Technical Arts 101
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Lecture – 25

So, we have 4 plus 1 more lectures to go and after that what? 4 plus 1, the fifth one, I will figure if I need it then I will give that lecture. Great day today, so hot outside you would have had your lunch. Now, here you are in L 7 and it so happens that, what should I say? So, you had your lunch, it is a hot day outside, you are here in L 7 its all chilled out here, air conditioned out here. In front of you are a bunch of ((Refer Time 01:30)) one slides which I am going to be you know going through ideal ideal situation for you to dose off, is it not?

Student: Yes

Is that the reason why you are here?

Student: Yes sir.

You know I would not blame you if I were you I would probably do the same. So, I try my level best to keep you awake. So, let get started this is about 10 minutes pass 2 almost. New topic; interpenetration of solids, as I said we are going to be covering 4 plus 1 lectures more. The fifth lecture is going to be very interesting already; the fifth lecture if I get a chance to take its going to be very interesting. It is got nothing to do with engineering drawing, but a lot to do with what you are going to be doing later on in your lives if I get a chance to take it that.

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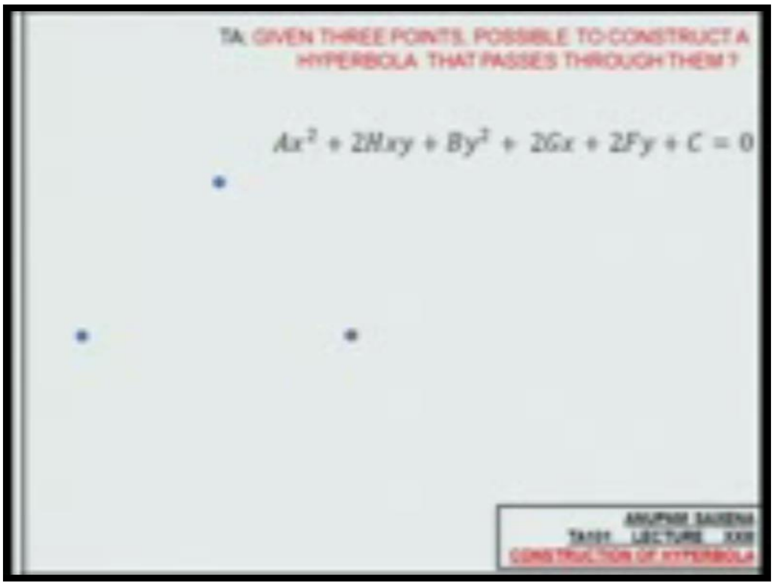

Anyhow so last time I ended up by posing this question to you. So, given 3 points is it possible for us to draw a hyperbola, is it? Yes or no? Who says yes? Given 3 points, is it possible for us to draw a hyperbola that passes through these points? So, you those who say yes want to think a little further generic equation of a conic 6 constants. How many conditions you have? 3, you think it is possible? Still I would not know let us see, how do you get a hyperbola? You have a cone well, so I am not really sure of I would be needing additional conditions to have a hyperbola pass to these points, but let us investigate further.

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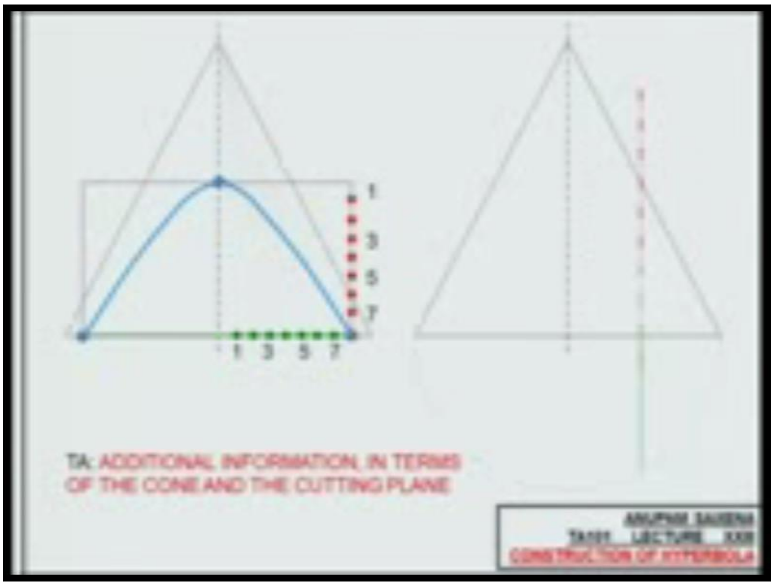

