Technical Arts 101
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Lecture - 7
Think and Analyze

Last time I asked you to come up with a single funniest ever statement that could relate four words late, elate, plate and slate anybody, anybody. Sir, I am late because, no how about Thursday, can I get some input from you guys? Thursday, a single statement that works as an excuse for you your late, because and you are going to be using all these three words elate plate and slate, let us see try to take of my funny bond. So, let us get started bored with orthographic views already, are you?

Student: yes.

Third angle first angle three views FTP, front top profile, want to learn something new? Yes, no.

Student: yes sir.

Maybe…

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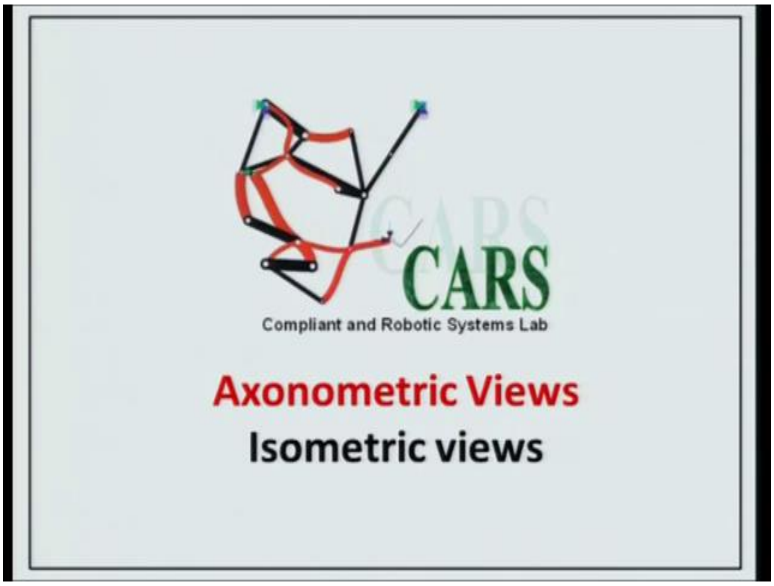

So, we will talk about axonometric views, and particular will talk about isometric views.

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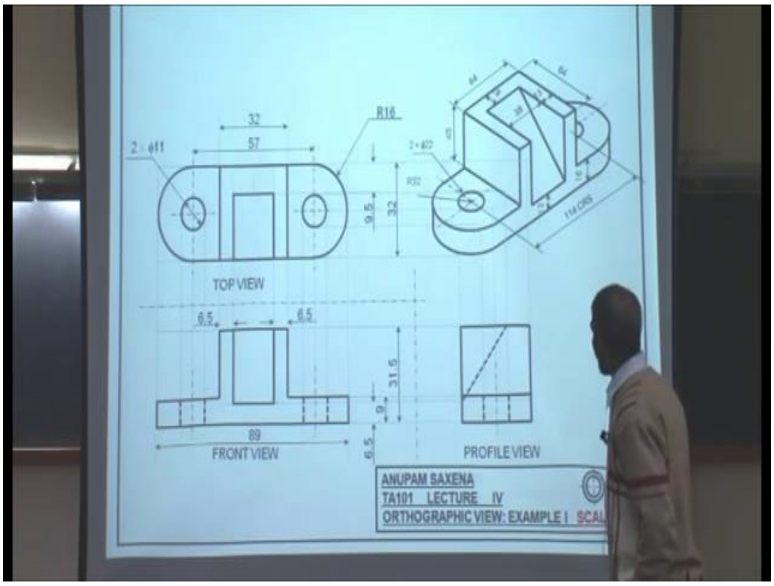

You have done this exercise before what did we do? We do this solid and we do the orthographic views in the third angle projection.

What if I do not give you this object what if Mr. Ashish can you come up and Vasu is there so what if Ashish gives Vasu these three drawings and asks Vasu to interpret these drawings would be difficult or would be easy, not that easy. So, would you need this drawing or would you not need this drawing, you do not know, thank you. So, the question is given the three orthographic views and nothing else is it possible, is it feasible for you to interpret the object? Who would say yes? Who would say no? Anybody who says yes would be difficult, yes or no?

Student: yes sir.

It will be difficult, but I show this object to you would like fine you have an idea of what can object would look like in 3 D. You also have an idea as to what the object would look like when look that from three different directions three different orthogonal directions. So, would you say that these pictures complete the three orthographic views and a pictorial view or three view would you say that, yes or no?

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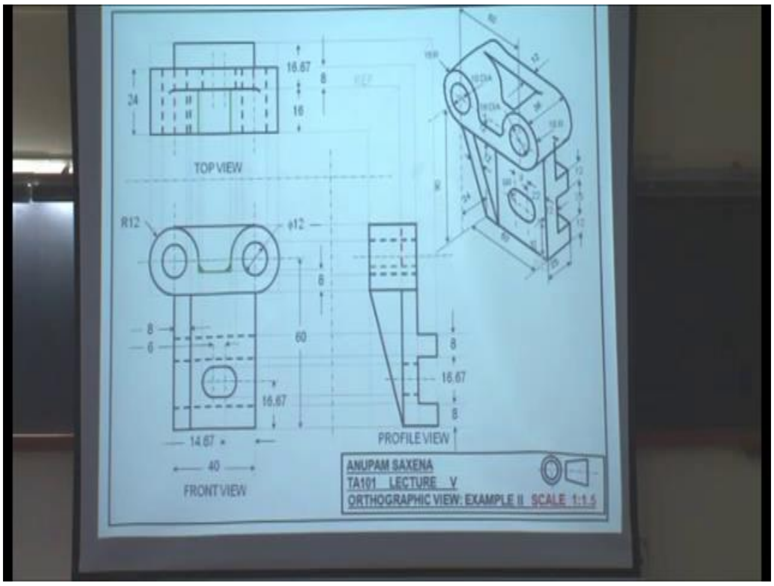

Another example what if I do not give you this even more difficult, but if I give you this, then you have a good picture of what an object would look like.

