BioMEMS and Microfluidics
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Lecture – 13

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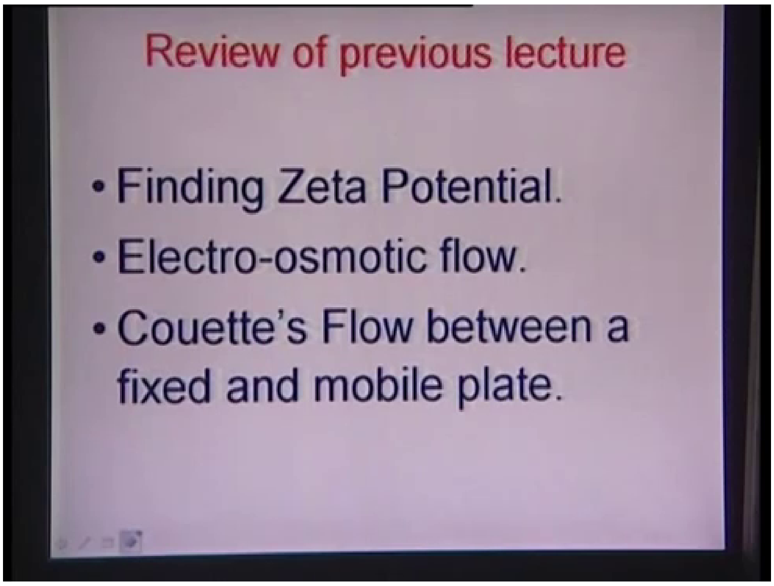

Welcome back to lecture 13's, I just like to begin with the quick review of the last lecture class we try to derive the zeta potential associated with the surface in contact with the liquid phase. We also try to kind of interpret how the dual layer comes it into existence. Why is that the charges in solutions get distributed and forms what you know as the bulk charge or the fused layer of a solution. And then, we try to also calculate the potential as a function of distance as you move away from this double layer into the bulk layer.

We try to use this concept of double layer charging in order to realize certain type of micro fluidic flow called electro-osmotic flow. Again just to review that if there is a surface which has certain set of dangling bonds like let say silanol bond silicon surfaces ion and it comes in contact with solution of a certain pH there is a tendency of the surface to acquire a charge that could be negative charge or positive charge depending on if the pH are acidic or basic.

Because of this charge, if you are able to place a solution close to this charge surface there is always going to be this defused layer which is formulated. If we talk about micro capillaries with in carved in such kind of surfaces with charges and try to flow fluids across it there is going to be some kind of comparative between the dimension of the channel and the thickness of the defuse layer. Now, this is adds to also this also tends to drag the defuse layer also tends to drag the fluid in such a micro capillary, when you put an external EMF across it.

So, we try to the derive some equations and formulations related to this flow process the electroosmotic flow process. And the basically we also try to look into you know the, the Couette flow just for a comparison of this electroosmotic flow with a flow, where in there is a, let say two plates with one fix and another moving and trying to drag of fluid layer along with it and then, compare it also to parabolic flow with there is a pressure gradient and two fix plates between, which the fluid flows.

So, we kind of saw that in case of electrokinetic flows there is always tendency of the flow to a develop a plug like behavior, so the velocity profile is like a plug. So, in case of special driven flows it is something like a parabola. So, if we consider and try to start from that aspect we found out that by solving Navier-Stokes equations.

So, basically we kind of try to make a comparison between the different profiles the velocity profiles within such channels, when the flows are even driven as suppose to maybe when the flows are electro-kinetic nature. And we also try to a certain, what happens in case of pressure gradient across such a capillary with two sides fixed at two plates fixed and try to derive an equation of velocity and the variation of velocity with respect to the y direction.

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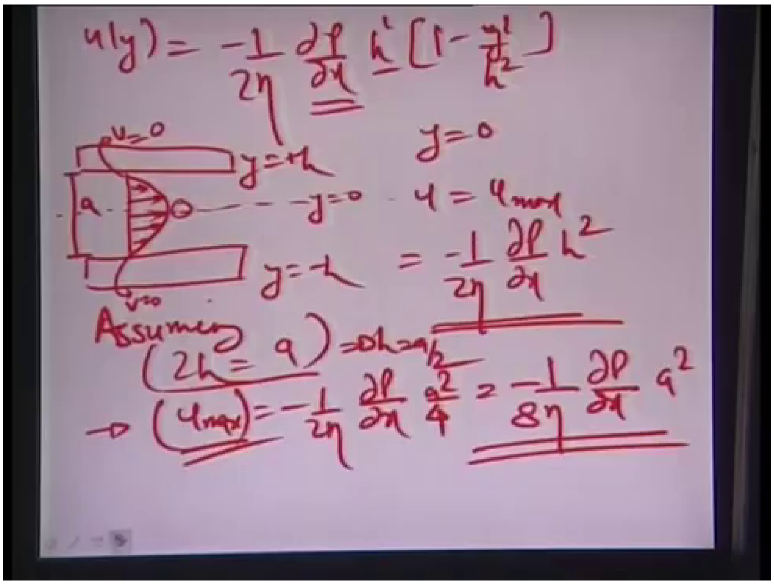

So, essentially in the function that we obtained for u on the velocity in that case we also

$u_y = \frac{1}{2\eta}\frac{\partial P}{\partial x}h^2\left[1-\frac{y^2}{h^2}\right]$ where $\frac{\partial P}{\partial x}$ essentially is a the pressure gradient across the capillary in that direction. And we assume that of the plates the two plates are respectively fixed at y equal to + h and y equal to -h the middle of such applied assembly is really the y equal to 0.