So, if you think about how we get the hyperbola this is how we do that. So, we have a cone and we have a vertical plane need not be vertical. If you have a plane that insets to the cone cuts it out. Then the intersection between the cone and this plane the red plane is going to be a hyperbola right. So, this plane can be vertical it can be parallel to be axis of the cone or it could be slant. What could it not be? It should not be passing through the, if I have this situation what do I get? We get a parabola otherwise we get a hyperbola right so this situation. So, we have a cone; we have a plane that cuts the cone and imagine that these 3 points they happen to be a result of this process.

So, the first point over here is obviously the intersection to in the cone and the plane. To get these 2 points we draw the base of the cone, we project this intersection point down over here we measure this distance. And if we rotate this cone by 90 degrees these 2 points are going to be line away from the axis of the cone by the same distance on both sides right. Now, if you have this sour speak additional information that you have a cone you have a plane. And you have 3 points which are a result of the intersection between the cone and the plane in red it is possible for you to draw or construct a hyperbola and this is how it could be done. So, construct first a rectangle, the length of the rectangle is the distance between these 2 points.

The width of the rectangle is essentially the height that this point lies above the base of the cone. The right vertical edge, divide that edge into equal number of parts. The bottom right edge of the rectangle divide that also into the same number of parts of equal length segments. Now, this is what is important from this point, first you number these points from the top of the rectangle to the bottom of the rectangle and from the center or the axis of the cone up till the end of the rectangle. So, the sequence is important top to bottom on the vertical side and left to right or center to right in the or on the horizontal.

And from this point, start drawing line segments that join this point to all these guys 1 2 3 4 5 6 7. What to do about these points? You start from the apex of the cone. So, from the apex of the cone join this point. Now, the point on the section between this line and this line would give you a point on the hyperbola likewise the point of intersection between this line. And this one here could give you a second point intersection between this and this would give a third point and so on. So, once you have the intersection points the points on hyperbola join these points and you will get a hyperbola. Now, this is only possible if these 3 points they happen to be such that you can extract the information about the cone and the plane that is cutting cone.

Let me ask you an inverse problem of course, this hyperbola symmetric so you would see the other side as well. So, do you think the additional information is required in terms of the cone and the plane that is cutting the cone in addition to these 3 points do you think that. So, if I give you 3 points and if I give you nothing else is it still possible for you to draw a hyperbola or if I give you these 3 points in such a way that you know the information about the cone. And the plane that is cutting the cone is available to you, you would be able to draw this hyperbola this is what my second sour speak inverse questions.