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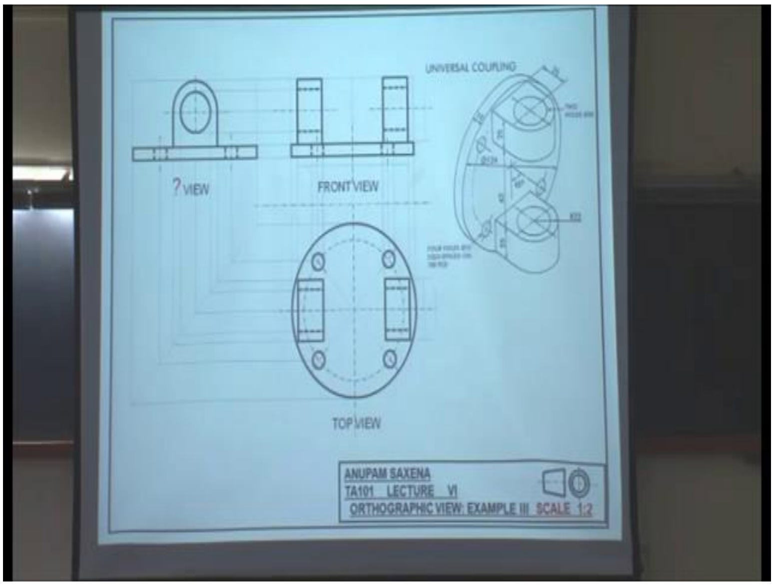

Third example if I do not give you this thinks might be a little difficult for you to interpret, but if I give you this, you are fine. You know what these are orthographic views what is this, what is that? This is what we are going to be learning today, isometric views to draw a single source be three dimensional picture of the object and I plan to you guys today with lot of math, well it is not a lot then little bit.

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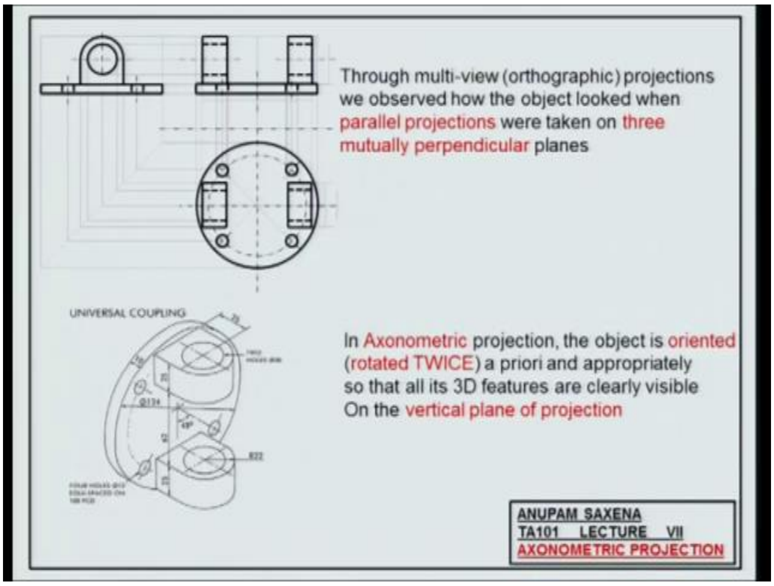

So, let me read what is written there through multi view orthographic projections. We observed how the object looked when parallel projections were taken on three mutually perpendicular planes and done below an axonometric projection. The object is oriented read as rotated twice a priori and appropriately, so that all its three dimensional features are clearly visible on the vertical plane of projection.

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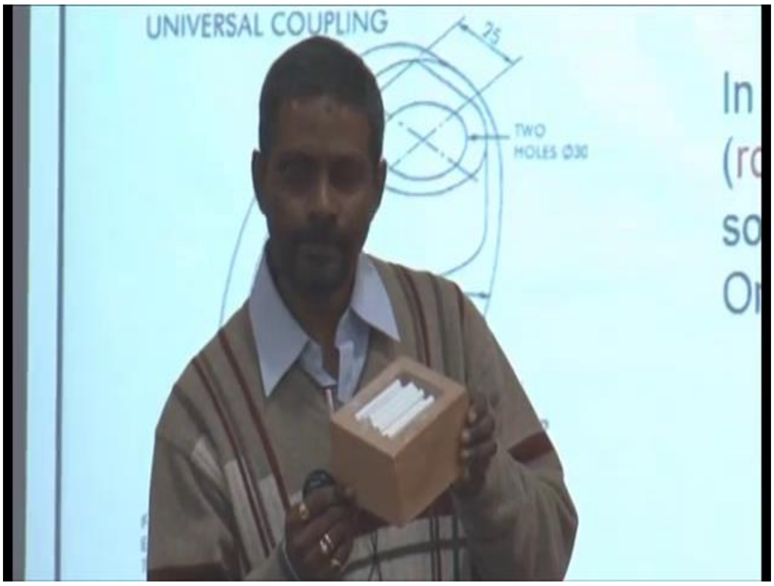

An example is a chalk box, what do you see? A rectangle so when saying back there is if I rotate this object once and rotate this object the second time. So, when I rotate this object once, how many planes are visible to you two, and then when I rotate this object the second time three. So, you see plane 1, plane 2 and plane 3 and you also see that there is a pot hole in this chalk box and a few chalks. So, the entire trick in trying to get the three the three dimensional picture of this object is to rotate this once rotate this twice so that all the three dimensions of this object are visible to you.

