

Technical Arts 101
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Lecture – 8
Think and Analyze

(Refer Slide Time: 00:20)

Unit cube

Rotation about 'z', then about 'x' and then projection on the y-z plane

$$G = PRxRz = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & 0 & 0 \\ \sin(\alpha)\sin(\beta) & \cos(\alpha)\sin(\beta) & \cos(\beta) \end{bmatrix}$$

$$|OP| = \sqrt{[\cos(\alpha)]^2 + \sin^2(\alpha)\sin^2(\beta)}$$

$$|OQ| = \sqrt{\sin^2(\alpha) + \cos^2(\alpha)\sin^2(\beta)}$$

$$|OR| = \sqrt{[\cos(\beta)]^2}$$

|OR| = |OQ| and |OR| = |OP| gives

$$\cos^2(\alpha)\cos^2(\beta) + \cos^2(\beta) = 1$$

$$-\cos^2(\alpha)\cos^2(\beta) + 2\cos^2(\beta) = 1$$

$$\cos^2(\beta) = 2/3 \quad \beta = \pm 35.26 \quad \alpha = \pm 45$$

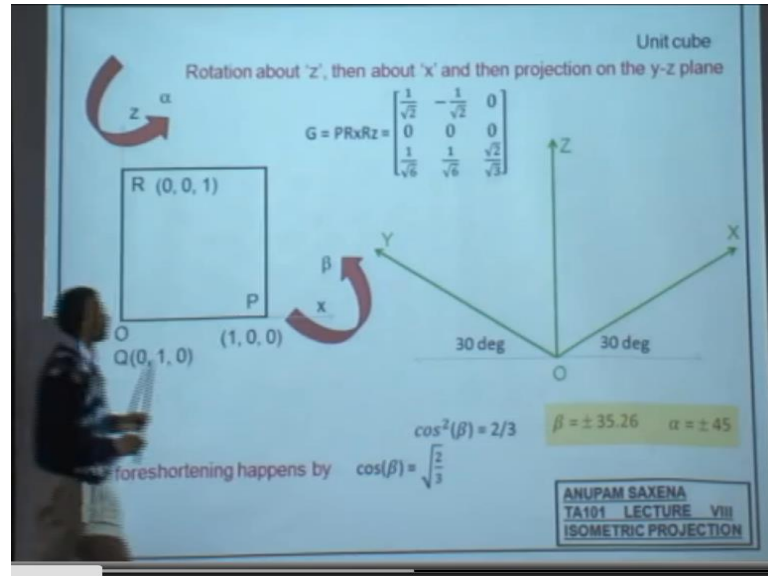
ANUPAM SAXENA
 TA101 LECTURE VIII
 ISOMETRIC PROJECTION

So, let us continue with isometric views so last time we did some math, so what we had was a unit cube and let me take a block in my hand, what we had was a unite cube. So, this was the x axis; this was the y axis pointing towards me z axis. And what we did was we rotated this unit cube about the z axis by an angle alpha. And then we rotated this cube about this horizontal axis by an angle beta, right. So, when we did transmissions, we rotate the object of the unit cube about the z axis and then about the x axis. And then we performed the projection on the vertical plane; that is the screen plane when we did the transmissions we pre multiplied the rotation matrix about the x axis with R z.

And then we pre multiplied the projection matrix and we got the new coordinates or the projected coordinates of the points P Q and R. So, this point P Q which is pointing inside the screen and R over here. We computed the lengths O P O Q and O R we equated them O P equals O Q O equals O R with this an algebra. And we eventually got our solution beta to be equal to plus minus 35.26 degrees and alpha to be equal to plus minus 45

degrees. So, if we plug in let us say the positive values alpha equals to 45 and beta equals 35 degrees 35.26 degrees over here in our transmission matrix.