So, if you plot the u with respect to you know the y essentially you get parabolic profile this square on the y. So, there is a flow velocity variation of this parabolic type and this make sense also, because the velocity here very closed this channel wall is 0 on either sides and it is maximum somewhere at the center of the channel. So, if y equal to 0 at the u becomes u maximum, which is

$$u(0) = \frac{-1}{2\eta}\frac{\partial P}{\partial x}h^2$$

This is the maximum velocity that one can have such a channel flow. So, if we assume a little bit of different sign convention in with the in which, the plates are laid in the way the plates are laid and assuming that 2h=a or that the distance between these two plates 2h is nothing but, a. So, that case h becomes equal to a /2 and u max this expression is

$$u_{max} = \frac{-1}{2\eta}\frac{\partial P}{\partial x}\frac{a^2}{4} = \frac{-1}{8\eta}\frac{\partial P}{\partial x}a^2$$

That is what the u max of the u maximum be in a case like this. Let us, now try to find out the maximum flow rate, which would happen, because of this function u.

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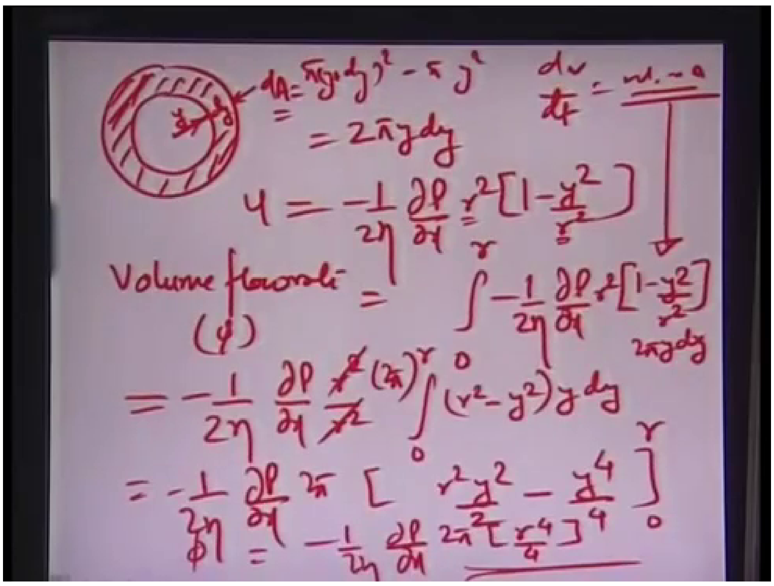

So, if you really look at the flow rate we can assume that in this particular case there is a circle of there is an angular of area a through, which the fluid is emanating out at a velocity u, which is given by on the function shown earlier. In terms of pressure gradient the square of the y and the square of the distance from the mean plane are y equal to 0, y equal to plus h minus h. So, if you look at the differential, let suppose this value here is y this distance here is dy and you want to find out what the area vector is.

So, the area here would essentially be

$$dA=\pi(y+dy)^2-\pi y^2=2\pi y dy$$

that is what the dA of this particular angular of fluid would be. And we already know that u essentially as a function of y has been represented earlier as

$$u=\frac{-1}{2\eta}\frac{\partial P}{\partial x}r^2\left[1-\frac{y^2}{r^2}\right]$$

r is a radius in this case by the by, which is also equal to h.

So if you want to find out what is the volume flow rate or phi this becomes equal to

$$\int_0^r \frac{-1}{2\eta}\frac{\partial P}{\partial x}r^2\left[1-\frac{y^2}{r^2}\right]2\pi y dy$$

as you also know dv by d t the volume flow rate is nothing but, the velocity times of area; that is exactly or this is about and the integral we assume takes place between 0 and r, which is the radius half the capillary.

So, essentially if you just a solve this equation you would left with, so you can take this

$$r$$

$$(|2-y^2)\,ydy$$

$$-\frac{1}{2\eta}\frac{\partial P}{\partial x}2\pi\int_0^r \square$$

$$-\frac{1}{2\eta}\frac{\partial P}{\partial x}2\pi\left[\frac{r^2y^2}{2}-\frac{y^4}{4}\right]_0^r$$

$$-\frac{1}{2\eta}\frac{\partial P}{\partial x}2\pi\left[\frac{r^4}{4}\right]$$

Now one thing, which is importance here to be seen is that you know essentially we a considered a channel at the very beginning if you consider the electroosmotic flow of length *l*.

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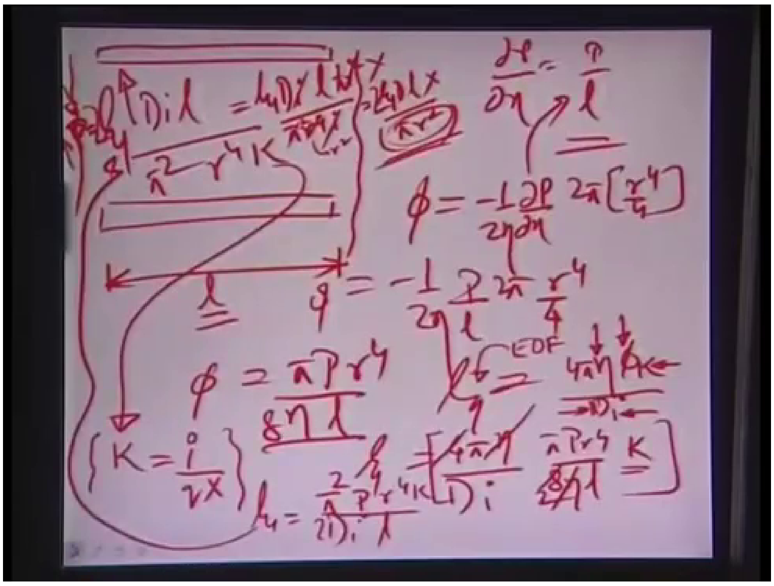

So, essentially you consider the channel of length *l* and if you assume that there is a pressure driven flow in this particular example, which is essentially between the length *l* and the pressure gradient in the case is P, then

$$\frac{\partial P}{\partial x} = \frac{P}{l}$$

So, if I put back the P by *l* in to this particular equation here for the pressure driven flow case phi becomes equal to

$$\varphi = \frac{-1}{2\eta}\frac{P}{l}2\pi\left[\frac{r^4}{4}\right]$$

$$\varphi = \frac{-\pi P r^4}{8\eta l}$$

Essentially, dp by dx is nothing but, P by *l* here, so we get pi is *1* by 2 eta P by l twice by r 4 by 4. And if we just readjust this equation pi comes out to be equal to pi p r 4 divided by 8 times of eta l that is, what the volume flow rate is in this particular example. And one more issue here is that if you consider, what happen in the electro-osmotic flow case the zeta potential for the electroosmotic flow the EOF this derived as

$$\frac{4\pi\eta}{Di}k$$

where k is the conductivity of the medium phi is a flow rate eta is the viscosity i is the current D is the dielectric constant.

