BioMEMS and Microfluidics
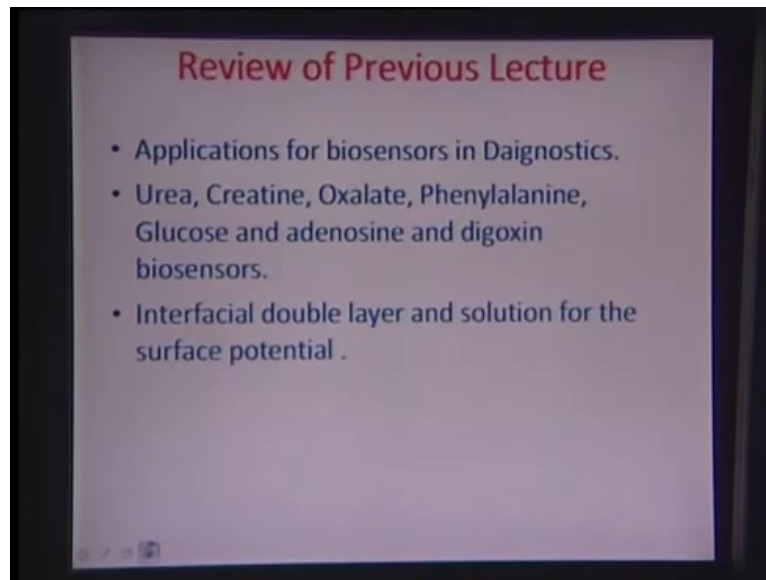**Prof. Dr. Shantanu Bhattacharya**
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Lecture – 12

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So, in this lecture 12 talking about the last lecture we try to cover the following topics; Applications of biosensors in diagnostics industry. Particularly for metabolize of interest like Urea, Creatine, Oxalate, Glucose, adenosine so and so forth. The different gas based sensors which are used for taking blood sample, doing quick analysis of one and more of these metabolize in the patient's blood sample. And essentially we also try to investigate details about how the double layer are between two phases of a solid, so solid in liquid or liquid in liquid formulates; and how there is a charge separation which would need to distribution of one phase as a counter ion charges in as bulk charge defuse layer within the, within one of the phases, one of the interacting phases.

So now, we would look into, and so we actually try to find out also the potential in one-dimensional model of an electrode contact with the solution and try to estimate the value of the potential from by solving the one-dimensional equivalent of the Poisson's equation, electric field, the forward for potential.
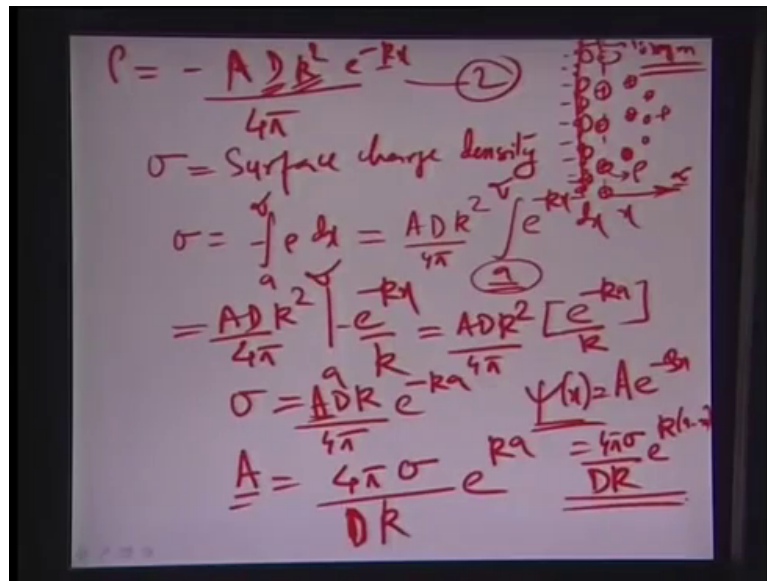
So, essentially we got density factor, the charge density, volume charge density rho; equated to

$$\rho = \frac{-ADk^2 e^{-kx}}{4\pi}$$

These the dielectric constant, k again is a constant which was put as an equivalent of the total amount of charges e^{-kx} ; x is the direction in which the potential has to be evaluated. Let me just write this little more clearly here, make it x and a of course, is the constant in the solution of the potential equation respect to x. So, if you really look at the surface charge density and the we also try to discuss this, that you know in the electrode a very critical parameter of importance's. What is charge density in the surface of electrode? When we are talking about micro flows, micro fluidic channels, the charge density essentially means the density of the dangling bonds which are there on surface. If you have a silicon surface and you s i o z, and get it converted into s i o h and then under certain p h this converts into s i o minus; the you know the charge density on the surface of such a situation would essentially be a function of the bond density on the surface 2.

So now, if you look at or if you assume sigma to be the surface charge density of an electrode. So, the relationship between sigma and rho would really be

$$\sigma = -\int_a^\infty \rho \, dx = \frac{ADk^2}{4\pi} \int_a^\infty e^{-kx} \, dx$$

One thing which I would like to illustrate here, is how this equation is coming out from the

fact that if you look into the principle of electrical neutrality. The charge which is there on the surface of the electrode should be equivalent to the total amount of charge and the volume of the solution which forms an interface with this particular surface. So, with that logic the surface density should be the negative of the volume charge density of the solution integral of that respect to the distance x from the electrode. And if you may recall the outer Helmholtz layer is certain distance equal to let say some value a which is the ion size parameter also. We can consider that to be what know is a beginning of this let say this is so called solvated ion cell, and then beyond that the potential extends solve the way up to infinity. And so, there is always density of charge mathematically possible up to infinity. So, charge density goes down of course, with increase in x, but it is almost 0 it goes to a certain finite amount which is close to 0.