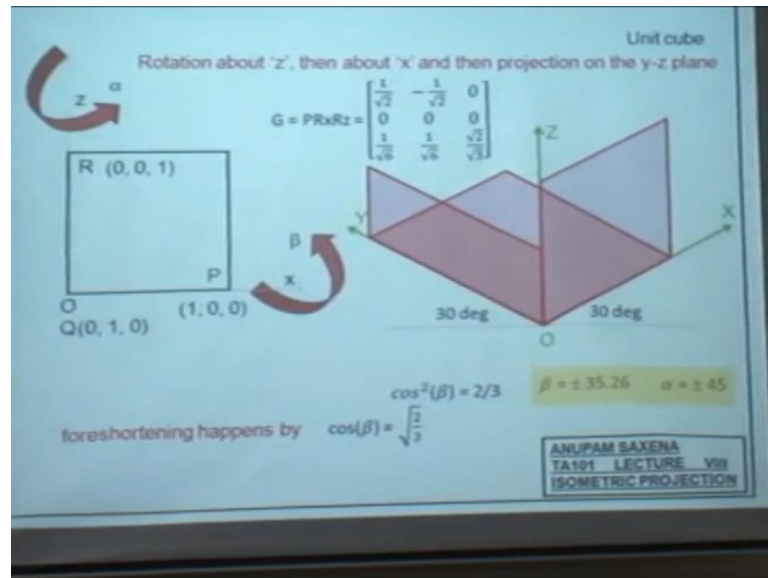
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We get G equals 1 over root 2 0 1 over root 6 minus 1 over root 2 0 1 over root 6 and 0 0 root 2 over root 3. And last time, we figured that the foreshortening the unit lengths O P O Q pointing inside and O R they all got the foreshorten by a factor root 2 over root 3 right. So, this is the x coordinate of point P on the projected x z plane your screen plane the z coordinate likewise; the x coordinate of point Q; the z coordinate x coordinate of point R and z coordinate.

So, we figure we figure that O P O P would lie along the x axis which is at 30 degrees to be horizontal likewise O Q which is pointing inside would lie along o y which is at 30 degrees from the horizontal. Again on the other side and O R which is vertical will remain vertical here let us say this is the unit cube 45 degrees 35.26 degrees. Alright, how will the bottom plane of this cube show up how will the bottom plane of this cube show up like. So, how would this plane show up in the figure like so right and how will this plane show up parallel to this plane?

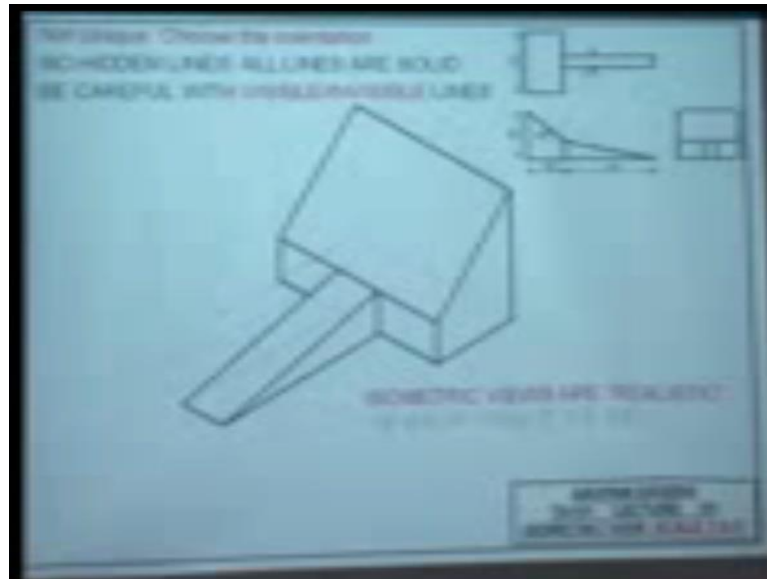
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Now, once you figured once you identify the O X O Y and O Z axis of the object it is fairly straightforward for you to identify a point. If I give you the coordinates for example, if I give you the coordinates of any point as 30 40 and 50 30 for the X coordinate 40 for the Y coordinate and 50 for the Z coordinate. How will you locate that point on the plane? What you would do is you would go 30 from the origin parallel to the x axis right let us say you have gone here. And then you would go 40 parallel to the y axis over here may be right you are still on the x y plane. And then to plot 50 you will stand here and go vertically upward.

That is how you would locate your x y z coordinates as 30 40 and 50 respectively with me not yet, good. So, says why go 30 along this 40 along this and 50 along this why I will not using this foreshortening factor. And he is absolutely right and at this point I would like to differentiate between something called isometric drawing. And isometric view in an isometric drawing you would be using that foreshortening factor in an isometric view. It is for you to use true length the object just gets scaled by root 3 over root 2 that is all the other way around.

