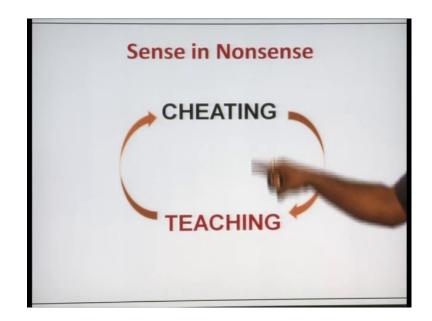
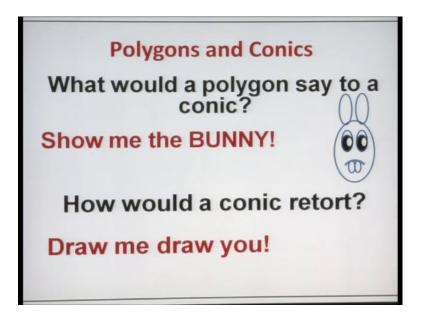
Technical Arts 101 Prof. Anupam Saxena Department of Mechanical Engineering Indian Institute of Technology, Delhi

Lecture - 2 Think and Analyze

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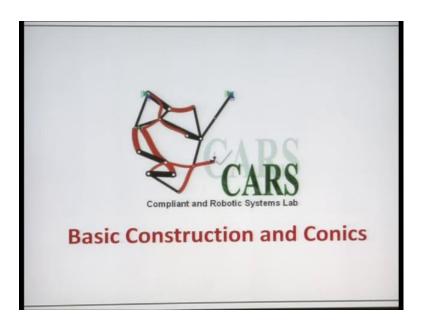


Lecture 2 T A 101 think and analyze; you know why always try to extracts some sense from nonsense. So, you know what this word is; you guys are students so you would be familiar with this word cheating. And I am a teacher so I am familiar with this word teaching. Now, what is common between these words cheating and teaching? You know if you rearrange the alphabets over here you essentially get this; you know what this called? So, these 2 are anagrams of each other; you know on the other side of the table you are probably be indulging in this; while if you are on this side table like me I would be indulging in this. And, my job is to have you guys not do this anyhow. (Refer Slide Time: 01:28)



Polygons and conics; what would a polygon say to a conic? So, you know this is this I am a movie buff and I was watching this movie j m and guyer. And, you know j m and guyer was managing one of the football players; one of the upcoming football players. And, 2 faces in that movie which I like very much which I guess you all may all so be familiar with anyway. So, what a polygon say to a conic show me the bunny in the movie it is like show me the money, show me the bunny. And, what would or how would a conic retort? Draw me draw you; let me help you draw me draw you all right. So, ((Refer Time: 02:23)) sorry.

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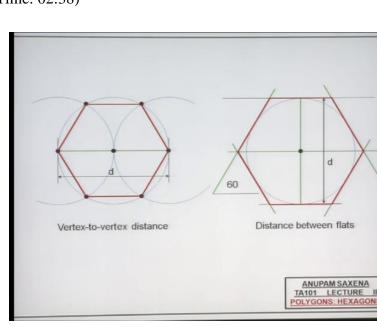


But anyhow it is a basic construction and conics. So, this what I am going to be covering in this lecture.

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Topic	Week (No. of Lectures)	Lab
ntro and Basic Constructions	Week 1 (2)	
Orthographic Projections	Week 2 (2)	J Lab 1
Orthographic Projections	Week 3 (2)	Lab 2
Isometric Projections	Week 4 (2)	J Lab 3
Missing Views	Week 5 (2)	J Lab 4
Sectional and Assembly	Week 6 (2)	J Lab 5
Oblique Projections	Week 7 (2)	J Lab 6
Perspective Projections	Week 8 (2)	J Lab 7
Lines and Planes	Week 9 (2)	Lab 8
Lines and Planes	Week 10 (2)	J Lab 9
Auxiliary Projections	Week 11 (2)	Lab 10
Intersection of lines/planes/solids	Week 12 (2)	Lab 11
Intersection and Development	Week 13 (2)	Lab 12
TOTAL	26	12

So, this is the second lecture and the first week introduction basic constructions all right.