If we just substitute this value of phi here assuming that we equate the electro-osmotic flow with the pressure driven flow, we have a situation well and we have this 4 pi eta D i times of pi P r 4 divided by 8 eta *l* times of k. And this essentially can be further a reconverted a little bit, if you remember from before the value of k is also equated assume to be ratio between the current i the area cross section q and the electric field X. So, that is how we found this relationship earlier from the famous r equal to row l by a term, then using v equal to i r Ohm’s law.

So, essentially if we substitute the value of k here, in this particular expression let us find out, what it would finally, look like one think, which is very important in the very critical to mention here is that since zeta here is equal to this particular term 4 pi eta D i times of pi P r of or 4 divided by 8 eta *l* times of, so here we can find out tentative relationship between zeta and the pressure.

$$\zeta = \frac{\pi^2 P r^4 k}{2\,Dil}$$

And then, of course, there is a k term here, which automatically means that the zeta potential in this particular case would relate to the pressure of the medium P by the relationship P is equal to zeta times of D i l divided by pi square r^4 k, what it is really mean is that if you consider a pressure driven flow analog of the electroosmotic flow.

The pressure driven flow analog of the electroosmotic flow, what I am trying to indicate is that if you have let say and electro osmotic force, which is in fluid the fluid in the capillary. And you consider that to be you know you consider that to be a essentially within the same flow rate everything contributed by an equivalent flow with the pressure gradient we can make a analogy between, what kind of zeta potential is needed for creating, what kind of flow pressures. And here as you see if the zeta potential of any surfaces more as in this equation the pressure of the flow would be more vice versa.

So, that is a very interesting observation, which will kind of carry forward later. Here if we substitute the value of k into this particular equation here, let us look at how this would really behave.

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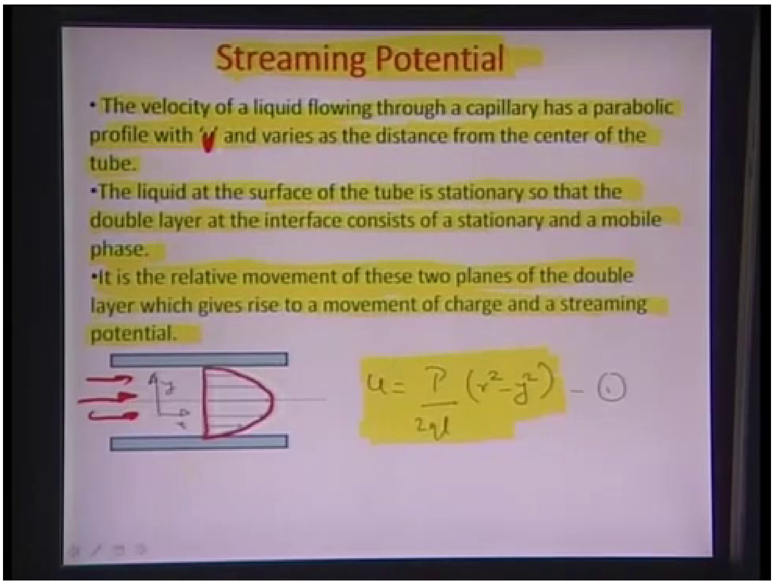

$$\zeta = \frac{\pi^2 P r^4 k}{2 Dil}$$

So, in this particular case and I need to actually substitute the value of k here we get t equals zeta D i l divided by pi square r 4 and k essentially i by q x and q is pi r square, so i times of pi r square and then, enumerator times of x. So, we can actually do this cancellation here or 4 cancels into r 2 and pi cancel out. So, we are left with essentially the terms zeta d l x divided by and of course there is going to be 2 here, which we forget, because essentially there is a 2 here in the denominator. So, there is a 2 D i l in the denominator here. So, twice zeta D l x divide by pi r square is, what this effectively would look like r i. So, essentially one important thing here is that pressure that is generated from a flow channel with the zeta potential, let say see is also inversely proportional to the area of cross section of the particular channel. So, the pressure head P important by the electroosmotic flow can be equated using kind of a formulations which I would just like to right.

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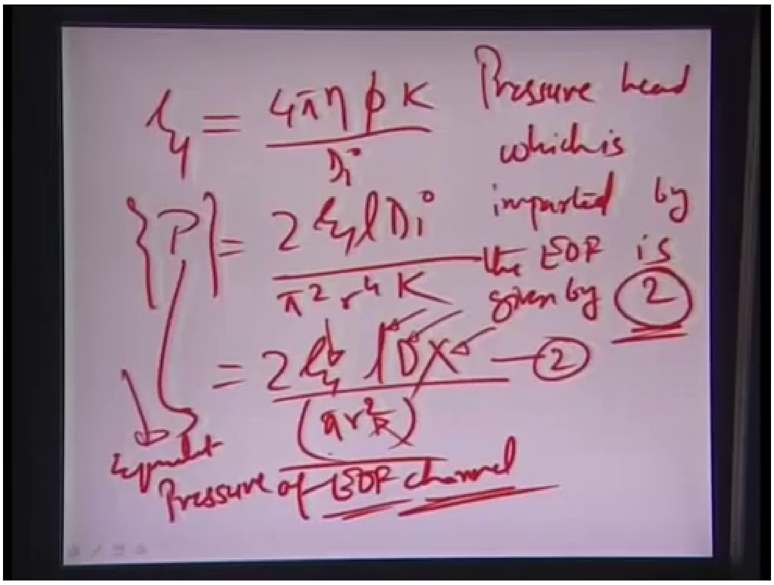

$$\zeta = \frac{4\pi\eta\phi k}{D_i}$$

$$P = \frac{2\zeta l D_i}{\pi^2 r^4 k} = \frac{2\zeta l DX}{\pi r^4 k}$$

So, therefore, in summary would like to write down that the zeta potential is equated to 4 pi eta phi k by D i is also equated to the pressure as P is twice eta l D i divided by pi square r 4 k is also equated again as twice zeta l d x twice zeta l d x divided by pi r square the area of cross section, this is essentially the equivalent pressure of an EOF channel. So, you can consider the amount of pressure head that is important on to the fluid by virtual of the EOF of the electroosmotic flow is essentially this P.