So, with that logic we can say that sigma is minus rho D x integral, x varies from a to infinity. So, if you put the value of rho here from equation two and try to solve this equation. We are left with sigma equal to $\sigma = \frac{ADk^2}{4\pi} \int_{a}^{\infty} e^{-kx} dx$. I just want to recall that, I would like to recall that a is the ion size parameter. So, if you really have an electrode with the negative ion say here and there is a plain here which contains all the different positive charges on the outer helmholtz planes. And as it goes down the density slowly reduces into the bulk volume and this distance is x. So, the x really starts from where the solvated ion shell, which is somewhere here starts which can be equivalent to the ion size parameter here a. So, from a to all the way to about infinity, we can find out what is that charge density by looking at really the rho the volume charge density and maintaining what you call the principle of electro neutrality.

So, these are all positive charges and these are all the solvated ion, solvated shell thin water molecule by the by this layer is only about 10's of nanometers. So, it is extremely small 10's of nanometers, you may clear about 10 nanometers. So, solve this integral here you are left with $\sigma = \frac{ADk^2}{4\pi} \left[\frac{-e^{-kx}}{k} \right]_{a}^{\infty}$; and on this will eventually convert into $\frac{ADk^2}{4\pi} \left[\frac{e^{-ka}}{k} \right]$. On other words find the converting into $\frac{ADk}{4\pi} e^{-ka}$. That is what this is, and from that we can estimate by looking at the surface tendency sigma, the value of A in the coefficient in the solution of in the coefficient that solution of the bosons equation.

$$\Psi(x) = A e^{-kx}$$

So, A here would be represented as $A = \frac{4\pi\sigma}{Dk} e^{ka}$. That is how A can be represented, and

essentially in the potential function x which is also $\Psi(x) = A e^{-kx} = \frac{4\pi\sigma}{Dk} e^{k(a-x)}$

D again is the dielectric constant of the medium, in which the solution is presents, the solution normally is a echo's medium also the medium is water and dialectic constant would be the dialectic constant of water in CGS units. That is what the potential function $\Psi(x)$ could be. Interestingly it would be almost very good thing to assume what the potential would be just closed to the surface. Just close to the surface means where the ion size parameter for the negative find ends almost. So it is very closed to the electrode surface where the solvated ion or solvated shell of this thin water molecules 10s of nanometers just about start and there is where the A actually ends; the n size parameter ends. So, at that particular parameter the potential that is available really is nothing but the surface potential of the particular surface and question. And so therefore, if you put x equal to A in this particular equation for the potential equation 3, we should be able to get that as an estimate.

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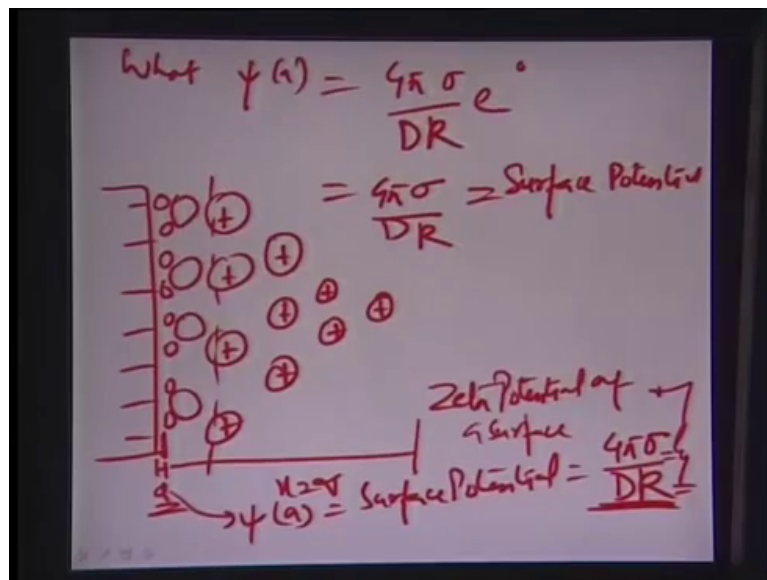
So, let us look at what in the potential function would be at x equal to a. So, should be equal

to $\Psi(a) = \frac{4\pi\sigma}{Dk} e^0 = \frac{4\pi\sigma}{Dk}$

So, this is also the surface potential close to where the salvation shell just about starts. So,

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this is the surface of interest and this is a salvation shell or a molecules who is start in this particular region. And this is let say the positive charges in the solution, and there is a reduced order of this charges it moves more in to the bulk. And essentially goes all the way up to 0, and so the electrode here is negatively charged and the ion size parameter can be somewhere very close to this surface let say this a. So, between a and x equal to infinity there is a charge distribution, but at a, the potential really is the surface potential. So, this concept should be very clear in mind because this, we need to use or will be using very often for determining what you call electro kinetic flows in some of the next modules.