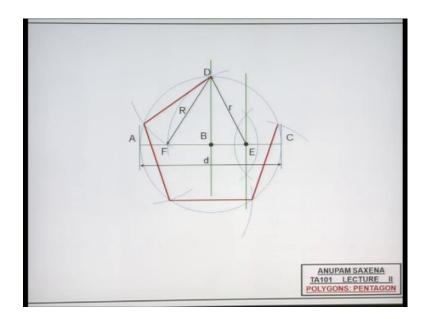
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So, we were be focusing on how to construct hexagon; you know it is in the family of it is a member of the family of polygons. And, there this 2methods that will be discussing vertex to vertex distance method and distance between flats. So, given 2 vertices; this one and this one and let us say distance d between these 2 vertices; how do I construct a hexagon? So, this what I will do; I would divide this line into 2 equal parts you know I will drawn arc with this is a center and radius as d over two. And, another arc with this as center and radius has d over two. So, that these 2 arcs they happen to meet at the midpoint of the line and also tangent to each other. And, with this midpoint as center I will draw a full circle of radius d over two; essentially this circle is going to be travels in through the end points of the edge.

So, focus on this end section point this one, this one and this one. So, and these 2 in addition; so 6 points they will essentially form the vertices of the hexagon all right. So, if you look at this circle and the hexagon; so this circle happens to be the circumscribing circle of the hexagon. Second method distance between flats. So, this is the flat over here, this is second flat over here you know this distance d is given to us; had we draw hexagon; mark the midpoint of this line segment draw circle of various d over 2 with this midpoint as center I will draw horizontal line. And, then I will draw a line which is tangent to the circle; and this line is such that it is making an angle of 60 degrees with the horizontal; I will draw a horizontal line tangent to the circle at the top; I have extend this horizontal line below again tangent to the circle at the bottom. And, then I have draw this line which is at 120 degrees to the horizontal and a line parallel to this line.

So, this point over here this point, this point, this point, this point and the 6 th point here; they are essential from the vertices of hexagon. So, you can actually joint these vertices and eventually construct a hexagon. So, the way this construction is done; the circle happens to be the inscribing circle; circle inscribed within the hexagon; vertex to vertex distance, distance between flats. In this case the parent circle or the circle happens to be the inscribing circle; in this case the parental main circle happens to be the inscribing circle.

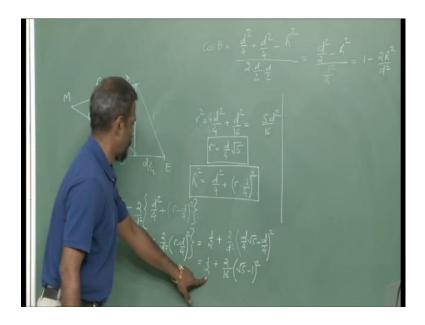
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Now, this is something very interesting; had you draw pentagon? So, given a line segment; whose end points are at distance d. So, the end points are marked as A and C. So, with the midpoint of A C line segment as center we draw a circle with radius d over 2. And, then what we do is we bisect this line segment B C; we draw a vertical passing through B intersecting the circle at d. So, once we bisect this line segment B C we will get a point over here this mark it E. Now, notice that we are going to be walking with distance D E; let us say this is of magnitude r. Now, with this distance and with E center we are going to be drawing an arc which is going to be cutting the line segment A C at F.

Now, this length over here let us say this is R; with this radius R and with let us say point D as center whether cut the main circle at this arc or at this point. So, this length over here from here to here is R. Now, with this as center and the same radius we will cut the main circle again at this point. And, we will continue doing so until we get 5 points on the circumstance 1, 2, 3, 4 and 5; these 5 points will constitute or will be the vertices of the pentagon; we are seeking. Now, think and analyze; how is it that you think that this is a pentagon; is it difficult for you to appreciate or perhaps not? Let us try to prove that this is really a pentagon; let me grab the pieces of chalk and let me focus on this region over here.