(Refer Slide Time: 07:06)



So, last time we were doing this example let us redo this again at this time let me say that this is the length direction of my object. So, stay with me here let me say that this is the length direction of my object let me say that this is the height direction of my object. And if I go to the top view this would be what the width once I identify my length height and width directions. It is much easier for me to draw the isometric drawing or isometric view. Let us do that what did I say this was length, how about this? How about that width good I am drawing x y and z . The other way around now this is my x direction which is still at 30 degrees to the horizontal my y direction which is still at 30 degrees to the horizontal and z direction vertical.

Now, I am keeping my length direction along the x axis. Stay with me here I am keeping my length direction along the axis that would mean that all features along this direction are to be drawn along this direction in the isometric drawing or view. And keeping my width direction along the y axis all features from here to here along that direction are to be drawn here and z I am keeping for the height. If you look at this object you will see that this is a fevicol joint. So, if you look at this object is composed of 2 features this one is the block a slanted block and this one is like a tapered. Let me draw the bounding box for the slanted box origin is the origin, what is this length? 40 wonderful, what is this?

Great, what is this? Wonderful. What are the coordinates of this 0.20 40 30 great not very difficult that is precisely how you need to draw isometric drawing or isometric

view. The bounding box for these slanted or sliced box ready. Now, let me draw the bounding box for the tapered rip from here to here the length is 60. So, I will go from here to here 60 and then from here to here on 5 15 1 5. So, I will go 60 from this point along this direction along the x direction and cell phones off please. And I will go 15 from here till here and then I will draw a line parallel to the y now, let us focus on this part here let us focus on this.

So, this phase of the rip is coinciding with this plane what is this length 10 10, what is this distance? 15 this distance is also 15 right. I join these 2 points likewise I have another square here. And I join these 4 vertices to get a block with in which the taper rip is gonna be. Now, note this that I am only using the construction lines I will have not yet began to draw the actual solid right there is a reason for that. And I will tell that reason to all in a little while what this edge be visible. I do not need to draw this, what this edge be visible no I do not need to draw this either how about this right my rip is ready.

You draw the isometric drawing or the isometric view, the way you see it no hidden lines absolutely no hidden lines right. How about the sliced block behind the back or at the back of the rip you see this edge, do you see this one you do? So, you would see only you would see that edge only partially not completely and since there is the slant. I get this since there is the slant which is very similar to this since this plane is a same since this plane is the same. This line will have the same direction cosines as this line am I done as simple is that no rocket science. What you see about this edge would you actually see this edge or no? Why do you why say that discontinuity?

Because the planes are different this plane this plane is not the same as this plane. So, you will see this line so a lot of people came to me last time and asked me; well, sir why only this orientation, why cannot have the object placed differently? Sure you can. So, instead of showing object like this we can turn it around about the z axis by 90 degrees and have this rip show along the y instead of along the z perfectly fine. Or you can turn it around by 180 degrees and show the rip going the other way around along the z along the x.

So, there are multiple possibilities in which you can show the isometric drawing or view. Word of caution, no hidden lines are to be shown like we do in case of orthographic views; no hidden lines are to be shown be stay as realistic as possible when we show the

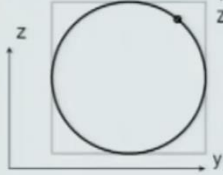
isometric drawing or view. So, when you drawing a isometric views you need to be a very, very careful you draw a solid line only when you are absolutely show that you will see that line. Otherwise you do not otherwise you will end up using your eraser and wasting time and spoiling the sheet. Isometric views are realistic, we show what we see and like incase of orthographic views where we show even those features that we do not see by means of hidden lines ignore that no that that scale is different that scale is.

So, may be what I did was I scaled this 60 to 40 maybe that is what I did so at this time ignore that. So, you draw an isometric view with the help of orthographic views right, but you do not have to strictly adhere to which is the left view right view which is the front view top view. Those 3 views are to give you an idea of how the object is gone a look like once you get the idea you can chose the orientation of the object. The way you want to show as many features as possible in solid lines. For example, had I shown this feature tapered rip at the back would it have been a good idea no, because when a major part of that rip would have been occurred major.