So, we call this zeta potential of zeta, of $\Psi(a)$. So, we call this the zeta potential of a surface. There ways and means now of measuring the zeta potential of surface of surface of interest. Initially for any solid surface the zeta potential would be a very hard parameter to find out, but there are meters now available where this a measurement can be made. Interestingly also for a nano particles, particles who sizes are lesser 100 nanometers. The surface potential forms a very interesting you know aspect, because it always is a result or it is it always is leading to the discretization of this particles and it always prevents aggregation.

So, very important aspect making nanoparticles, this how we can change the zeta potential of the surface and you can use variety of surfactants or a variety of materials which kind of code the surfaces in a manner that you have a certain potential associated with the surface. And if you are have neatly similar potentials across all the other nano particles which are all have hardly calculate. So, just give you an idea of how important this zeta potentials whether it is the plain surface of the surface of a nano particles or any other surface, zeta potential extremely importance. So, with this we find out have very good idea now, how surface would behave with respect to you know the solution.
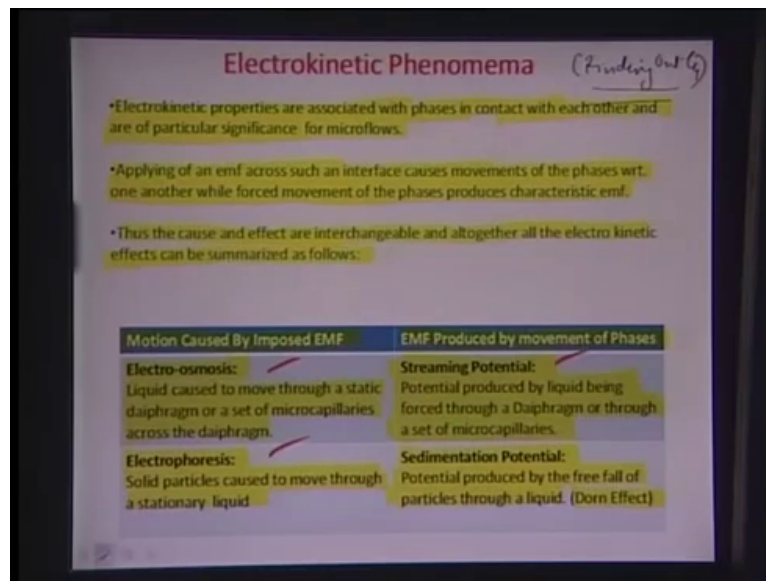
I would now go to the directly almost into the various electro kinetic phenomena there are available and that can be used for a lot of a you know micro flow related situations. So what is, what are those electro kinetic phenomena? So, let us look at some of the definitions in order to begin with this new topic and it is very much related to you know the earlier topic on zeta potential of a particular surface. So, electro kinetic properties are associated with the phases in contact with each other and order particular significance micro flows. So, whenever there are two, two or more phases which have been contact with each other there is almost always. This property of charging which comes into picture. So, one or more are these interfering or participating surfaces, interfacing surfaces develops some kind of a charge.

So, applying an EMF across such an intercept, interface causes movements of all the phases with respect to one other. While force movement of phases produces characteristic EMF. So, thus the cause an effect are interchangeable and all together all electro kinetic effects can be summarized as an either motions caused by imposed EMF, or EMF produce by the movement of phases. So you know one cause or in one kind of phenomena you impose in EMF or put a potential or subject a potential and try to study the flow or the most in to the charges are the charge separation of the double layer which happens. And other instants you have basically producing an EMF by forcing a certain interface over in other which double of its dual layer charging.

So, there are 4 different mechanisms which can categories into these two sub types: one is motion caused by the impose EMF, this category involves or it includes electro osmosis. And definition really electro osmosis really principle by which liquid cost move through a static

diaphragm with micro capillaries across the diaphragm occurs. So due to electro osmosis it generates some kind of driving pressure which causes liquid to move automatically through static diaphragm or set of micro capillaries. So we will investigate this process little bit in details in the following slides. The other phenomena of motion caused by imposed EMF is really a motion of charge particles in medium, on subjecting to an electro fields. So, this is also known as electro freezes.

So, solid particles caused to move through this stationery liquid when it is subjected to such an external EMF for such an external electric field. In other 2 phenomena of importance and these 4 phenomena really important for sensing and micro fluidics applications. Is basically those which are categories in the sub type of the case where EMF is produced by forcing using pressure driven flows one fluid or one interface on other surface. So, one such phenomena is call this streaming potential. So, it is a potential produce by liquid being forced through a diaphragm or through a set of micro capillaries in the diaphragm. So, if you are forcing this liquid phase over solid phase essentially, with some kind of a diaphragm which have all this micro capillaries. So, you have forcing them with several micro capillaries, it causes of potential because of the flowing fluid, it is also known as streaming potential.