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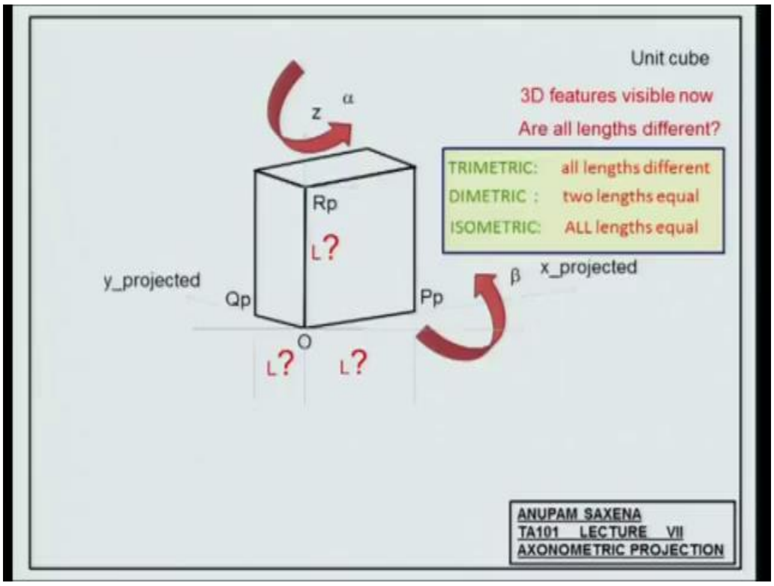

So, we will try to learn the math behind this let us say have unit cubes how many edges cube twelve lengths of them one, what you see is the vertical plane of this cube. Let us call the plane as the x z plane one of the vertices of the cube is at the origin of the axis this the x this z where is y outside or inside right hand rule. Let us practice, let us this cube in such a way that you are able to see the three planes, right now you are seeing just the one plane, let us first rotate this cube about the z axis when angle alpha. When I do that I see two planes right fine I am calling this x projected, but it is essentially containing the projected length of this edge and also the projected length of this edge right, but the actual reference happens to stay stationery it is fixed and then.

So, you would actually see two of this vertices p p and q p at first you are seeing only this vertex right now you are seeing two planes, and then by the way, let me ask you this let me go back rotate. So, this length is one right when I rotate what happens to the length does it get decreased or increased by what amount good decreased by cos alpha or decreased to decreases to cos alpha to cos to cos alpha. Now, you take the object about the x axis when we do that rotation angle beta, we get to see the third face of the cube. Do not worry about that, and then you also see three vertices very distinctly p p, q p and r p would you agree with this are three dimensional features of the cube visible to you.

Starting with neither the single plane to neither two planes to nor three planes yeah yes or no, what is happening to these three lengths of the unit cube? When you perform one single rotation you said the length reduced to cosine of alpha in this case what any idea no idea we need mathematics for this, but would you say that these lengths are equal to 1 or less than 1 or greater than 1, all three of them less than 1, yes or no? Who says no? Why would say no, z component is not changed. Is this too length? Yes, vertical if I rotate it towards myself what happens to the length? It decreases, by how much, by cosine of this angle ha by cosine of this angle.

So, all length will change and it so happens that all lengths will get reduced, now depending on what angle alpha is and what angle beta is we will have to figure out by how much do these lengths actually gets reduced. For that, we need mathematics you review with this all lengths are different for all three angles of them better. Now, it so happens that for a certain sect of angles alpha and beta you get these three lengths to be totally equal and when you looking at that projection it is called the isometric. ISO means same metric means scale isometric projection when all the three lengths are equal when two of these lengths are equal it is called diametric projection di 2 metric sides.

When none of these lengths are equal, it is called trimetric projection, you guys are getting bored already as this, so many. So, this is something that you would keep in mind, but in particular we are going to be looking at isometric projections as a part of this course.

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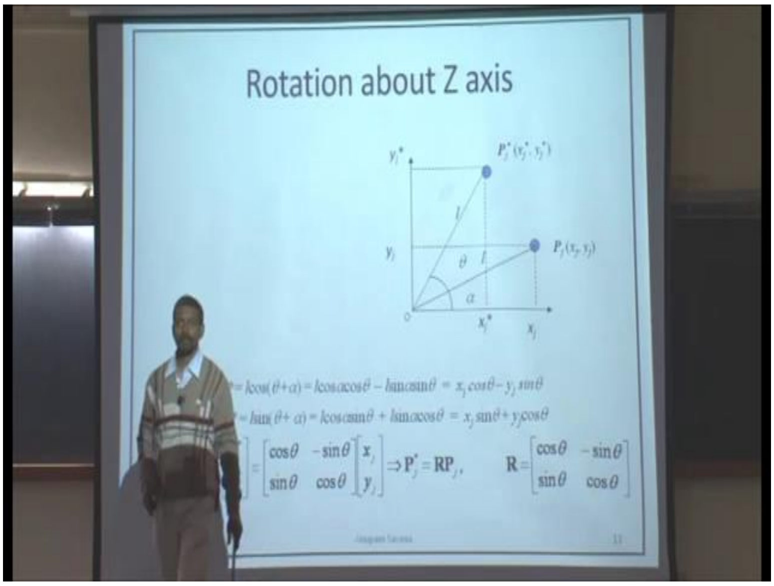