Or possibly all I do not know would it have been a good idea for me to rotate this by 180 degrees and may be not see this slant on the plane no. So, you need to orient the object in such a way that you get maximum; you get see maximum features isometric drawing foreshortened isometric view or isometric sketch true values. Something very important so remembering the first class so the second class and definitely in the first lab you did lot of exercises on ellipses we are going to a, be drawing a lot of elliptical arcs or ellipses in isometric views. You see that unit cube bottom left imagine that you have a circular feature.

(Refer Slide Time: 18:46)

TA: What happens to the shape of a circle on the y-z plane ?



$$G = PRxRz = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} \end{bmatrix}$$

$$\begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 0 \\ a + r \cos \theta \\ b + r \sin \theta \end{bmatrix}$$

$$-\sqrt{2}x_n - a = r \cos(\theta)$$

$$\sqrt{6}z_n - a - 2b = r \cos(\theta) + 2r \sin(\theta)$$

$$(\sqrt{2}x_n + a)^2 + \left(\frac{\sqrt{3}}{2}z_n - b + \frac{x_n}{\sqrt{2}}\right)^2 = r^2$$

$$Ax^2 + 2Hxz + Bz^2 + 2Fx + 2Gz + C = 0$$

$$A = 2 + \frac{1}{2}; H = \frac{\sqrt{3}}{2}; B = \frac{3}{2}; H^2 - AB = \frac{3}{4} - \frac{5 \cdot 3}{2 \cdot 2} = -3 < 0$$

Ellipse on the 'isometric Xn-Zn' plane

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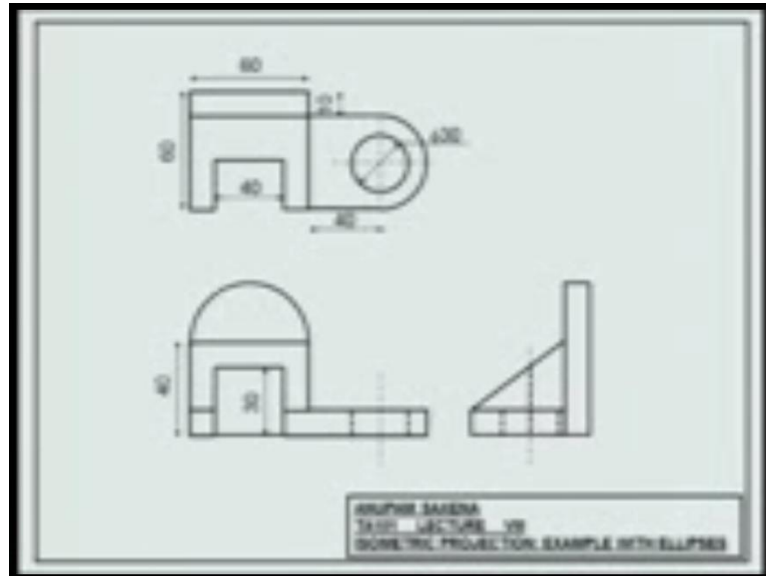
Imagine that you have a circular feature here you have rotated this object twice and projected it on a vertical plane. Now, let me turn this object around you will have a circular feature here how would that circle look. Are you sure absolutely positively definitely thought just in case you were not sure I would back myself of with a little bit of math. So, this was the projection matrix alpha equals 45 beta equals 35 minus 35.26 if we have circle on the x y plane which is the x y plane which is the which is the bottom plane of the cube. The coordinates of a point on a the circle will be A plus R cosine theta of B plus R sine theta and 0.

A and B are the centers of the circle the center coordinates of a circle as the radius plug in these values over here pre multiply this thing by this matrix get the new coordinates do some algebra eliminate theta. And then compare it with your generic equation for a conic get what A is what H is what B is and the test is the litmus test is for an ellipse is less than interesting right. Chose any value of A any value of B any value of R H square minus A B will always be minus 12 in case the circle is on the bottom plane so a circle gets converted into an ellipse right.