The other important factors sedimentation potential where is used very often in the mechanical industry. So, this is the potential produce by the free fall of particles through liquid. So, it is a gravity driven fall of particulate matter in a liquid we should cause certain amount of potential, then this can given information about the weight of those particles are the rapidity which they would sediment a settle down. This is also known as down effect. So, we would be more interested in studying just few types, actually these 3 in more details about how a motion can because by a EMF or EMF can be call by motion.
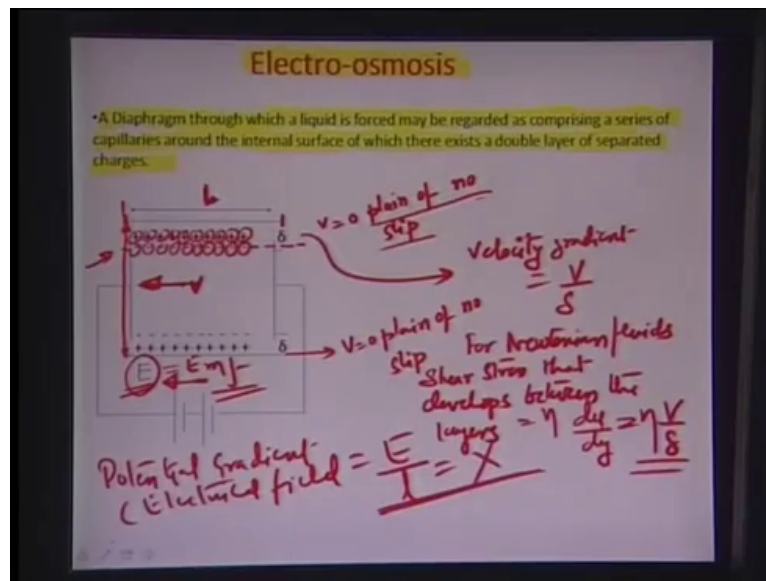
So, let us looking to a little more details of the first process that is electro-osmosis. So, by definition again a diaphragm through which a liquid is force maybe regarded as comprising of series of micro capillaries. So, we have a situation here, where you are pushing fluid through set of micro capillaries. So, let us look into this first process electro-osmosis and as you know by definition electro-osmosis really is a case when liquid is push through a set of micro capillaries. Diaphragm it can be consider as a bunch of different micro capillaries through which a liquid is forced into. So, this around the internal surface of this capillaries there exists a double layer of separated charges because of the flow of fluid which is push through this set of micro capillaries. So, this situation can be represented here as you can seen in this figure here; that you have a flow which is of this water and this is let suppose the span of the capillary. So, this essentially is the span of the particular micro capillary in to question, let me just put let more appropriately from here all the way to here; is what the radius of the capillary is? In the length across this you want to investigate this dual layer charging let say L from one into the other end of the capillary. So, what would happen immediately is as you know that these are pressure driven flows and it causes and no slips on. So, very near to the surface in this particular area regime there would be 0 velocity, so there is a plane of no slip.

That is true for this other end of the surface is well. Where there is no velocity or there is no slip between in the fluid layer and the liquid. Now let suppose in the dual layer really formulates a little bit away from this particular positively charge surface, let suppose that the charges on the surface are all positive in this particular case and the dual air is formulated by this negative charges which almost always comes in to this solution and extends back in to

the bulk. So, you have to be careful though in this case that you know positive really means electron less. So, there are the surfaces divide of electro-osmosis negative in the solution would be always in an ionic states. So, the ions are almost always bigger then if there are holes. So, the same phenomena would happen if the electrode is a positively charge electrode and the surface or the solution has negative which are ions.

Whatever is in the solution is of a bigger size if you consider the charge equivalents, there is almost always the formation of a diffuse layer; because whatever is in the solution whether it is a positive charge or the negative charge is always going to be of bigger size. If the electrode is positive, it is by positive by virtue of the electron because it is electron divide, by virtual of being electron divide. So, essentially it is whole, it is an electron less state. So, therefore, there is no way almost that you can neutralize in the total charge whether it is positive or negative in surface of the electrode, by the charge that it is an a solution. Solution charges are almost always of finite size, whether it is a positive or negative ion. And a negative ion is much much bigger than the neutral state of the species itself. Whether positive charge is a little smaller, but then it is no way comparable to the size of an electron. So, there is almost always the development of defuse layer of charges.

So, here we consider let say an EMF which is applied external to this capillary. So, there is an EMF because of by virtue of this battery which is applied as you can see here, by means of series of batteries in the circuitry. So, what you thing will happen? There would be a tendency of these negative ions to move towards the positive electrode. So, as the fluid is moving in this particular direction with you know V N, it has occupied the whole capillary and after that let say we have remove the pressure, and so there is no driving of the fluid anymore. When we apply the EMF in the negative charges should start flowing towards the positive of the battery, and let us assume that in the velocity of flow in this case is V. So, let say that the velocity of flow in this particular case is V, because it is a negative charge and it is flowing in the opposite direction this velocity is V. So, from a point of no slip very close to this surface, this particular outer Helmholtz layer where the first line of charge in the counter ionic charge comes into existence and the salvation here just extended moves at a velocity V; so the velocity gradient here. If we assume in the total double layer thickness to be delta, so the

velocity gradient is essentially $\dfrac{V}{\delta}$.

We know for Newtonian fluids. In the shear stress which develops between the different

layers, is actually equivalent all its proportion to the velocity gradient; $\frac{du}{dy}$ in this case. And the constant proportional it is there nothing but then none other viscosity of the medium in the eta, so that is what Newton's law state. Let me write this more clearly viscosity to the velocity gradient.