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So, this is my point B here is my point D; D B is half of d diameter of the circle; so it is d over 2. Now, this distance is again d over 2 and B E is half of B C; so B E would be d over 4. And, I have look at this distance D E; let us first try to figure what this distance is? So, this is r then will have r square by Pythagoras theorem as d square over 4 plus d square over 16 this is equal to if I am multiply and divided by 4 here; 5 d square over 16 implying that r is d over 4 times root of 5 all right.

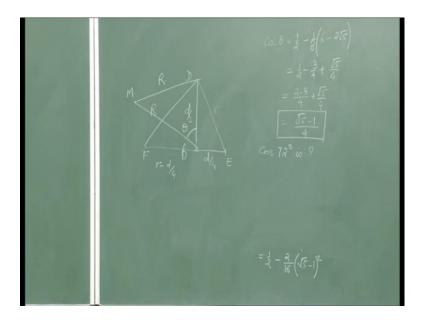
Now, let us try to figure what this distances r; if I go left towards from B I am looking at point F here. Now, by construction; so with this point E as center what I had done was I had cut the diameter A C with the arc of radius r. So, this distance here would also B R. And, therefore F B will be r minus B E which is d over 4. So, F B is r minus d over 4 and so this R here D F can also be determined using Pythagoras theorem. So, R square equals d squared over 4 plus (Refer Time: 11:43) this r minus d over 4 the whole square.

Now, let us ((Refer Time: 11:56)) this thing in this squared form because we needing that. Now, focus on the triangle; well let us call this point M, M B D. Now, this distance M D is a same as R this distance is d over 2 the radius of the circle and this distance is again d over 2 the radius of the circle. So, let me draw this point here somewhere M; so this distance here is R this is d over 2 and M B would again B D over 2. And, let me say ((Refer Time: 12:56)) trying to find what this angle theta is? So, we can use the cosine rule and the cosine rule is such that cosine of theta equals d squared over 4 plus d

squared over 4 minus this distance over here R square over 2 times d over 2 d over 2 which is equal to this is d square over 2 minus R square over; this is again d square over 2 which is 1 minus 2 R square over d square; I hope I am doing this right; I can substitute this value over here. So, cosine of theta; let me write the same term here equals 1 minus 2 times R square is d square over 4 plus small R minus d over 4 the whole square over d square which is equal to 1 minus within parentheses this would be 1 over 2 yeah minus 2 over d square R minus d over 4 the whole square.

Let us substitute this value over here and let see what we get? This is equal to well half plus 2 over d square and this is d over 4 root of 5 minus d over 4 the whole square right. And, so this would be equal to half plus 2. So, this d would come out and get cancel with d square here; from here we will be 16. So, let me write this thing down over here over 16 and then this is root of 5 minus 1 the whole square; let us see what this is? Well, let me erase this and take this thing up over there.

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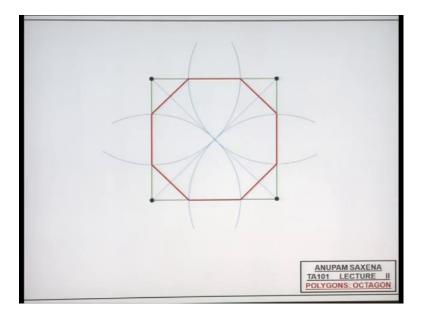


So, this is half plus 1 over 8 and this is 5 plus 1 6 minus 2 root of 5; seems a ((Refer Time: 17:04)) let see were takes me half plus 3 over 4 minus root of 5 by 4 this is equal to if I multiply and divided by 2; I have got 5 over 4 minus of root 5 by 4. So, I just realized that I made a mistake here make I make low mistake; so instead of the sine over here I should have really had a negative sign. So, I will go back in correct my mistakes. So, this is a negative sign here because of which this would be negative because of which I guess

this would be negative and this would be positive. And, so this actually would be equal to you know 2 minus 3 over 4 plus and root of 5 over 4 which is root of 5 minus 1 over 4. And, if you go back and try to figure out cosine are 72 degrees is than you would see that this is nothing but this value here. So, in the sense through geometry constructions what we do is we essentially divide the circle into 5 equal part in such a way ensuring that this angle here theta is equal to 70 degrees; this angle.