So, you have a plane x y you have a particle or a point which is at fixed length with respect to the origin. So, this length is fixed you can consider this to be an edge and you are going to be rotating this edge about the axis that comes to you. For some angle, so the coordinates of that point are x j and y j the original coordinates the new coordinates are x j star and y j star the initial angle made by this edge o p j is alpha o p j is of length l. This edge is being rotated about this axis, the axis that comes to you by an angle the (( )) right, have you come across this before yes or no? Yes straight forward, so this projection this horizontal projection is x j the x coordinates of p j this vertical projection is y j the y coordinate this horizontal projection is x j star this is y j star.

Given the angles theta and alpha can you relate these projections you can x j star is a times cosine of theta plus alpha which is if you expand that is l cosine of alpha cosine of theta minus l sin of alpha sin of theta l cosine of alpha is x j l sin of alpha is y j. So, x j star is x j cosine of theta minus y j sin of theta, likewise you can find y j star as l time sin of theta plus alpha at the end that is x j sin of theta plus y j cosine of theta. A matrix form you can write x j star y j star equals cosine of theta sin of theta minus sin of theta cosine of theta times x j y j that is something that I think you may have come across before you have, so far so good.

So, this matrix this matrix is called the rotation matrix cosine of theta minus sin of theta sin of theta cosine of theta what happens to the z coordinate of the point. If this is the x axis, this is the y axis what happens to the z coordinate of the point, it may same or different same.

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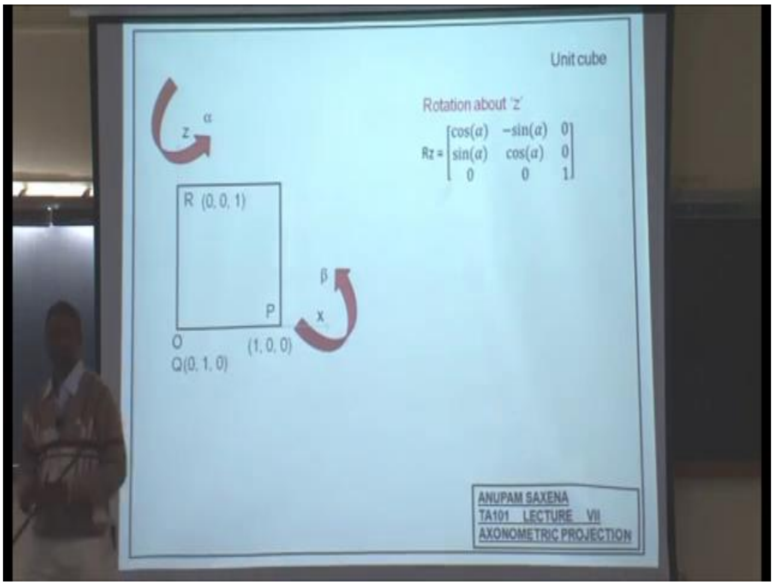

Coming back to the cube one of the vertices is called O, let us say this vertex is p with coordinates 1 0 0, Q is inside the stream with coordinates 0 1 0. So, y axis is pointing in words and r is 0 0 1, we are going to be applying the result the rotational result on this rotation about the z axis. If you look at this block there is 2 by 2 block that would take your x and y component and your z component is the same. So, you have 0 0 1 here and 0 0 1 here, so the old value of x coordinate the old value of y coordinate. The old value of z coordinate three multiplied by this matrix will give you the new value or the x y and z, everybody with me yes or no? What if I want to now find the question about the x axis?

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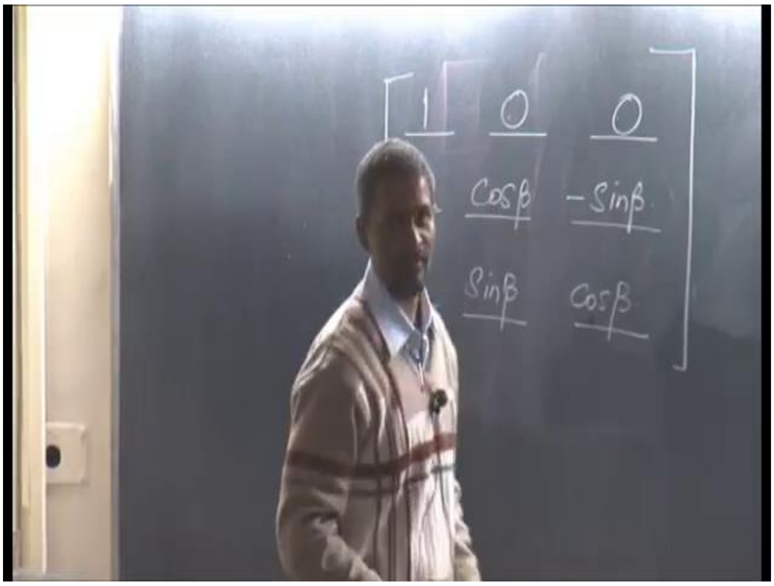

What is the entry here? 1 because is it too trivial?