So, I can say that the pressure head, which is important by the EOF is given by equation 2. So, there can be design problems, where in you want to find out, what kind of pressure you know I any EOF flow gaps. And, so essentially you need the parameters like zeta potential of the channel the length of the micro channel the dielectric constant of the medium the field, which is a cross this channel and then, find the radius of the capillary micro capillary into question.

So, let us shift our attention to the second electrokinetic phenomena there is the streaming potential as we talked about before the velocity of a liquid flowing through a capillary as a parabolic profile with y, which where is that the distance from with v that, where is distance from the center of the tube to the sides this is essentially is v this is v the velocity. So, if you just remember back the various ways and means of a predicting you know the various electrokinetic properties there can be a case, where you produce or give pressure driven flow and it creates a set of charges on the walls are set of currents and there is a case when you apply and an EMF from outside and it generates a flow.

So, in one case the causes the EMF and other case the causes the flow. So, flow generates EMF inside the channel a flow generating EMF inside the channel the phenomena is also known as streaming potential. So, the liquid at the surface of a such a flow being stationery leads to the double layer at the interface consisting of a our stationery phase and a mobile phase in the solution and the relative motion and movement of these two planes of the double layer one stationery with respect to the wall and other moving with the fluid and would give raise to the movement of the charge and generate a potential.

So, let us derive what this potential level would be in such a micro channel case. So, let say we have a parabolic flow taking place within this particular architecture here and the velocity flow profile is also indicated by this particular parabola maximum velocities at the center and then, this velocities at both sides of the channel essentially in the no slept zone the 0. So, u here the velocity is a kind of also defined as P times of r square minus y square by 2 eta l, P is the pressure r square minus y.

So, this is a again what we find out from the parabolic flows P by eta l r square minus y square. So, if we put this liquid through a pressure difference or push this liquid through a pressure difference across this channel there is going to be a double layer, which is generated.

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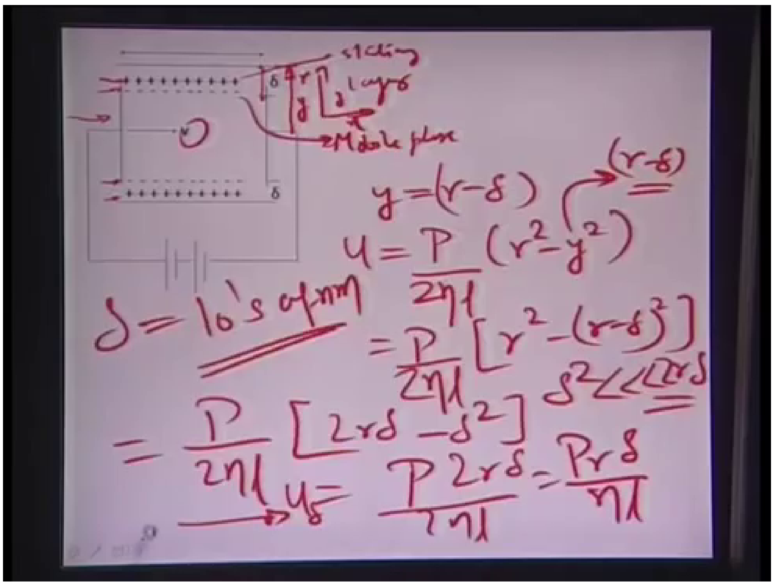

$$u = \frac{P}{2\eta l}\left(r^2 - y^2\right) = \frac{P}{2\eta l}\left(r^2 - (r-\delta)^2\right) = \frac{P}{2\eta l}\left(2r\delta - \delta^2\right)$$

$$u = \frac{P\,2r\delta}{2\eta l} = \frac{Pr\delta}{\eta l}$$

In that case we can model the flow pretty much like this figure right here. So, if you see here there is a double layer of charges, which are created on both sides one is a stationery layer, which is the positive layer in this case. So, this is essentially the stationery layer of charges and the other is a mobile phase, which is the negative you know charges on the solution. So, whenever there is a fluid flow by means of a pressure these charges this positive layer being stationery and the negative layer been mobile them move relatively with respect to each other.

So, let us assume that this delta be this thickness of the double layer and let us also assume that r be the radius of a this particular channel. So, essentially r is the distance from the center of the channel all the way up to the wall of the channel and we decided for a parameter x here, where x equal to essentially r minus delta and in this particular case the u the velocity, which is actually represented as P divided by 2 eta l times of r square minus y square can be represented in this case if I substitute the y by the equivalent r minus delta.

So, this is the actually, let us say this is y in the notational just for notional consistency this direction in the y direction in this is the x direction. So, this is actually y and this is essentially is also y equals r minus delta for notional consistency. So, r square minus y square can be represented as P by 2 eta l times of r square minus r minus delta square. And if we do this if you try to calculate this we are left with 2 r delta minus delta square and delta being very, very small we have already mention that essentially the delta charge the you know the dual layer thickness is around tens of nanometers.

So, therefore, this delta square is very, very, very small in comparison to twice r delta can be say for in the neglected and the u essentially in this particular case would be nothing but, P times of twice r delta by 2 eta l r P r delta by eta l that is what the u is going to be the u delta is going to be in this particular case the velocity was this double layer.