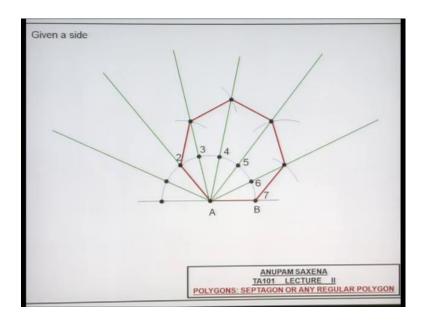
So, once we ensure this it becomes little easy construct again ((Refer Time: 19:06)). So, this was an algebra, this was the algebra adjust to you know support the construction. But you would have realize that we did not actually algebra and construction of the pentagon; anyways let us move forward. So, these 5 vertices is there essentially be a part of this pentagon; how would you construct an octagon.

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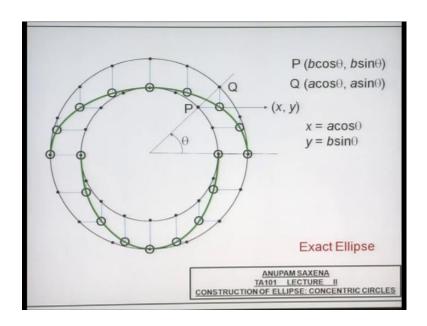
So let us say you have square of dimension sides all right draw 2 diagonals indicate this vertices from this a center and this length as radius draw an arc; do the same with this center and the same radius again the same radius with this center. And, finally with this radius with this is center and same radius. So, draw these bunch of arcs and wherever these arcs gone a been intersecting the edges so here, here, here, here, here, here, here and here; there essentially have 8 vertices join them and you will be getting an octagon. So, pretty simple construction I think an analyze you know through algebra or otherwise if this is really an octagon knock.

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How about a regular Septagon or any other regular polygon for that matter; given a side you know. So, let us say you have given a side you have been given a side all right; so you have this point over here this distance is known. So, draw a semicircle and divide this semicircle into; so since we focusing on Septagon. So, divide this semicircle into let us say 1, 2, 3, 4, 5, 6 and 7 equal parts and name them in such a way that you let go of this first point on the left. So, start naming them as 2, 3, 4, 5, 6 and 7 right. And, then draw this happens to be the centered of this semicircle over here; name that center as A. And, one of the ends of the semicircle as B and draw and draw radial lines; in such way that they happened to be connecting this point A here and all these points on the semicircle. So, once you have done with that take this as your distance, your reference distance you know mark that on your compass with 0.7 as centered mark an arc on this edge; with this point as centered with the same radius mark a point on this line segment, with this as centered same radius mark a point over here and keep going forth until mark a point over here. And, then once you have gotten all these points vertex 1, 2, 3, 4, 5, 6 and 7. So, these are the 7 vertices that will essentially constitute your ((Refer Time: 22:40)); so this is a generic method that allows you to draw any regular polygon.

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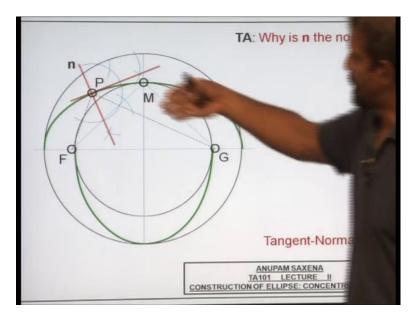


Construction of an ellipse by means of concentric circles. So, you got this larger circle, you got this smaller circle they happened to be concentric same centered divide both circles into equal number of parts. So, I would have divided the outer circle as or into 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16; 16 equal number of parts and likewise the inner circle also into 16 equal parts. Now, this ((Refer Time: 23:27)); from one of these points draw horizontal, from one of these points corresponding points draw vertical or vice versa; let us see. So, I draw horizontal from inner circle and vertical from the outer circle corresponding points over here horizontal, vertical, horizontal, vertical, horizontal, vertical from the inner and vertical from the outer; horizontal, vertical horizontal, vertical you know horizontal. And, these points they essentially will be lying on an ellipse all right; I could do the other way around.