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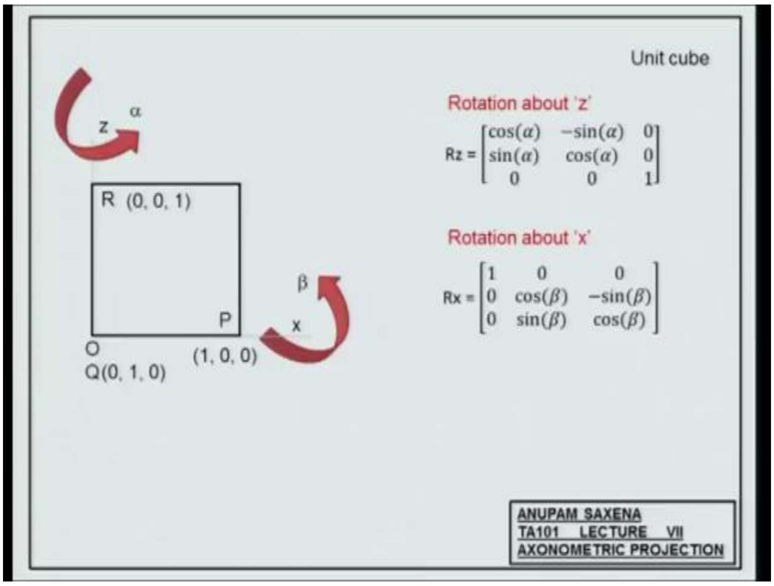

Anyhow, so remember let us not lose the physics come back, a single plane rotation about z by some angle that rotation given by r of z. Then, this is my x axis rotation about the x axis the second matrix gives you the rotation and the third step is whatever you see here you have to project that on the vertical plane, what happen? Then, your vertical plane is what, exit plane, your vertical plane your screen plane is the x z plane. So, your x coordinate were remain the same your z coordinate were remain the same what happened to your y coordinate 0, so this your projection matrix yes or no.

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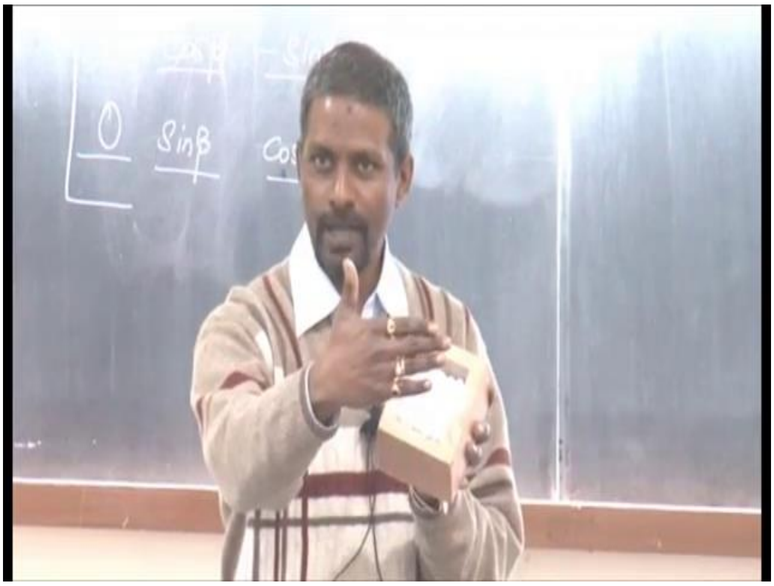

No step 1, step 2, whatever the x y z coordinates are this is my x z plane, this is my x z plane where y coordinate is 0. If I project if I project this sky over here x z coordinates from these two rotations and the y coordinate would be 0. Now, this is the tricky part when you perform transformations, you have to follow a certain order first your performing rotation about z second your performing rotation about x and third your projecting.

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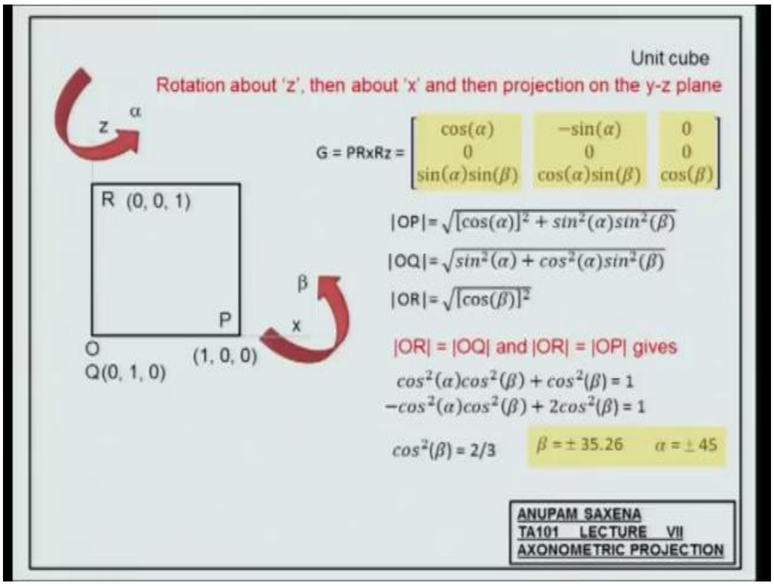

When you follow this order, when you follow this order rotation about z, then about x and then projection of the y z plane you have pre multiply a matrices. The first matrix on the right is rotation about z, then rotation about x and then projection of the y z plane and when you multiply all these three matrices. You will get the result on the right hand side top row cosine of alpha minus sin of alpha 0 mirror row 0 0 0 explains that you are actually you have actually gotten the image of the object on the x z plane. Therefore, all the y coordinates is 0 and the third row sin of alpha sin of beta cosine of alpha sin of beta and cosine of beta.