Same thing the circle is on the y z plane the x direction the y direction; the z direction; the circle is here ellipse. Do the same exercise the y z plane your Y coordinate will be non 0 your Z coordinate will be non zero. Do the same exercise and you will find that your litmus test holds in this case H square minus A B is equal to minus 3 which is

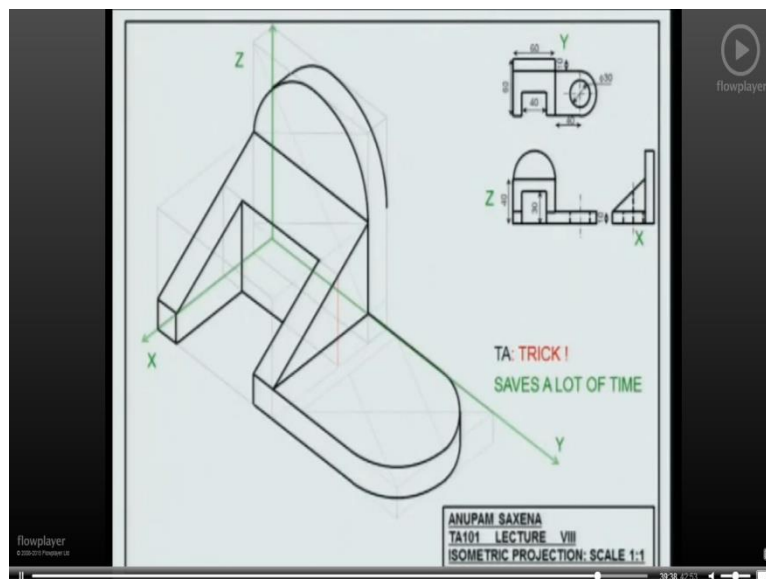
smaller than 0. And the same thing if the circle is on the x z plane same thing the interesting part is irrespective a is what B is what R is these values remain constant x square minus A B it remain constant.

(Refer Slide Time: 22:16)



Let us get to a tougher problem shown on the screen is the third angle projection of an object I will say you the effort.

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I will tell you that the object looks like this 10 seconds to clear your trots. Now, if you realize you can actually break this object into 3 parts, the part on the back which is

resting feature a part on the right. So, this is the first part this is the second part and this part would be the third part. You can break this object up into 3 parts divide and rule that helps you can have a bounding box corresponding to the parts which is resting at the back. And you can have one or this one or you can have another bounding box for this here. Now, for us to understand let me explicitly mention the X Y and Z axis now let the X direction have all the features along this direction.

Let the height direction have all the features along this direction and let the length be drawn along the y axis. Let us get started lets first draw the bounding box corresponding to this guy here what is this height? 70, what is this? 60, what is this? 10. Once you have identify the 3 dimensions of a box or of a block you can draw parallel lines along the X Y and Z and go ahead and draw the block. So, I am drawing a block that would in compass this feature here, what is this length? 50 right so 60 60 minus 10 what is this height? 40. The same dimension got one phase if I got one phase all I need to do is draw lines parallel to the Y. And get this block how about the third one this length is the same as this length.

Now, what is this length here from the origin, what is this length? 70. I am working at all this length from here to here 40 and what is the radius of this? 25 so 25 plus 40 is 65. So, this would be at 65 from here may be is it gone a be difficult for you to get this block no fairly straight forward alright. Now, let us try to get this slant here this slant here and that would happen or that would appear on this phase the corresponding slant will appear on a phase parallel to this on the other side right. Now, focus your attention on this line is this line here likewise this line is this line here these vertical lines are this and notice that there is the slot here.

So, I have taken away a piece of block from this the width of the block is 40 the height of the block is 30 30 and 10 well I said the block well. So, if you if you trapezoid but its equivalent to taking a block of 40 by 10 40 by 30 by whatever distances anyhow. So, this is the critical aspect of the drawing identify this length or this height as 30 go from here parallel to this identify. This height get this point and get this point likewise go parallel to the y axis identify that vertical line get that phase. So, this phase would correspond to the phase correspond to this phase here at the back.

Now, start drawing now start drawing I am working with features on the slant surface. Now, I am ready to start drawing solid lines I will see this, will I see this? I would see this phase, I would see this phase here on the slant this line this line this line this line this line. I worked out the details on this slant here I will see this vertical line I will see this vertical line. This one I will see this phase I will see this vertical line, I will see this vertical line, how about the line here? I will see a partial line corresponding that not a complete line.