For example, instead of drawing the horizontal from the inner circle I would be drawing the horizontal from the outer circle. Well, this is the construction of the ellipse for the first case as I said I would be drawing horizontal from the outer circle and vertical from the inner circle, inner circle this time all right. So, horizontal, vertical, vertical, horizontal, vertical, vertical, horizontal, vertical, horizontal, vertical, vertical, horizontal. And, thus I will essentially beginning these points and they will happened to fly on an ellipse which essentially will be you know oriented 90 degrees. So, these are the points on the ellipse and I will join these points ellipse and get another ellipse; another question think and analyze for you is this or this an exact ellipse or not let us try to find it out?

So, let us join the corresponding points on the inner and outer circle have this horizontal, have this line segment over here' let this angle. So, let this point P the let this point be P and the coordinates of P would b cosine of theta and b sine of theta; where these the center of radius of the inner circle likewise point over here Q; will have coordinates a cosine of theta a sin of theta; where a is the radius of the outer circle. And, if you focus on this point what would be the X coordinate of this point and what would be the Y coordinate of this point; can you guess? Yeah, these values over here. So, the X coordinate of this point is essentially will be the X coordinate of Q and the Y coordinate of this point would be Y coordinate of the ordinate P. So, essentially if this point is represented as X and Y; then X will be a cosine of theta Y will be b sine of theta. And, of course if we go back to your 11 th grade or 12 th grade coordinate geometry you would readily confirm that X Y is nothing but a point on the ellipse. And, therefore this ellipse is an exact ellipse yeah all right.

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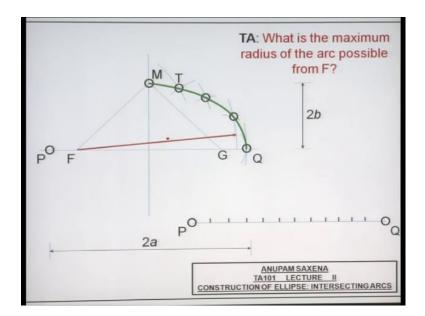


So, if I want to draw a tangent or normal at any point on the ellipse what do I do? Let us say have this point over here; I would like to draw a tangent or a normal at this point P. Now, while be doing as I will be using one of the properties of the ellipse; first I try it you figure the focal or the 2 foci. Now, let me go back let me draw the horizontal and let

me also draw the vertical I will set at this point let us say. And, I would also I would rather already know the length of the major axis of the ellipse; I will take half of that distance with this as centered I will draw 2 arcs over here; and here this point being M I will mark a point over here. So, this distance as half the major axis and I also mark a point on the other side of the major axis. So, these 2 circles would represent the 2 foci of the ellipse. And, the property I am using is straightforward. So, if I take any point on the ellipse; so the distance of this point from this center and this center if I sum these 2 distances that some will always be equal to the length of major axis. So, 2 foci F M G and this is the property I just had mentioned.

So, being a point on the ellipse distance F M plus distance M G has to be equal to 2 a; where a is the length of the major axis. So, having found the 2 foci I will join or I will draw line segment joining F and P I will draw another line segment joining P and G; I will take the angular bisector of the angle F P G you know how to draw the angular bisector. And, this line over here essentially will be the normal at point P; all I need to do is you know using a the compass or using a ruler I have to bisect this angle; this angle is 180 degrees have to bisect this. And, I will do exactly the same thing you know bisect the angle; once I do that this line segment here will be the tangent to the ellipse, normal to the ellipse. Now, think and analyze T A why is the n you know this line segment normal to the ellipse.

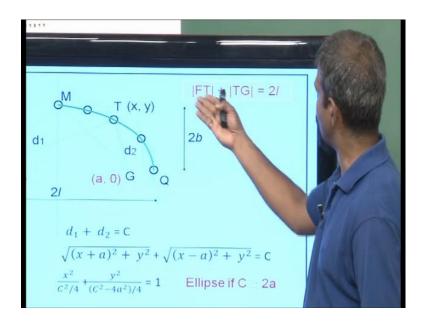
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Construction of ellipse by intersecting arcs; again I use the same property I had discussed before. So, if I have a point M on the ellipse and if I have these 2 foci of the ellipse. Well, this is not these are not the foci but they happened to be the points at the of the extreme ends of the semi or of the major axis. So, if the length of the major axis is 2 a and the length of the minor axis is 2 b should be this way I guess never mind. So, the property is any point well let me see if F and G are 2 foci and if T is any point on the ellipse then F T plus T G has to be equal to 2 a all right. So, what I will do is I will divide 2 a into equal number of parts let us say; the entire length this length over here it just given to me into equal number of parts. Let me mark the extreme points on the ellipse as P and Q there I go. Well, and then what I have to do is if I choose let us say this length and draw an arc where the center F and this length.