Remember, mark would be boring what do you have to say about this these are the coordinates of what these are the coordinates of p these are the coordinates of q. These guys are the coordinates of r right if I want or if I ask you what the new length of o p s what would you have say new length of o p x square plus y square plus z square cosine alpha square plus sin alpha square times and beta square. New length of o q square this half square this half and new length of o r right enough from math or little more axonometric projections you have three categories isometric dimetric and trimetric, what happens in isometric? All three lengths are equal what happens in diametric two lengths are equal trimetric, all lengths are different.

If I want to work with the isometric case, I have to equate or i have to ensure that o p is the same as the o q and that is the same as o r, how many equations 2, 3, 2, how many unknowns. So if I say that o p equals o q and o r equals o p I should be able to solve for alpha and beta how many solutions do you expect? Would you expect a unique solution? Would you expect more than one solution? If so how many more than one if you do the algebra which you can in your hostel rooms, you would see that data comes out to be plus minus 35.26 degrees and alpha comes out to be plus minus 45 degrees.

So, in fact you have four solutions, in fact you have four solutions right, so we have positive rotation in corresponding we have negative rotation in both directions for alpha and beta enough. So, this is the crafts, but in TA101, you do not worry about the intermediate steps, we only worry about higher result.

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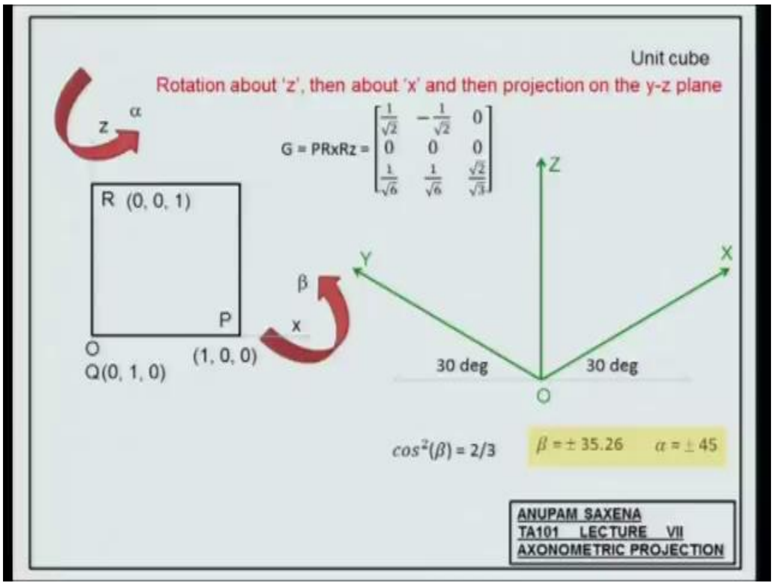

This was the final result is that if you incorporate those angles alpha and beta, what it essentially means in terms of drawing is you get to know the orientation of the x axis, the y axis and the z axis of the object of the cube. The x axis is oriented at 30 degrees from the horizontal, the y axis is also oriented at 30 degrees from the horizontal and the z axis remains vertical lunch followed by mathematics is not a good combination. So, in your labs or when drawing isometric views, do not worry about the math.

You can just start from here pick up point on a sheet draw x axis at 30 degrees from the horizontal draw y axis at 30 degrees from the horizontal draw the z axis which remains vertical. Once you have identified the z axis, you should be able to draw the three dimensional picture of the object. I will see one example yeah louder plus minus plus minus 35, 30 in what range, cos theta would be. If you have the solution to that cos theta, there should be four values corresponding to those values, corresponding to those values you will probably figure that you will get the one of the axis.

No matter what you will get, the other axis is this no matter what and your z axis will remain vertical no matter what you can tried out. So, it could be as well 35.26, there would be something l s upto 6, so in that sense maybe it is an approximation, but cosine of beta is of 2 by 3 that is the precise, no this exact how to be get that you see this result.

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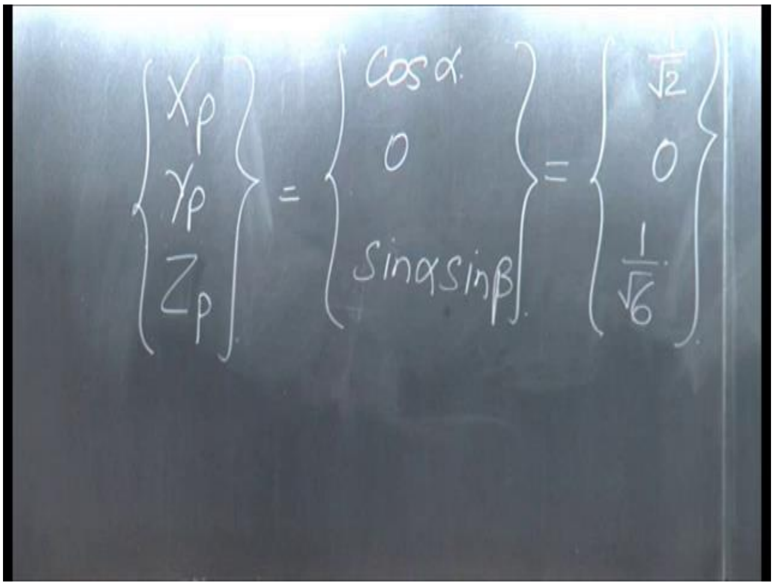

