen at time t equal to 20 seconds, ok. Assuming, that the initial curve xy equal to 16 satisfies, you know, a streamline, which is formulated at time t equal to 0 seconds.

(Refer Slide Time: 51:17)



So, therefore if we integrate this particular equation, let us say dx by x on the left side, a dt on the right side from 0 to time t, assuming the initial position to be x0 and the final position to be x here. And similarly, this is for the velocity of the particle in the x direction and for the y direction. We, similar, we do a similar thing as dy by y from y0 to y equal to minus a d t from time t equal to 0 to time t equal to t. So, therefore, l n x by x naught here is equal to a t and l n y upon y naught here equal to minus a t, ok. And hence, really it is exponential relationship, so x becomes equal to x0 equal to the power of a t and y, new coordinates become equal to y0, e to the power of minus a t.

$$x = x_0 e^{at}$$

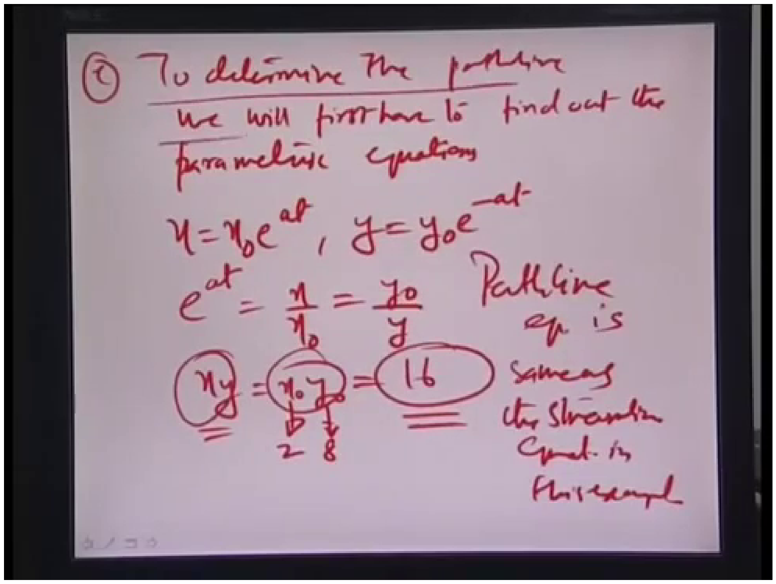
$$y = y_0 e^{-at}$$

We already know the value a to be 0.1 second inverse and the time here really is from 0 seconds to 20 seconds, which means, the new x coordinates here assuming this to be 2 x0 to be 2 and y0 to be 8 as the two different points through which this earlier streamline, which have passed from, can be found. So, the, so the new position x and y could be found out as 2 times of e to the power of 2, ok. e square, which is about 14.8 meters and y, similarly, would be 8 times of e to the power of minus 2, which is about 1.08 meters. That is what the new x y coordinates of this particular, you know, point is really.

So, if the particle is moving at the velocity a x i cap minus a y j cap and it was supposed to be at time t equal to 0 along the streamline or at a point (x0, y0, 0) at the particular point at the streamline and x equal to xy equal to 16. Then, from the relationship between xy and t we find out, that the new positions x and y of the particles would be represented by 14.8 meters and 1.08 meters. So, therefore, the new streamline equation, first of all, the new velocity, that the particle will possess in this instance v nu of the particle is equal to essentially, $1.48\acute{i} - 0.108\acute{j}$. Mind you, velocity, again relationship $ax\acute{i} - ay\acute{j}$ p holds true. The only difference here is, that the x and y are the new values of x and y, essentially, ok, which are 14.8 meters and 1.08 meters respectively. So, this again is in meters per second.

Also, we would like to determine the path line equation for this particular example and for that we also need to find out first the parametric, parametric equation, That means, the relationship between the between x and t and y and t as we had done in the earlier example and then try to solve for time, essentially, ok. And from there we can find out what in the path lines are really, ok. So, definitionally, again what are path lines really? Path lines are typically lines, which are a kind of paths of trajectory traced out by moving fluid particle, ok, and to make up. So, this is something, that you know, a particle at a particular point of time when introduced into the fluid would follow temporarily as it goes ahead inside its, its domain, ok.

(Refer Slide Time: 55:32)



So let say in order to determine this path line, we will first have to find out the parametric equations x equal to x0 e to the power of a t and y equal to y0 e to the power of minus a t. So, from this expression really what we get is a relationship between x, x0, y, y0.

Let us say, e to the power 18, this case is x by x0 and in this other case is y0 by y, right. So, therefore, x y really equal to x0 y0, which is again equal to 16, x0 being equal to 2 and y0 being equal to 8 from the earlier example. So, therefore, what we find out here is, that if suppose by definition, what, what a path line means really is the location of the particular point at a time instance, ok. So, you have to make if you want to join all such, you know, points after all such particle, which have moved past the certain point. You have to make them independent of time and in order to make them independent of time. You find out the equation, which has come out is same as the equation of the streamline as before. So, therefore, really the path line equation derived here is same as the streamline equation in this example. So, this kind of is just to get a feel about all these different, you know, different concepts of how to kind of, kind of trace trajectory or lines of fluid particles within fluid flow systems. These will be immensely helpful later for understanding of micro flows because they are highly streamlined and laminar in nature.

So, with this I would like to kind of close today's lecture and then start with derivation of the stress field and then correlation of all these parameters together to derive something, which is very fundamental and unique to the fluid mechanics called the Naviers Stoke equation. So, we will cover this in the next lecture.

Thank you.