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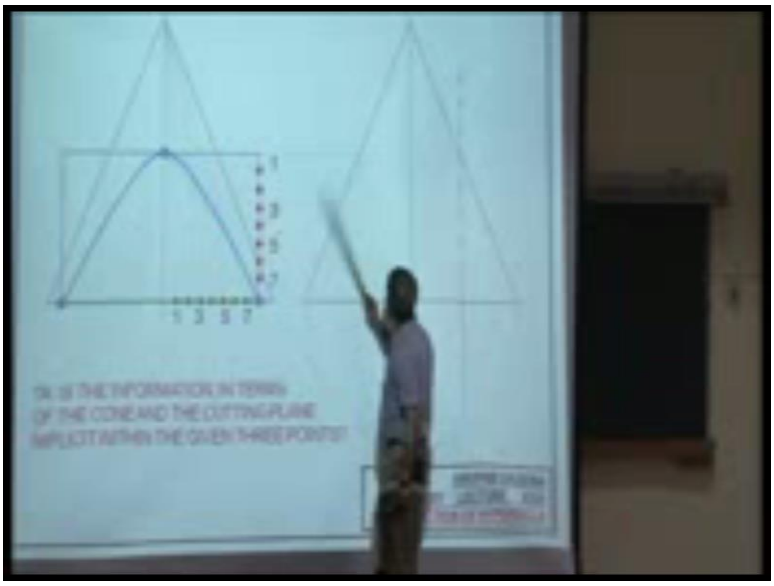

Now, given these 3 points, is it possible for you to find a unique set of a cone and the cutting plane? In other words, is it possible for you to figure if the information about the cone and this cutting plane is implicit within these 3 points that is the inverse problem and think about that, because you know last time a bunch of you guys came to the board. You know to the mike and started describing different techniques to construct a parabola given the c points So, I thought maybe it was possible think about that. So, for today's lecture, I am going to be using a special pointer so this my tea cups. And this is my pointer and the point that I am trying to drive through is interpenetration of solids.

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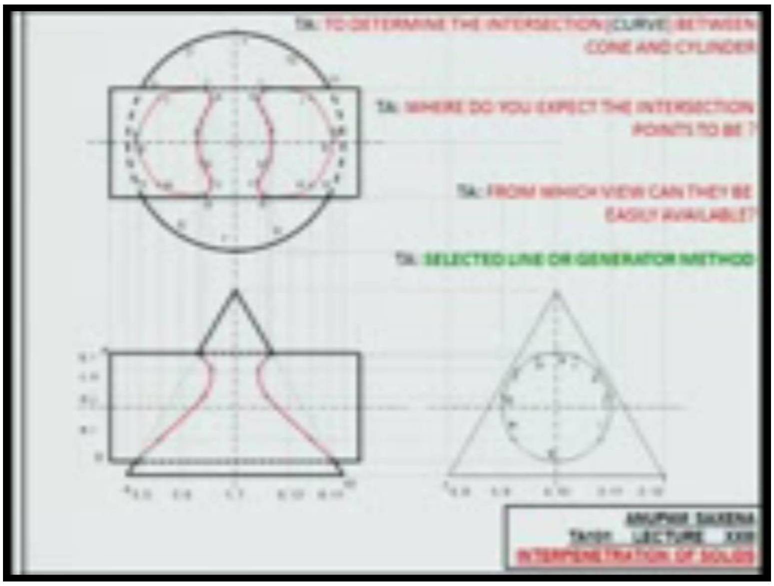

This is the first example that we have going to be drawing. So, imagine that this is a cone imagine that this is a cylinder. And imagine that you have put through the cylinder I mean put the cylinder through a cone. So, of course, you would have made some cuts, on this side of the cone and this side of the cone the problem is very simple. Figure out these cuts what the shape is? An extension of this is what you are going to be possibly doing in t a a to o 1 when you are working with sheet metal So, a cone is a developed surface right? Is it? Yes or no? So, it is cylinder. So, you can actually cut the cone out and spread it out on a plane, how was that going to look like? It is going to look like a big sector na. What it is going to look like big sector likewise a cylinder.

For example, you can cut it out and lay it out on a plane and its going to look like a rectangle. So, first things; first given the positions of the cone and the cylinder depicted by these 3 views in. It does not really matter, first angle or third angle; you know cone cylinder here; cone cylinder here; cone cylinder here. Figure out the intersection curves or in a single word interpenetration between the 2 solids. A question that you might want to ask yourself, where or which view would you possibly use to figure out where the intersection points would lie? In other words let me rephrase this question. So, which view will tell you the best or which view will give you the best information about the intersection points between the cone and the cylinder? Would it be the top view; would be the front view or would be the side view?