Better, let us focus on let us focus on the p coordinates p coordinates I given by cosine of alpha 0 cosine of alpha 0 and sin of alpha, sin beta, thank you. This guy right, let us, let us just focus on p, so if you plug in the value of alpha what you get? Let us just take the positive values for now what is cosine of alpha 1 by root 2 by 0, what is sin of alpha? What is what is sin of alpha times sin of theta? So, this would be 1 by root 6 fine, so my point p will be here somewhere.

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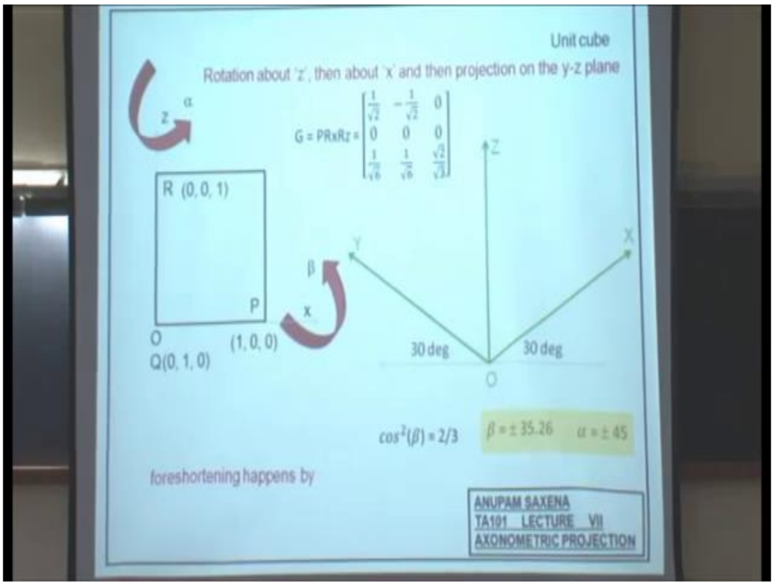

Remember, that this is the x z plane the vertical plane is x z plane, my point p will be here somewhere, what is the x coordinate of that one? By what is the z coordinate of that, so what is the cosine of the angle what the cosine of the angle which is what 30 degrees.

Student: yes sir.

You get it and likewise if you concentrate, q you will get 30 degrees on the other side. So, the lengths get shortened or for shortened by yeah have you got the word last for a what step one rotation about x rotation about z, then rotation about x and then projection on the x z plane. So, the screen plane is the x z plane, so you are actually seeing all three dimensional pictures of an object on a vertical plane which is the x z plane, do you agree with the math? If you plug in the values of alpha and beta, let us take just a pastor values, you will get the new coordinates of p as 1 over root 2, 0 and 1 over root x, new coordinates of q as minus 1 over root 2, 0 and 1 over root 6.

New coordinates of r as 0, 0 and over root 3, so your p will be lined here, your q will be lined here right and your r will be lined here, but this is what x coordinate of p z coordinate of p. So, the x coordinate is one over root two the z coordinate is one over root six this is one over root two this is one over root 6, so this yeah, so this over this is what 10 of this angle, which is what which is what you can you can same for cube.

So, the x coordinates minus one over root two the z coordinate is 1 over root 6, 10 of minus 1 over root 2 which is 30 degrees on the other side, what happens to the lengths? They get shortened, they get poor shortened by what factor or two or factor how. So, o r folks can I have all ears here, so the initial length of o r was 1, the final length of o r is root 2 over root 3. So, o r has shortened by root 2 over root 3 and so has o p and so has o q. So, the lengths in all three dimensions they have shortened by equal amount which is root 2 over root 3. So, this is something for you to understand, so through this lecture you can understand the math, but the take home message is this part.

When you start drawing your isometric views pick up point on your sheet draw, the x axis at 30 degrees to the horizontal draw the y axis at 30 degrees to the horizontal for the x axis, draw the z axis vertically of message 1, message 2 that your two lengths. Then, it to be factor, then it to be multiplied by root 2 over root 3 to get the isometric lengths and we are going to be working on how to draw the isometric scale horizontal.

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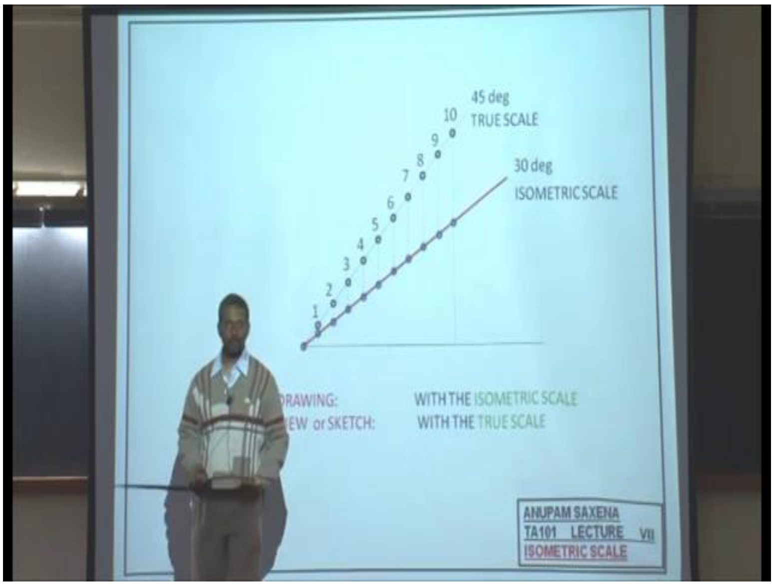

We have a line you have a line this line is at 45degrees this line is at 30 degrees mark the two lengths on your 45 degree line 2 lengths on the 45 degree line. If you take the horizontal projection of let us say length l which is the too length on the 45 degree length 45 degree line then the horizontal projection will be l times cosine of 45.