So, do appreciate how judiciously I am drawing my solid lines without spoiling my sheet. And that can happen only if I am prepared if I am not prepared I will start making mistakes. Now, here comes the interesting part this circle of arc will be appearing as an elliptical arc on which plane on this plane here on the x y plane or a plane parallel to the x y plane. Do you remember the 4 center method you do? Yes for that I need to draw a rhombus that would bound the circle or bound the ellipse right. The length of the rhombus is gone a be, what the length of the each edge of the rhombus gone be what 50. So, this length is 50 I have one length this is 50; this is 50; this is 50.

I have got a rhombus on the top phase of the, identify the longest diagonal from this vertex join the midpoint of this edge from this vertex. Join the midpoint of this edge you have 2 centers; you have the third center here you have the forth center here with this as center this as radius draw an arc with this as centered this as radius draw another arc. So, since this is a circular arc I do not need to draw the entire ellipse I only need to draw half of the ellipse. The rest I can join using straight lines with main part here comes the trick, what do you say, what will you gone a be seeing here a parallel ellipse now.

But you are a little ahead of me, now do you think that a drawing a rhombus to draw that ellipse would be a good idea no absolutely. So, what you do and then you gone a be drawing only half the ellipse you have the center for this one which is here. And if you have to draw an arc which is parallel to this arc what you need to do you need to go down by what distance by 10. We need to go vertically down by 10 identify this center here the same radius from here to here draw this arc If you start making a rhombus to draw this ellipse you will be wasting time that is a trick that you need to keep in your minds.

Likewise from this center go down by 10 go down by 10 identify the center with this as radius draw an arc. Now, this arc is not gone a be complete this arc is gone a be partial this arc is gone a be partial, because that this vertical line here. And then join this the rhombus that we use to make the ellipse what it mean to get the ellipse here this length is 50; this length is 50; this length is 50; this length is 50; this length is 50 This angle is 60; this angle is 60; this angle is 120 degrees; this angle is 120 degrees, you have all the conditions necessary for you to have an ellipse enclosed with an rhombus.

What lengths you got the block you have this block here and of course, what is your question again if the diameter of a circle is 50, what would be the length of the bounding square of the square? That is bounding the circle 50 length and width would be the same in case of rhombus absolutely same in case of rhombus, because what is happening is if you are rotating an object about the x about the z. And then projecting your square is getting shaped into an into a rhombus so far so good it has to be tangent to the arc. So, if you draw this arc using soft pencil using construction lines your arc will probably be here it will probably go some somewhere.

Like this will go somewhere like this and the point where the vertical line intersects with the arc that is where you need to be stop I would need 10 more minutes be patient. So, this vertical line is tangent to both these arcs hold on, you can do precisely the same thing for this block here what would be the length of the rhombus for this 60. So, this I know is 60 I can identify were 60 from here I can identify well 60s from here I can draw this line which is parallel to this line I get another rhombus. I join or I identify the longest diagonal the same method 4 center I get 4 centers here first 2 centers the third center the forth center.

Again I need to draw only half an ellipse with this point as center this as radius I draw this arc with this point as center this is radius I draw another arc. And to draw the same thing at the back or on the back face I need to essentially control c and control v control c is for copying control v is for pasting, but I cannot do that here. So, what I would do is this was my center for this arc I would go 10 parallel to the x identify this as center with the same radius. I will draw an arc at the back from here I will go 10 again parallel to x here I will draw an incomplete arc. And then try to figure out a line which is tangent to both the arcs.

And then this vertical line is going all the way down see this feature here please repeat are you with me. Do you think now it is gone a be easy for you to draw the ellipse or rather ellipses corresponding to this void; this circle. All you need to do is locate the bounding rhombus same method 4 center method get the ellipse am I done there would be one more elliptical arc. And that would give the impression or the perception of the depth of the void for that the same trick. I go 10 down with 2 centers what you mean for the second arc it is a void n a, it is a cylindrical void along the z direction I do not need line a few last things.

Actually I never needed this, this was only for our understanding and appreciation that I had marked the X Y Z coordinates or the axis explicitly. We do not needed this, we do not need this in your actual drawing or in reality on your sheets. This is how the isometric, what did I draw drawing sketch or view, view isometric sketch or isometric view. Because I used the true scale I did not used the isometric scale that is even harder that is for the reason why we at times. Somebody asked me right that is one of the reason why we preferred to use true scale.

Thank you.