So, let us think about this case here where this particular Helmholtz layer has started moving. So, before it got to start to move it would have to undergo the necessary shear stress to create this velocity gradient $\frac{V}{\delta}$. So, essentially we can calculate that you know this in this particular case can be represented as $\eta \frac{V}{\delta}$. So this is the amount of stress that is needed, the amount of stress or amount of force per unit area that is needed. To shear the out the outer Helmholtz plain with respect to this layer of no slip, which is present close by a you know salvation cell close to the surface. So, we have single molecules of water which is very close to these bunch of positive charges and that is the inner salvation shell with separates on the counter ionic cloud the negative counter cloud from the positive vacancies on the top of the electrodes. And so, V by delta is essentially is the velocity gradient and eta times of V by delta is what the shears stress would be for such a layer separation to occur, when the fluid starts moving.

So, let us assume also potential gradient and if the length here is L and you have applied voltage between these two points same as the voltage given by the batteries here as E. Then the electric field in that case on the potential gradient which is also the electric field, electric field is E by l. E is the EMF, you have to remember this is not the electric field but the EMF the voltage that is applied between the two points of the capillary. This end and the other end here, of the capillary. So, that is what the electric field really S, let us delete this portion away, that is what the electric field really is between these two ends. So, this can be represented by some term X here. So, in order for the potential gradient to move the fluid the shear stress should really be equal to the electrical force to the charge per unit area.

So, let us suppose the charge density on the electrode surface, in this case the surface of the channel or a micro capillary is a sigma. So, sigma times X really is nothing but, the electrical force to the charge per unit area. Let us assume, let us let us try to imagine the dimensional consistency here is sigma here is charge per unit area, it is the surface charge density and X here is the electric field. So, electric field which is X in this case times of charges a forces and so this is force per unit area. So, in order for this layer to be mobile the total amount of electrical force per unit charge area to the charge per unit area which is needed would be equivalent to the shear stress that would allow the separation to happen and the fluid in the micro capillary to move ahead. So, all set in done, we can find out relationship between sigma and all the other parameters are n V by delta X here. And also from the earlier derived equation on zeta potential that we have found out just about few slides back.

$$\sigma X = \eta \frac{V}{\delta}$$

$$\Rightarrow \sigma = \eta \frac{V}{\delta X}$$

As zeta came out to be equal to

$$\xi = \frac{4\pi\delta}{D}\sigma = \frac{4\pi\delta}{D}\frac{\eta V}{\delta X} = \frac{4\pi\eta V}{DX}$$

, which is the zeta potential on the surface.

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So, the velocity of flow in this case V would really be equal to nothing but zeta potential times dielectric constant times the electric field, external electric field times of divided by 4 pi into the viscosity eta of the medium.

$$V = \frac{\xi DX}{4\pi\eta}, \text{If } X = 1\, volt/cm$$

And if we assume the field to be one volt per centimeter or we assume an electric field of unity value then we are left with velocity which is also known as the mobility of electro-osmoses or the electro-osmotic mobility. $u_0 = \frac{\xi D}{4\pi\eta}$. So, it is essentially the velocity per unit electric field, so it gives an idea of how mobile the liquid phase within the micro capillary would be by this principles of electro-osmoses per unit field external field there is applied to the capillary, this is also known as u_0 .

We can further do some analysis because when we talk about micro flows and you know capillaries associated with this kind of flows, we need parameters like the flow rate. And velocity essentially is nothing but the flow rate per unit area as we all know from our fluids background. So, volume fluid q divided by let say some area would represent the velocity. So, here also if you assume phi to be the flow rate, the volume flow rate let say volumetric flow rate and let say small q to be the area of cross section of the small capillary. So therefore, V essentially is $V = \frac{\phi}{q} = \frac{\xi DX}{4\pi\eta}$

So, this is how you can represented in terms of flow rate and area of cross section, in any micro capillary the relationship between the mobility and you know the flow rate and area of cross section.

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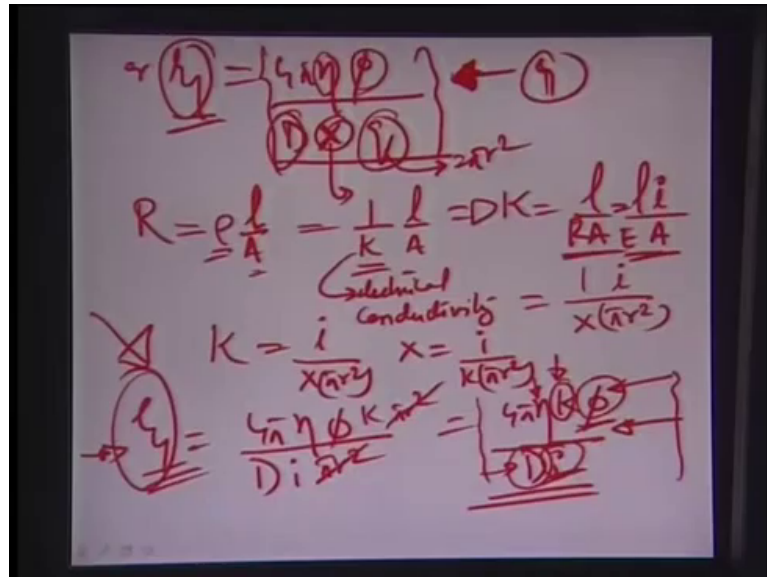
So, we can further make it or change this equation to gaze the zeta potential, zeta as

$$\xi = \frac{4\pi\eta\phi}{DXV}$$

So, these are all measurable quantities except this X here, which I would just be changing in little bit; and zeta essentially can be found out from this equation for any surface in combination with any liquid. So the, if for a certain commentarial of two interfaces we can find out the zeta potential by this method; where eta is the viscosity of the liquid phase, phi is the flow rate which is obtained because of electro-osmoses. These are the dialectic constant, q is the area of cross section of the micro capillary or the channel through which all these flow is taking place. X here essentially is the field and we would just an about little bit see how this field can be predicted in terms of current per unit the conductivity.