What is this length, this length over this length is cosine of 30, so you can find what that length is l cosine 45 over sin of sorry over cosine of thirty which is l times root 2 over 3.

So, in effects what is happening is that you are scaling the lengths on the 45 degree line by root 2 over root 3. If you are taking vertical projections of those points on the 45 degree lines on this red line mark, these points draw verticals on the red line from the green line. So, this is the true scale this is the true scale and this is the isometric scale, so this is the second take home message from today's lecture as first one or one is concerned, you do not need to worry about the math that we had done.

What you need to worry about is how to draw the x y and z axis on your sheet and how to come up with the isometric scaling. So, in your lab assignments and homework assignments, when I say isometric drawing, I want you guys to draw with isometric scale with this scale. When I say isometric view or isometric sketch, I would want you guys to draw isometric view, but with true scale, so keep this in mind.

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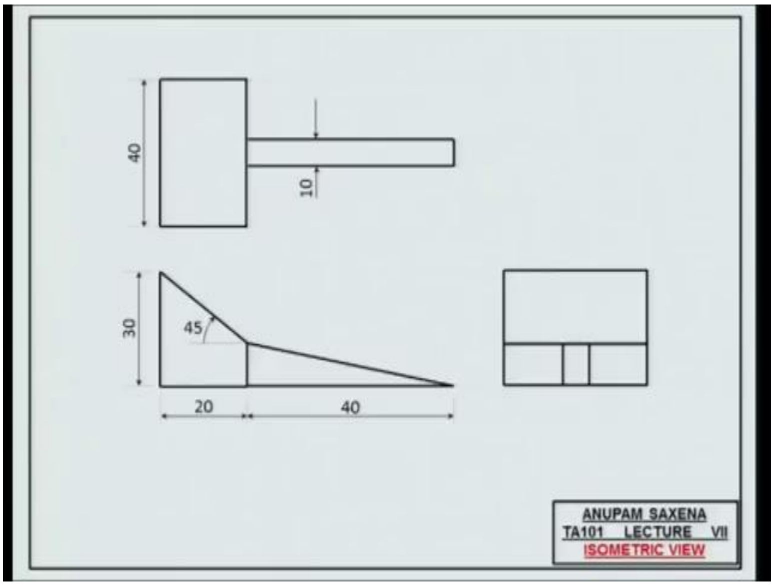

So, let us draw the isometric view from these three orthographic views given third angle, first angle, third angle have you say that third quadrant, how do you think or what do you think the object is going to look like? How do you think the object is going to look like in three dimensions?

Let us get back to your exercise first thing get one of the axis the x axis now realize how I am drawing the axis, I am drawing the axis opening towards you not opening away from you, x axis, y axis and the z axis. First thing both x and y third degrees from the horizontal the axis vertical, I know the dimensions from the orthographic views, I can draw the bounding box, I can draw the bounding box, the bounding box.

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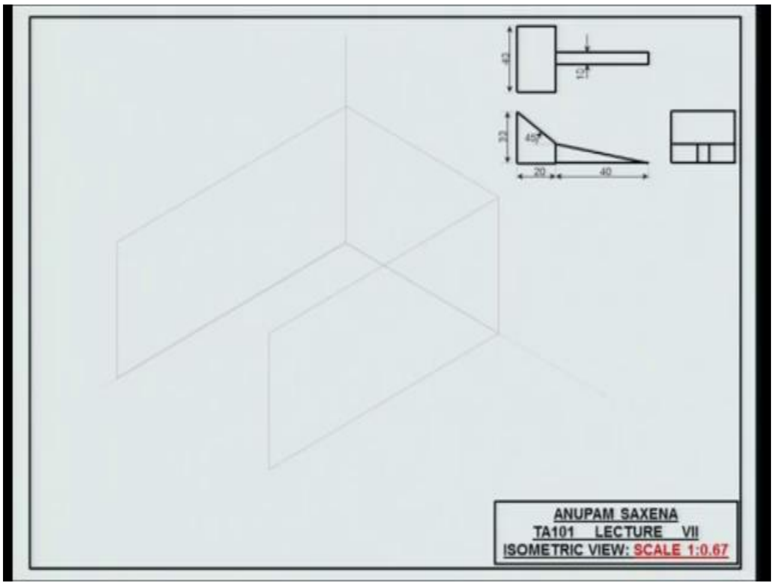

Three dimension bounding box which plane is this plane, which plane is plane x axis z axis this is y axis, so this plane here is the y z plane, how about this plane x z x z or x z. So, we can draw isometric views is to identify three different planes distinctly and work on this things. From now on I will not attar a word, I will just have you seen what is going on.