Student: Side view.

Side view; wonderful. Where would the intersection points lie? So, you say side view, but where would the intersection points lie? Would I be correct if I say that the intersection points are going to be lying on the circle right? Now, the second question is how do I extract these section points? How do I extract these intersection points? I will have to do something to represent my cone right? I will have to do something to represent specifically the slant surface of the cone. To do that this is what I will do; I would divide the base of the cone in the top view into equal number of parts. And this is where things become very important; this well labeling becomes very important. So, you need to be very careful when you are labeling, because if you are not then you will mess things up.

I number these points anticlockwise 1 to 12, once again I divide the base of the cone into let say 12 equal parts could be 15 could be 9 could be 10 whatever they have to be equal, right? They need not be equal as well. Now, once I do that I start taking projections of these points on the base of the cone on to the front view be very attentive be very careful. And then I start labeling the points on base of the cone. Now, which is this point great how about this point wonderful 2 6 great what 1 7 next 8 12; next 9 11; next 10. Now, do the same exercise I will represent these points over here on to this base of the cone in the profile view and for that I am going to be using this 45 degree line. So, essentially I am going to be taking these projections, you know and transferring them on to the profile view. The reason why I said the numbering is going to be very important, because here would you know that the numbering will change of course.

So, when I do that I am going to be join the apex of this cone to all these points on the base of the cone same thing. Now, what would this point be? Great 8 and 6, 8 and 6, next 5 and 9, next last, great. Now, what you have noticed this 2 things? One; that is going to be very easy for you to extract the points of intersection from the profile view, the points of intersection are going to be lying on the circle. And once you have these, what I call generators or select lines they essentially represent what the slant surface of the cone, right? Your intersection points are going to be the intersection between the slant surface of the cone and the circle he is a little late oh that is intersection points are going to be the intersection between these lines which represent the slant surface of the cone.

And of course, the circle your first point of intersection will be here, what is this point intersection between which line on the cone, on the surface of the cone 4 in 10 and the circle project that on to collate pointed project that on to the front view. Now, in the front view, how would this show up, how would this show up in the front view? It would be on what slant lines 4 in 10 were is 4 here were is 10 there. So, point a is going to be on these 2 lines in the front view right. So, of course, in fact, its timing was just perfect so imagine that this is a cylinder imagine that this is a cone. And this is how you are actually see the interaction between the cone and the cylinder like. So, there would be a point of intersection here and they would be point of intersection here right there.

So, what I would do is I would keep this on the chair in front of you. So, that while I am working with the slide you can imagine how the intersection is going to shape up maybe I could use the table better. Now, the other 2 intersection points are going to be here and here. So, point b would lie on the slant line 5 9, so go on to the front view 5 is here 9 is here and it is at this height. So, you will have point b here and corresponding a point b there likewise there would be another intersection point on the other side. So, let me call this point l l will be at the same height from the base of the cone and l is also going to be line on 3 11, so 3 11 is here. And here so be very careful were the intersection points are going to be lined, because tracking that is very important once you understand this.

Third intersection point on the circle lies on leader 6 8 project that call it cylindrical project that on to the front view. Identify were leader 6 is or were generator 6 is here mark that were generator 8 is, mark that there would be intersection point on the other side of the cone. Call it k and k would essentially be line on the same generators here and here in the top view. Now, this is so in the in the third angle projection this is what the profile view right. So, if you turn it by 90 degrees this way this part is something that is going to be facing you here. So, this part is something which is going to be facing you this part is going to be behind you so understand that. So, this, these 2 intersection points they are facing you and correspondingly other 2 intersection points are behind the cone.