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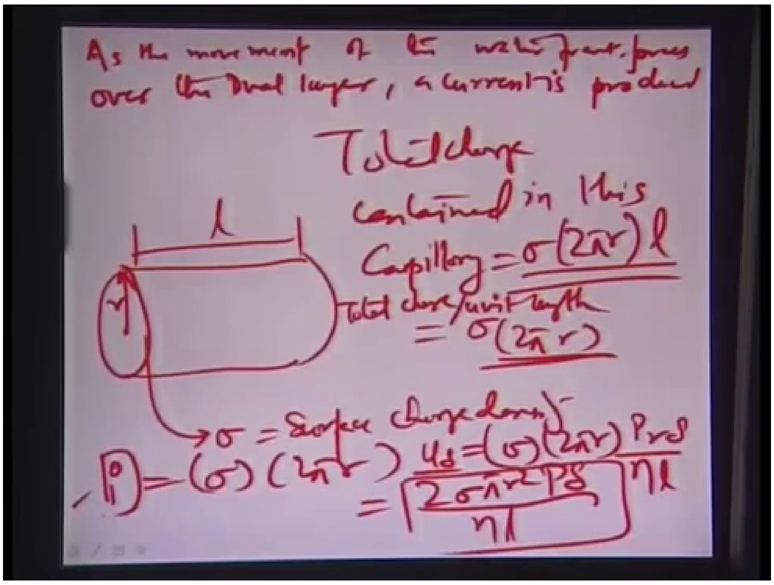

$$Capillary=\sigma(2\pi r)l$$

$$i=\frac{2\sigma\pi r^2 P\delta}{\eta l}$$

Now, as a we need to consider there are the moment of in front of a liquid forces over this layer of charges are, which on the surface essentially produce as a current. And this current is also given by the product of the total charge around a unit length of the tube and the velocity. So, if you have let say certain surface charge distribution in terms of charge permit area sigma and in a unit length; that means, you have the charge density in terms of permit surface area, but essentially the total amount of charge we are considering on the on a unit length.

So, that particular charge times of velocity; that means, how many such unit lengths the relative moment of charges happening over the surface would come price of the current. So, let me just explain this in a little more bit, so let me explain in this little bit details here that suppose you have a capillary of length l and radius r, which is the radius r length is l and we have a surface density of charge inside this capillary a sigma surface charge density.

Now, the total charge contained in this capillary is given by sigma times of 2 pi r times of l 2 pi r l to surface area the internal surface area of a the particular capillary. So, if I consider a per unit length of this total charge. So, the total charge per unit length that comes out to be sigma 2 pi r and this essentially if multiplied by the velocity would mean if suppose velocity is x meters per second, so this per unit length moves, so many times in a second. So, essentially this charge per second and that is, what current is defined as.

So, therefore, let me just right this down in totality as the moment of the water front forces over the dual layer a current is produced and this is also given by produce of the total charge around a unit length of the tube and the velocity of the moving part of the layer. So, the total current i in this case would be sigma times of 2 pi r charge for unit length times of lengths per unit time u delta substituting the value of u delta from the derivation made earlier we are left with u delta is P r delta divided by eta l as you found out from the case of the pressure different flow. And therefore, this current i in the case is nothing but, twice sigma pi r square P delta divided by eta l.

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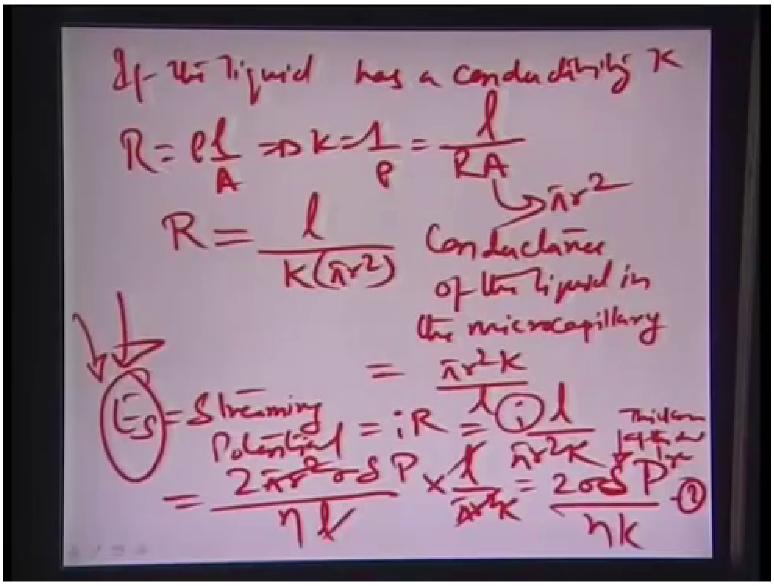

$$R = \rho \frac{l}{A}$$

$$R = \frac{l}{k\pi r^2}$$

So, let us assume that if the liquid in question as a conductivity k liquid has conductivity k and we already know the relationship R equal to rho l by A and essentially k, which is 1 by rho is nothing but, R l divided by R A. So, A in this case of course, nothing but, the radius square times of pi in the cross section area of the circular capillary and essentially the k the resistance R of the channel is l divided by k times of pi r square.

In other words, the conductance, which is the reciprocal of resistance of the liquid in the micro capillary is just the inverse of that, which is pi r square k by l. So, the streaming potential E S develop, because of this current and current is formulated by the motion of the charge across the dual layer is given by i times R i times l by pi r square k. And if we further try to put the value of current here from the previous derivation, what we which we obtained as 2 sigma pi r square P delta by eta this term here was the i.

So, here the final expression would come out to be 2 pi r square sigma delta P by eta l times of l by pi r square k these go off and the l's go off and we are left with the term twice sigma delta P by eta k, so that is, what the streaming potential would really be. So, one important point here to be mention is that you easily find out from this potential the length delta the thickness of dual layer, which can be of immense utility in almost all electro chemistry.