So, you as we know from Ohm's law, you know the way that resistance is defined is

$$R = \rho \frac{l}{A}$$

ρ is the specific resistivity, l is the length, A the particular medium; A is the cross section area of medium between two contacts, of which the resistance has to be measured. Now this

can also be written down as the $R=\dfrac{1}{k}\dfrac{l}{A}$. So, this is essentially the electrical conductivity, is the inverse of electrical resistivity. Which in this case would be close to; so we can change at a little bit. So, K here can be represented $k=\dfrac{l}{RA}$ length by radius into area. And what is resistance really? Resistance is nothing but the EMF there is applied divide by the current that is created that is how you can have the resistance. So, this is I am sorry, just may little type over here.

So, the resistance is the EMF juts by Ohm's law, the EMF divided by the current i that is created in the particular medium. In this case there is a small thin liquid column within the micro capillary and you are creating or you are measuring the current i as you apply an external electric field E. So, $\dfrac{E}{I}$ is the resistance in this case. So, essentially again as you know E by l is X, and area here is nothing but let say pi r square; we assume that r is the radius of thin micro capillary. And i is the current and length here is always accommodated in this X factor. So, K the conductivity is essentially

$$k=\dfrac{i}{X\left(\pi r^{2}\right)}$$

. There is a way that we can express $X=\dfrac{i}{k\left(\pi r^{2}\right)}$. I could put this back into equation 4 here which we and that way zeta can be come out to be

$$\xi=\dfrac{4\pi\eta\phi k\left(\pi r^{2}\right)}{Di\pi r^{2}}=\dfrac{4\pi\eta\phi k}{Di}$$

Now, would like to draw your attention to that very fact that these are all measurable quantities. You could measure the current value that electric constant is almost always known K is the property of the medium there are standard tables, so that phi is a flow rate, n is again the viscosity is again the property of the medium. So, with all these different properties are aspects of the medium and assuring the you know the flow rate per unit current you could always gaze what is this zeta potential of a particular surface. On this is of immense importance in electro chemistry as well as micro fluidics. The way that zeta potential changes the you know affect the lot of behavior. So, this kind of gives us you know way out of trying

to analyze electro-osmotic flows within micro capillaries. Now let us actually also for comparison sake, try to compare at what would happen if these capillaries would have pressured even flows. So, essentially we start investigating these micro capillaries with Navier-Stokes equations for different situations of pressured raven flows.

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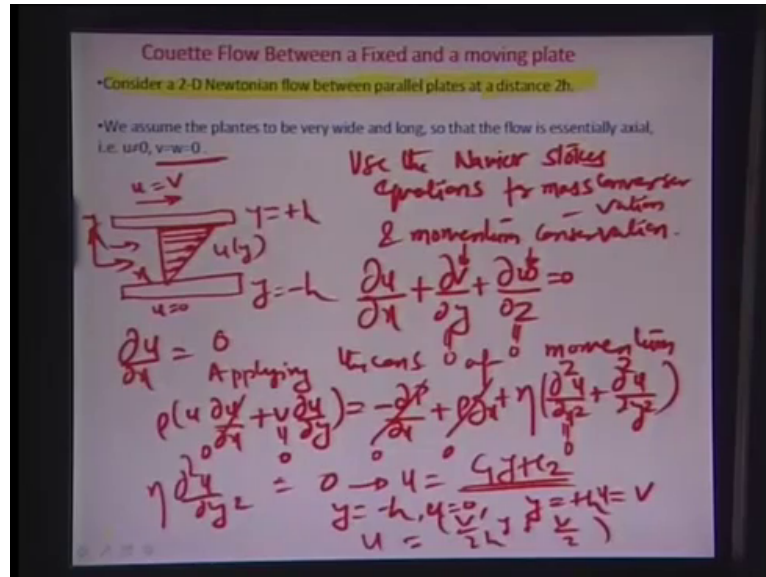
So, before doing that, just we illustrate that little bit of a conceptual you know things. We would investigate the Couette Flow, which is essentially between you know 2-dimensional plates or surfaces parallel plates at a distance of 2h; which has fluid in between. So, let draw this out. So, this is like, you have two surfaces; surface one and two, and we have fluid in between. And what you are doing is, you are trying to move this surface at a velocity V, assuming there is no slip zone. The fluid kind of develops a velocity profile starting from 0 here, V equal to 0 here. All the way to about, you let say the velocity of the fluid is a parameter u for convenience sake. So, you have u equal to 0 here, because of this zone of no slip. And then the velocity slowly increasing to equal to V the velocity if the plate here, so you have the zone of no slip here, you have a zone of no slept here. So, you can assume it as if you know this zone of no slip leads to the fluid moving integrally as one part, fluid layer two move integrally as one part with the plate and this zone of no slip kind of rested back and then there is a velocity gradient which is obtained.