So, understand that rest is a little mechanical so d j they lie on 6 8 project that identify 6 and 8 t. And I they would lie on 5 and 9 3 and 11 5 and 9 would be here 3 and 11 would be here. And eventually the last point g that will be lying on slant line 4 and slant line 10. So, you are pretty much your intersection profile pretty much is might going tough no you are in section profile pretty much is done in the front view join these points. And this is how your intersection profile will show up. So, once you have drawn these curves transfer these points on to the top view once again be very, very careful these points they have to lie on specific slant lines on the cone or generated lines on the cone.

For example, this one were would this lie 4 great this one would lie again on 10. And this is the points a how about this one 3 and 5 careful, how about this one, one would be on 5 the other one would be on 3 likewise this one would be on 9 and 11 so 9 here. And if you project them upwards lead is important b would be on 5 and 9 respectively. And l will be on 3 and 11 respectively project this point they would that be. So, think about this so this point this point they actually represent 2 points c and k right. So, this point would lie on 2 and 6 let see where it is 6. And if you go on to the profile view point c lies on 6 right and k would lie on 12 right no k would lie on 2 likewise from the right these points would lie on leaders or generators 8 and 12 and so on so forth.

So, keep projecting be very careful those in section points they have to lie on specific slant lines I will go little slow. So, that you can follow and labeling is very important so if you mess up the labeling then you are going to be messing up the entire procedure. So, labeling is very important so looks like you have all the intersection points in top view. Now, let us look at the profile view now if you are looking at the top of the assembly of the cone and the cylinder, which points are visible to you which of these points are visible to you the points above the. So, these points all the points above this axis; this horizontal axis of the cylinder they are going to be visible all the points below this axis they will be invisible right. So, this part will be visible a b c d on both sides and this part will be invisible or hidden.

Now, if you going to figure of the rest of the visibility of the cone cylinder, this how is going to be. So, this part of the cylinders visible of course, this part is visible this part is inside the cone so it is hidden. Of course, these vertical edges or these vertical circles are visible, visible, visible this part is invisible again because this is inside the cone. And this part of the cone is visible. Now, notice that I have not shown these edges using hidden lines why is that why is that they have to be removed. So, that the cylinder is accommodated within the cone likewise the same thing top view this part is visible; this part is visible of the cylinder this is inside the cone. So, that first hidden visible, visible visible, visible, this part is inside the cone again so it is hidden. This part of the base of the cone is visible; this part is below the cylinder so hidden visible and hidden.

Now, notice what is happening here and likewise what is happening there the state of the line is changing right. So, the cylinder is visible up till this point and then it is going inside the cone. And then it is coming out of the cone and it is visible again and this is happening were at the junction. Of course, were the cone and the cylinder are interacting right at this intersection point just a double check so this called the select line method or generator method. So, once again to summarize start by looking at the 3 views carefully, start by observing the 3 views carefully number 1, number 2. Figure out which view will give you the best information about the intersection points, most direct information about the intersection points in this case it was a profile view.

So, once you have figured that thing out in this case represent the surface of the cone using these generator lines. So, they are essentially the slant surface of the cone they would represent the slant surface of the cone. And of course, the points of intersection would be between this circle in this example and these slant lines once you get the points of intersection label these points very carefully they have to lie on you know certain lines. Once you label them carefully transfer them on to the front view and the top view straight forward the same example using a slightly different method. And that is the method that I am not quite sure. If you guys are comfortable with the cutting plane method you guys probably did not do very well with the quiz with the cutting plane method.