So, that essentially is what the streaming potential is in case of a flow just flow and through the pressure radiant over a surface over a dual layer of the surface. So, when we talking when, so we have already derived the streaming potential here is in terms of 2 sigma delta P by n k. Let us modify a little bit of in terms of the zeta potential try to involve what the zeta potential of a such a channel would be the surface would be.

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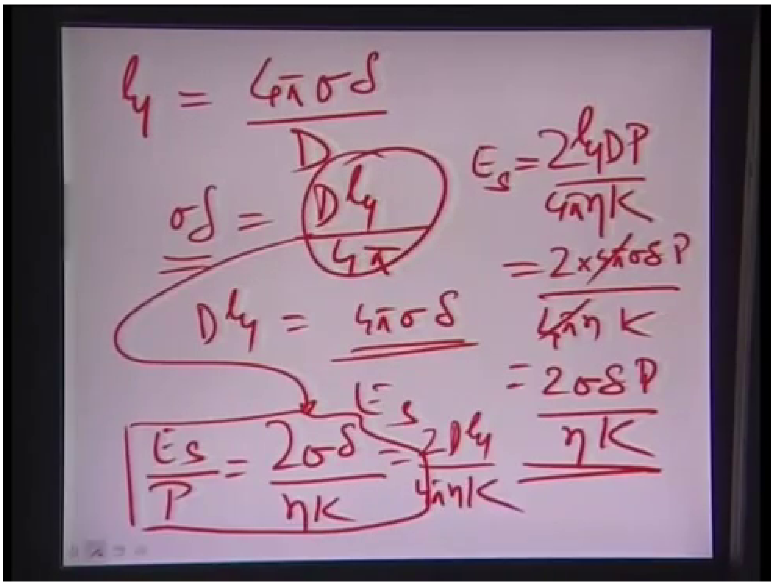

$$\sigma\delta = \frac{D\zeta}{4\pi}$$

$$\frac{Es}{P} = \frac{2\sigma D}{4\pi\eta k}$$

So, as we know from earlier equation that the zeta potential of a such a system of micro channel in contact with the liquid phase is also given by $\zeta = \frac{4\pi\sigma\delta}{D}$ sigma is surface of chagrin delta is a double layer charge, double layer thickness D is the dielectric constant of a the medium of interest. So, if you just do a little bit of mathematical manipulation here sigma s sigma delta comes out to be equal D times of zeta by 4 pi and essentially if you have had a look earlier at the E S the streaming potential it came out to be twice zeta the P the pressure divided by 4 pi eta k.

So, if we just try to substitute the value of this delta in to sigma in to this particular equation here left with the let say D zeta is also equal to 4 pi delta sigma. And essentially from the E S value if you substitute this 4 pi sigma delta P divided by 4 pi eta k, k you are left with twice sigma delta P by eta k. So, essentially that is what E S are streaming potential good result in and we can also write this down as E S streaming potential per unit presser, which is given as difference between both kinds of capillary, which is cosign it flute to is also twice sigma delta divided by n k. And essentially that is what is the final form of the ratio between the streaming potential the special.

So, another interesting think here to find out is that of you really want to write this whole terms, in terms of quantities, which are measurable. So, let us actually try to understand this more in terms of measurable quantities like a flow rate phi, then you know thinks like measurable quantities like current across the micro papillary so on, so forth. So, a would be a little modification though, which will be needing here, let us re substitute back this value of sigma delta back in to this equation you are left with essentially twice D zeta divided by 4 pi eta k.

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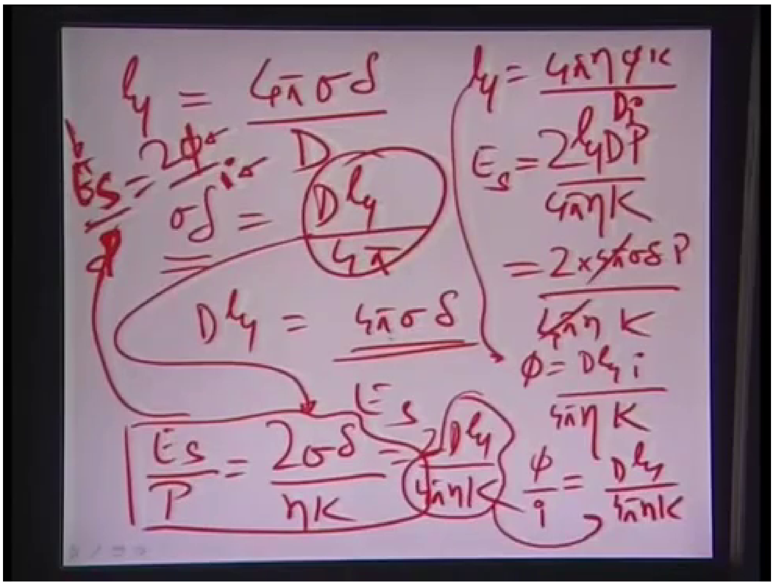

$$\zeta = \frac{4\pi\sigma\delta}{D}$$

$$\sigma\delta = \frac{D\zeta}{4\pi}$$

$$\frac{Es}{P} = \frac{2\sigma D}{4\pi\eta k}$$

$$\varnothing = \frac{D\zeta i}{4\pi\eta k}$$

We can easily find out a correlation between the potential phi the flow rate phi and in the current i by looking at the equation that we had the right earlier, which talked about the relationship between the zeta potential and these other parameter 4 pi eta pi k divided by D i. So, if you try to you know kind of readjust this parameter shear you are left with the value of pi the flow rate as D zeta i divided by 4 pi eta k. And this can also be represented as phi by i is D zeta by 4 pi n k this quality this kind of similar what this is So, essentially E S by P here is nothing but, you know twice this quantity here 2 phi by i. So, if you can measure the flow rate in such a streaming flow and also measure the amount of current that is produced this can easily give you this radio E_S, which is by P. So, ES, S is a streaming potential and P is the pressure.

So, if the let say the pressure radiant that what driving the pressure difference that what driving the flow is P and phi was the flow rate, which was created and it generated current i easily calculate that calculate, what is the streaming potential in that particular application. So, that is what is use sometimes of sensing mechanism for investigating the combinations of surfaces and their behaviors with the respect to flowing solutions.