So, let us assume this to be ui as a function of y. So, where y varies between plus h or minus h. We place of the coordinate system in a manner, so that origin is somewhere in the middle of the plate between the plates. So, this is the x y direction. So, we use on the Navier-Stokes equations to get an idea of the velocity profile or the velocity equation. As I told you earlier

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we will be only discussing the conservation of the mass and conservation of momentum equations from micro fluidics. So, for the conservation of mass equation we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

v and w do not make any sense because we are assuming the v and w to be 0. There is no velocity along the z direction which is let say into this slide and also along the y directions. There is really know circulating flow in the whole of this domain. So, there is only a unique directional flow and u is basically nonzero and also a function of y. The way that it varies, u varies is a function of y. So, essentially this and these terms would be 0 and so we are left

with $\frac{\partial u}{\partial x} = 0$. Similarly if we apply the momentum equation, so applying in the conservation of momentum. We have

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{-\partial P}{\partial x} + \rho g + \eta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

So, here as we know V 0 as we have made in this assumption for the covert flow and d u by d x is already been, obtain to be 0 from the earlier conservation of mass equations. We assume there is no pressure gradient just a moment of fluid trapped between two plates with one moving one fixed. And there is no inertial component of the flow also and

$$\eta\frac{\partial^2 u}{\partial y^2} = 0$$

$$u = C_1 y + C_2$$

So, solutions for this equation would be y equals C 1, $u = C_1 y + C_2$. So, that what

essentially $\frac{\partial^2 u}{\partial y^2}$ equal to 0 would mean. And now we evaluate this particular equation and

the boundary conditions where we already know, therefore

$$at\ y = -h, u = 0$$

$$at\ y = h, u = V$$

So, on the evaluation of this equation we really find out u essentially as

$$u = \left(\frac{V}{2h} y + \frac{V}{2}\right)$$

So, this is the case of the Couette flow. We also want to determine what happens if you have a pressure gradient which exists between these two flows and then you know essentially we want to plot the flow of file in that case and compare it with flow profile of a electro kinetic flow as we have seen earlier. The electro-osmotic flow as we have seen earlier.

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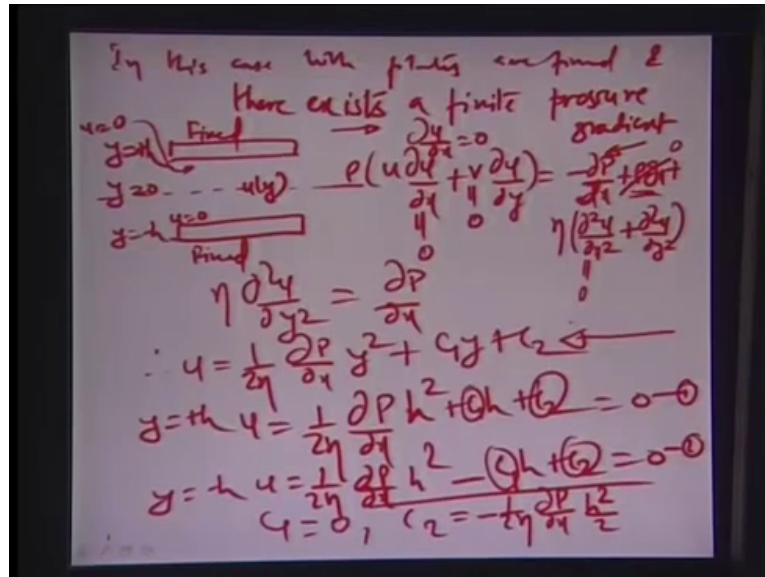
So, here in the other case we have this the pressure driven flow, where in this particular case both plates are fixed and they are exists a finite pressure gradient. That is why it is pressure driven. So, essentially you have two fixed plates; this is fixed and this is also fixed, u as a function of y in between and so really the you know from the conservation of a main term

equation can be written as $\dfrac{\partial u}{\partial x}=0$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{-\partial P}{\partial x}+\rho g+\eta\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\eta\frac{\partial^2 u}{\partial y^2}=\frac{\partial P}{\partial x}$$

So, just you know from the conservation of mass equation is we know already in the earlier slide we have seen that d u by d x in that cases you have. So, this component is 0, there is x actually know V which is access in this particular case. Is again unidirectional flow we only assume flow along the x directions.

So, there is a velocity along the x direction, but along the x direction as you see by the conservation of mass, there is no variation in u, u on the variation in the y direction. So, there

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is a finite pressure difference in this case, this is really not a nonzero term. We assume no inertial effect so, rho x is 0. By the by one of the reasons y no inertial effects assumed in case of you know especially micro fluidics or micro channel flows because of the volume of the mass that is involved is a pretty low. So, therefore, the inertial effects sometimes really not that significant to the prominent. As it is corresponding macroscopic counterpart.

So, micro fluidics we normally assume until it is absolutely needed. We assume the inertial part of the flow the, you know the gravity term of the flow to be you know kind of negligible.