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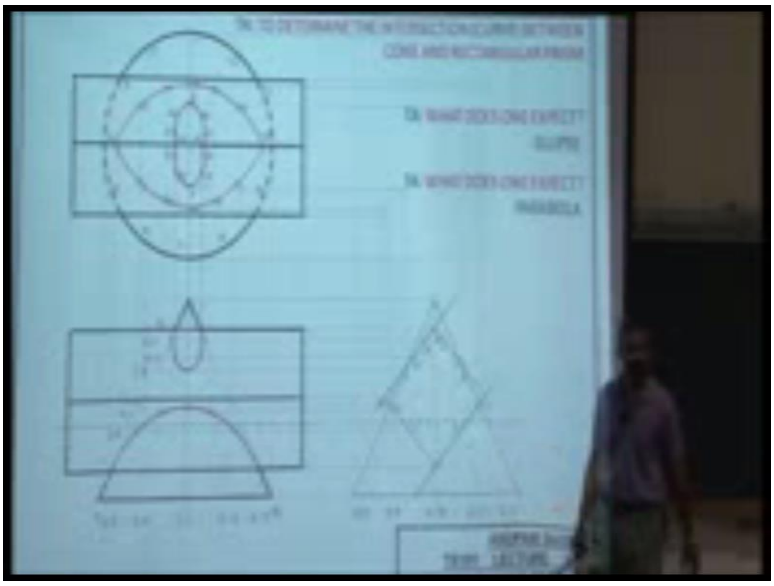

So, here it goes I will go a little quick with this so the idea is very simple so you have the assembly of the cylinder and the cone just slice it using a bunch of parallel planes slice it using a bunch of planes. Now, if you slice the cone with a plane parallel to the base of it, what do you get a circle and if you slice a cylinder with a same plane what do you get a rectangle? So, your intersection points will be what your intersection points will be essentially the intersection between the corresponding rectangles and the circles this, what the basic idea is. So, you have this view for example, slice it with the bunch of horizontal planes the green planes are those cutting planes number them 1 2 3 4 5 6 7 8 9 from top to bottom. Now, these planes are going to be intersecting the cone in certain at certain heights project. These guys up in the top view the intersection between the plane and the cone will appear as a, what you guys are dosing off as a what great.

So, each plane will intersect the cone at a circle sketch those circles in the top view. And of course, do not forget to name or number those circles. So, from inside to outside they are numbered as 1 2 3 4 5 6 7 8 9. Now, from the profile view from the profile view you know that these planes are also cutting the cylinder. So, you would have this intersection point these 2 intersection points you know and so on so forth. So, this portion over here will essentially be a rectangle project this rectangle on to the top view for each and every plane each and every cutting plane. So, this point here will essentially be a line rectangle with no area this would be rectangle b b rectangle c c rectangle d d. And finally, rectangle e e and remember that the same rectangles will be below the axis as well. So, once you have identified or named those rectangles name them in the top view. So, a would be a line b, b would be this one c c would be this one d d would be this one and e e would be essentially the rectangle corresponding to this plane here.

Now, your first point of intersection is going to be between some circle and some rectangle. Can you give me that circle 1 and rectangle a first point; second point is between 2 and b b. How many points do you expect 2 1 at the bottom? The other one at top third one is between circle c and rectangle c c or circle 3 and rectangle c c 1 at the bottom. The other one at top fourth one is in between 4 and d d bottom and top and fifth one is of course, in between circle number 5 which is here passing through the axis of the cylinder and this rectangle here and do not do not stop here keep going down. So, this one is in between let say d d or maybe c c right rectangle c c and circle what circle what 6 6. Next one b b and 7 is a b b and 7 no it was c and 7 this guy here and this guy here c and 7, so b b and 8. And of course, eventually a single point here a, and plane number 9; this guy is symmetric.

Once again it is going to be these intersection points above the axis they are going to be visible the intersection points below they will be hidden they will be invisible. So, this is solid and this part is hidden so the intersection is symmetric mirror image. And drop the corresponding intersection points down on to the front view where would you want drop this to which plane which plane? So, here you have plane information here so which plane number, what plane number 9 these 2 guys here which plane where do they lie? Were these points lie on which plane do they lie what plane number 8 great these guys plane number 7 plane number 6 key working backwards eventually up till plane number 1. And of course, you have this intersection point well this intersection is again symmetric. So, get the mirror image get this intersection point or intersection curve right. So, do you expect the 2 methods to give you the same result do you expect the 2 methods to give you the same result?