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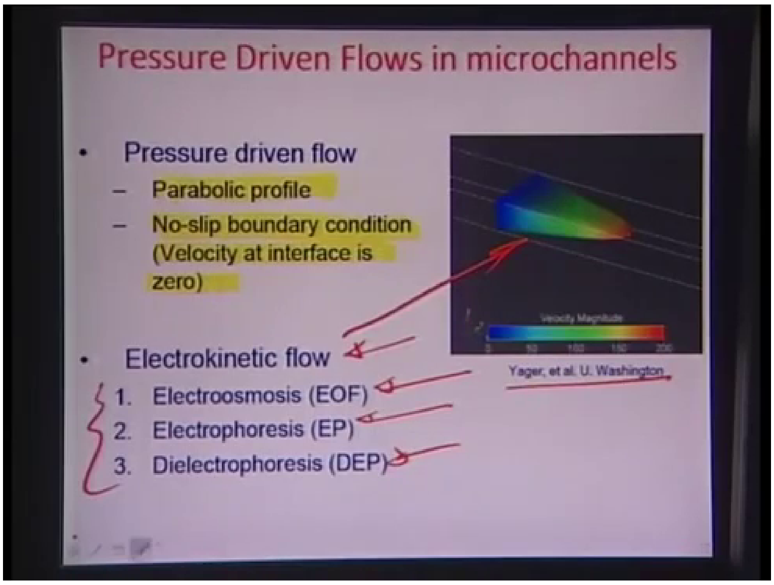

So, let us try to summaries that in pressure driven flows essentially the flow profile is parabolic in nature and there is a no slip boundary condition at all the walls are the edges of such channels the flow profile pretty much looks something like this right here right you have like parabola the velocity, which is maximum at the center and, which is actually 0 at all these difference walls.

So, electrokinetic flows in the other hand electrophoresis electroosmosis and dielectrophoresis and these are some of the mechanism for doing like electrokinetic flows this right here is the simulation by Yager's group apart the University of Washington.

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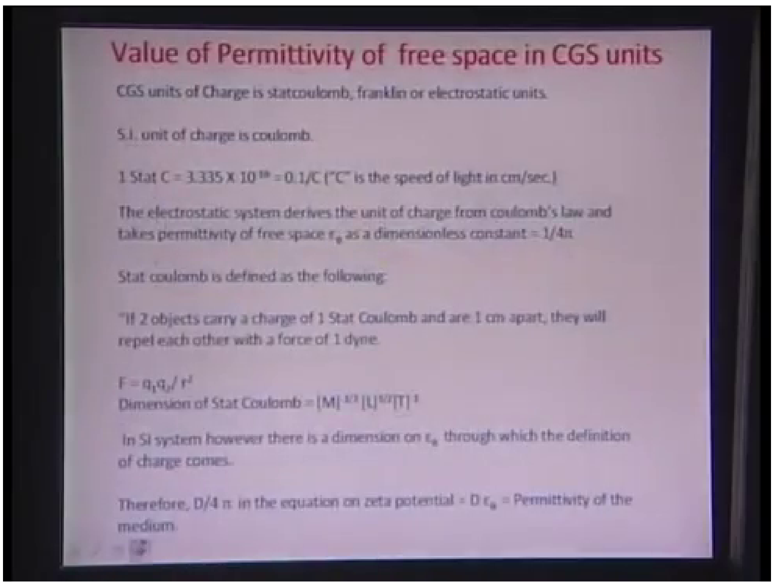

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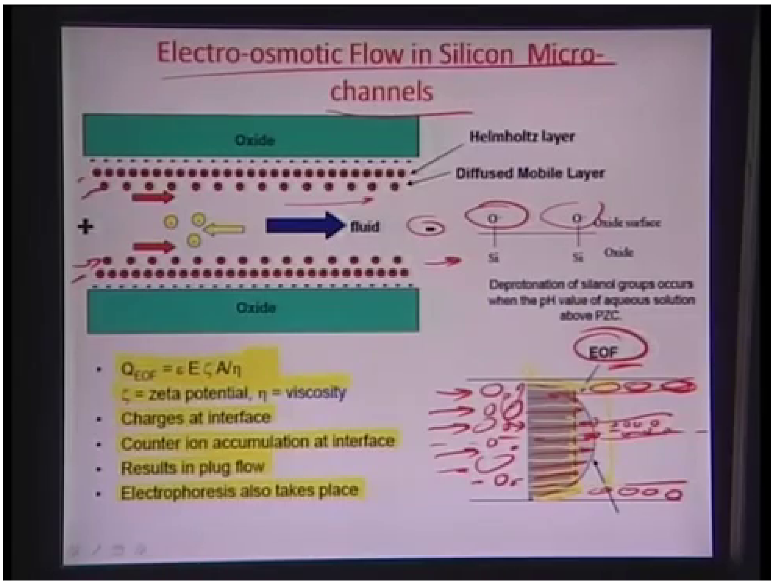

Essentially, if you just compare you know the flow profiles, which are generated from EMF are parabolic flow if you may recall in a pressure driven flow, the flow profile is something like a parabola of this sort. So, the velocity kind of the velocity vectors kind of keep on maximizing as they go from walls towards the center of this particular capillary. On the other hand for the EOF flow or the electroosmotic flow as we know the flow really takes place as a pluck flow from the start of one double layer, which is essentially the this side here all the way two the other double layer, which is in the other side of the channel.

So, this area is having a constant velocity, so let me just use a different color here to represent a flow rate related to are the flow velocities related to the you know this the electro somatic flows. So, if you see here as represented by these yellow color arrows these are the flow vector between the double E S essentially in an electro somatic flow. So, it is like a flow all the velocity is between these two double layers starting from here all the way up to here are uniquely similar to each other suppose to the parabolic, where there is a slow increase of velocity as it goes from sides all the way to the center.