And so, we are left with again $\frac{\partial^2 u}{\partial x^2}$ is 0 because $\frac{\partial u}{\partial x}$ itself is 0. So, you left with d 2 u by d y 2. So, in this case in the equation takes the form eta del 2 u by del y 2 is equal to del p by del x; the pressure gradient. So, there is a finite pressure gradient in this particular case. So, if we just try to solve this particular equation, we are left with solution for u; which is equal to eta, Which is equal to

$$u = \frac{1}{2\eta} \frac{\partial P}{\partial x} y^2 + C_1 y + C_2$$

1 by 2 eta del p by del x, the pressure gradient times of y square plus some constants C 1 times y plus C 2. Let us investigate in this particular case, what happens? Really if you have a central axis here and y equal to 0 here. And we assume y equal to plus h as the location, special location of upper plate and y equal to minus h this is the special location the lower plate. That case both the velocities here, because they are fixed have r 0. So, u essentially the velocity close to both the plates; topper and the lower plates both the zeros in this case. So u equal to 0 here, u equal to 0 here; both the planes essentially have zones of no slip.

So, in that case at y equal to plus h, $at\ y=h, u=\frac{1}{2\eta} \frac{\partial P}{\partial x} h^2 + C_1 h + C_2 = 0$. And y equal to

minus h, $at\ y=-h, u=\frac{1}{2\eta} \frac{\partial P}{\partial x} h^2 - C_1 h + C_2 = 0$; So therefore, the on the way you know you could. So, you have to D psi for what is the C 1 and C 2 are in this particular case. So, from these two equations, you essentially have 2 nouns and you know two different equations. So, serf linear equations you can solve them to find C 1, C 2.

$$C_1 = 0, C_2 = \frac{1}{2\eta} \frac{\partial P}{\partial x} \frac{h^2}{2}$$

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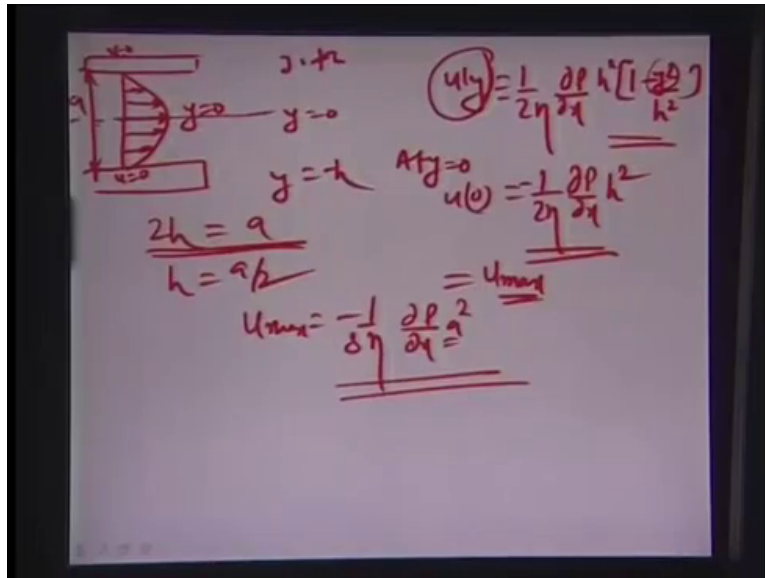
So, if you put the C1, C2 values back into this particular equation here, you finally get the final form of the equation as equation as

$$u_y = \frac{1}{2\eta}\frac{\partial P}{\partial x}h^2\left[1 - \frac{y^2}{h^2}\right]$$

So, if you plot this equation is really the equation of parabola, because u is the barring as square of y. So, essentially if you look at the flow profile here within this particular channel that I would be just strong in about minute. So you have two fixed plates here, and you have for central axis. You have a y equal to 0 along this, y is plus h here, y is minus h here. And so essentially it is a parabola.

$$At\ y = 0, u(0) = \frac{-1}{2\eta}\frac{\partial P}{\partial x}h^2$$

So, u i is a barring as a square of y. So, essentially you can draw the profile something like a parabola. Where there is a zone of no slip, v u 0 here and there is another no zone of no slip here; and this profile here is the plow profile of parabola with velocities starting from y equal to 0 to y equal to plus h and y equal to minus h. And u 0 at y equal 0 would really be equal to minus 1 by 2 eta del p by del x h square is also the maximum velocity as in this particular case. You can see in the parabola kind of as a point of inflection here, where slope changes and so the maxima here in this case is corresponding to y equal to 0.

$$u_{max} = \frac{-1}{8\eta}\frac{\partial P}{\partial x}a^2$$

So, you know if you assume different sign convention, different special location in the

manner is if a let say this 2 h term, this whole 2 h term here that you get as a dimension or diameter of the capillary is equated to something like A; so that h is a by 2. So, in that case the u max would just would be written differently as 1 by 8 eta del p by del x times square of a; where a is the diameter. So, circular cross section of a diameter a would essentially give a u max which is equal to minus 1 by 8 eta del p by del x into a square; a is the diameter of this particular capillary. So, let us now try to calculate the volume flow rate because this should be important from perspective of a later on you know trying to integrate that onto the electro-osmotic flow equation. And we can also compare the pressure driven flow with respect to the electro-osmotic flow, which was really be objective. And in terms of profile as you will find out and just about little bit the electro kinetic flow is really, the electro-osmotic flow is really a plug like flow is suppose to the parabolic flow which is introduced by you know the pressure driven flow.

So, basically this brings as to you know the end of this particular lecture. Next lecture will be talking more about how to integrate this pressure driven flow kinetics and especially the flow rate with the electro-osmotic flows. And then we will also be beaming into streaming potential as a essentially EMF caused by velocity of motion of a charge layer with respect to a surface.

Thank you.