Student: Yes sir?

Compare now, you have done AutoCAD, if you do 3 d modeling or auto cad take the cone same size, take the cylinder same size perform mid section get the cylinder in to the cone perform mid section. And if you try to compare that with what we have your identical. So, why is this exercise important? Take a look at this example. Now, what I had done is I had made a little modification to this cone I have cut a certain feature of this cone I have cut a certain feature away. And this feature is such that it allows this hexa hexagonal the prism to go very nicely into this cone with any effort I am making some effort. So, if you do not know how to do this or if you cannot do this, this is not possible for you. So, this method permits you to figure out the intersection between the hexagonal the prism and the cone and eventually figure out the part that needs to be cut away from the cone.

So, that when you fold it up its a lot easy off for you to let this hexagonal the prism pass through this cone something that will be possibly doing in t a t o or 1. And of course, this the portion which is cut away from the cone. Another example, so this time we have a rectangular a prism rectangular represent passing through the cone. Once again question number one which view will give you the right information about the intersection points profile view where would the intersection points lie? Where would the intersection points lie? They will lie on these edges or these edges. So, essentially the problem is to figure out the intersection between the slant edges of the cone slant edges that lie on the surface of the cone and these edges right same thing as in case of example number 1 previous example.

So, this your first intersection point it lies on this horizontal surface I am going to go a little fast so just follow this. So, they are 2 intersection points over here the second intersection point comes from here the interaction between the generators 5 9. And this edge correspondingly there will be 2 intersection points on the front view b c. The third one comes from here lies on 6 and 8 fourth one would come from here possibly that would lie on number 7 and 1 7 1 would be at the center here. So, keep following this procedure so h and I would lie on 6 8 and to 12 j and k would lie on 5 9 3 11. And eventually the bottom intersection point would lie on 4 in 10 what so if you think about this phase of the prism if you think about this phase of the prism.

Of course, there would be another intersection point that comes from here and here visibility a part of it is outside a part of the pyramid is outside the cone some parts is inside. Some part is hidden this part is visible this part of the cone is visible and of course, this edge is visible. Now, if you think about this plane what would this plane give you? So, this plane on the prism, what would this plane give you? Would give you ellipse, what would this thing give you? It will give parabola. So, the top part is the electrical part and the bottom part is the parabolic part this thing should be parallel to the what? So, what you are saying is this plane should be actually here somewhere this will give you an ellipse?

So, both are ellipses are you sure what it is a it is a half ellipse. So, you guys are not sleeping after all good, so this is so. So, project these guys up on to be respective slant lines on the cone be very careful with regard to where these intersection points are going to be. You can figure this thing out this not this is not very difficult you can figure this thing out where the intersection points are going to be. Now, which part will be visible in the top view which part of the curve will be hidden in the top view which part of the intersection curve will be visible which part will be hidden? Would this part will be visible yes; yes would this part be visible?

Student: No sir.

No; so everything from here up will be visible everything from here to down will be hidden. So, this part of the prism is visible that is hidden this is visible, visible, visible visible that part of the base of the cone is hidden part of the base of the cone is hidden visible, visible. So, the center part is going to be visible, because and the bottom part or the other part will be hidden. So, the part correspond to this region over here that be hidden. Once again if you go back and if you work it out on auto cad this is what would you see coming back you know I mess up my coordinate geometry.

So, a plane that is parallel to the base of the cone if it cuts the cone it give you a circle a slant plane will give you an ellipse right how do you get parabola one at time if it is. So, if it is parallel to the slant line anywhere you will get a parabola here you will get a hyperbola this one will give you a hyperbola right. So, it need not be vertical, so a plane a plane need not be parallel to the axis of the cone to give you a hyperbola. So, it could also be like so till it is again parallel to the slant line that is when you will get a parabola. Now, of course, this will give you a what? This will give you an ellipse. So, fix this.