Other interesting point here is the behavior the flow rate around that the surface in question till the double layer starts. So, this is essentially the Helmholtz plane we have been talking about this a lot of none the Helmholtz plane. So, this particular layer here as you see is though having kind of parabolic profile, which means that this is the layer, which is shear to give way to this pluck like flow. So, the velocity really close to this surface here is 0 and the velocity here some value maximum value v and the flow profile between two are really parabolic and both sides.

So, that is how we interpret both these electro somatic flows and the parabolic flows, now there is there are several important issues, which immanent from this that the electrokinetic flows being plug like flow there is always almost you know a continuity of or a uniformity of velocity vector across such a channel. For characterization say particularly when you do particle image velocimetry it may be of immense utility if you in the flow across section of such a channel is having all uniform velocity.

This also would mean that these kind of flows do not lead to any particulate separation particularly when the particles concerned and all different sizes. So, if there are parabolic flows as we have seen in cases of channels, which are pressure driven essentially one aspect that comes in to picture is that if suppose there are different set cells of all different sizes moving across such channels with the, the continuity and with this velocity variation in the parabolic profile of the heavier masses tend to move to towards the psi.

The lighter masses tend to kind of move more towards the center by the principle of conservation of momentum ultimately the retain at here at the center as all consisting smaller entities and the retain at the sides here all containing the bigger entities there is known as the this also known as a margination in the human body the micro capillary is essentially do this job, where leukocytes, which are heavier in comparison to the RBC's would migrate slowly to the walls of the capillary and would be rich on towards the vasculators walls essentially.

So, that is sometimes an advantage, but then in some cases it may be desirable to move these kind of different sizes and masses throughout the micro channel uniformly without the separation coming in to picture. So, electrokinetic flows are the best in those kind of cases, because the essentially do not have any you know any velocity variation it is a plug like flow like behavior. So, this kind of talks about, what happens on a silicon dioxide surface have been repeating at off and on.

So, you can see here then a silicon dioxide surface first of all the surface gets hydrolyzed and then, later on forms minus and there is a negative charge on the surface due to, which there is a positive charge of counter ions, which is developed in the bulk as bulk charges are defused layer charges and then, when you apply a potential across side all these bulk charges try moving towards negative electoral cross dragging fluid around it. And, so therefore, electro-somatic flow in silicon micro channels is great area of study..

So, some important observation here a summary the fluorides of such flows like propositional to the dielectric constant epsilon the electric field external to the channel the zeta potential of a surface and also propositional to the area and inversely proportional to the viscosity of the particular medium, which is flowing through these surfaces the. So, essentially depends on the charges at the interface and there is a counter ion accumulation and, which actually drags along the fluid along with it and results in a plug like flow. Electro produces on other hand is motion of touch particles directly in a fluid medium and will be dealing with this little bit later little more details.

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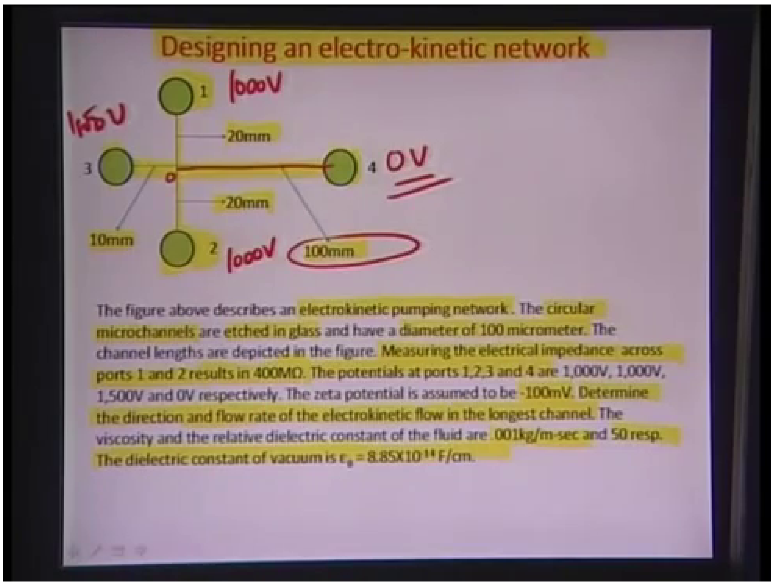

But, let us look at one illustration here where we can use this electrokinetic techniques for designing micro pumps. So, in this particular problem here we want to design somehow electrokinetic pumping network. So, using this principle of double layer and formulation charge etcetera on the surface in the bulk of the fluid we would like to design a pump network here. So, the dimension there are given is basically that the circular that the micro channels are circular in nature they are etched glass of course, so they will be a SIO minus layer and they have a diameter of about 100 micro meters.

So, this here and this right here are the two channels and the channel lengths are a actually depicted in the figure. So, the length here is 10 mm between let us say 3 and there is a point of intersection let say 0. So, between 3 and 0 the length of the channel is 10 mm between 4 and 0 it is 100 mm between 1 and 0 and 2 and 0 both are same 20 mm each. So, that is how this public network is being late out these essentially are the reservoirs 1, 2, 3 and 4 and we have the following measurements.

So, if you measure the electrical impedance across ports 1 and 2; that means, this port right here and this port right here it results in 400 mega arms of resistance and the potentials at ports 1, 2, 3, 4 are respectively 1000 volts 1000 volts 1500 volts and 0 volts respectively, so let me just right that down here. So, the potentials at port 1 is about 1000 volts of applied potential same goes to for port 2 another 1000 volts and then, a port 3 it is about 1500 volts and port 4 is about 0 volts.

So, there is essentially no potential applied on port 4. So, the zeta potential here is assume to be about minus 100 mill volts as can be defined by the surface in connection to another liquid phase and you have to determine the direction on the flow rate of the electrokinetic flow formulated in this case the longest channel that is the one, which measures about 100 millimeters are this channel between 0 and port 4. So, the viscosity and relative dielectric constant of the fluids are given to be 0.001 k g meter second and 50 respectively and the dielectric constant of vacuum is found to be 8.85, 10 to the power of minus 14 farad per centimeter.