Then, I will have to choose this length for the radius of the other arc that I am gone be drawing with G as centered that would be my point; one of my point stay on the ellipse. And, I will do that you know for as many points as possible; once again if I choose you know any other point over here let us say this one if I choose this length with this length as radius and this is centered I will draw another arc. And, with this as the radius and center as G; I draw the second arc the intersection these2 arcs will give me another point. So, I will keep getting these points and that will essentially help me draw the ellipse straight forward. Think and analyze what is the maximum radius of the arcs possible from point F or G; that is for you; think always a good idea to think.

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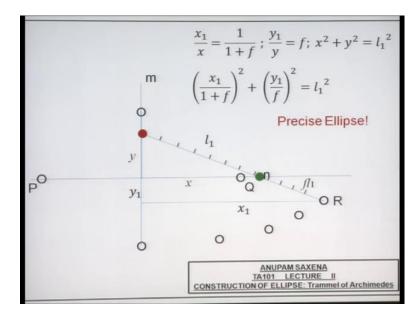


Let us do a little bit of algebra; choose any point over here lets mark F T as distance d 1 and let us mark G T as distance d 2. Let the coordinates of T be X and Y. And, it so happens that point T is a result of this property that d 1 plus d 2 has to be equal to the length of the major axis of the ellipse 2 l. And, essentially d 1 plus d 2 is a constant because 2 l is a constant for given ellipse. Now, you know for simple case I choose G such that as coordinates (a, 0) then F would be such that it would have coordinates minus a, 0 with this point as center.

And if I do a little bit of algebra; so d 1 essentially would be you know X plus a the whole square plus Y square and d 2 will essentially be X minus a the whole square plus Y square and that is equal to constant, and with a little bit of algebra I will get this equation X square over C square over 4 plus Y square over C square minus 4a square entire thing over 4 is equal to 1. And, it so happens that if we chose you know if you represent this you know sum a 1 if you represent this as sum b 1 then it is like X square over a 1 square plus Y square over b 1 square is equal to 1 which is the equation of an ellipse. So, apparently this happens to be an exact ellipse. And, precise these so this if constant C is greater than or equal to 2 times a I think yeah.

So, you can use the same property; let me go back it is the same property have a pin here at one of the foci have another pin here at the second focus; and you know you can have a loop of string. So, this distance and this distance making sure that sum of these 2 distances is a constant you can draw an ellipse.

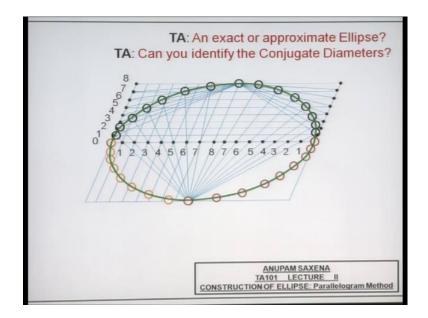
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Trammel of Archimedes another way of constructing an ellipse all right. So, let us say you have 2 line segments m and n. Well, so you have this length equals 2 a you know have 2 points red and green. And, have this entire ruler place on the vertical with the same 2 points red and green. And, start moving this ruler in such way that the green is always on the horizontal and the red is always on the vertical. And, let me go back see what is happening? Focus on this point green always on the horizontal red always on the vertical you know this point is actually leaving some traces there over here. And, these traces are nothing but points which are gone a be lying on the ellipse. So, you can do the algebra and figure that this also kind of grids an exact ellipse; let me skip this algebra or let me go back and let me try to work out the algebra for you all right. So, for any position of this line you will have this as distance y will have this as distance x; join point r with the vertical over here.

So, this triangle and this larger triangle they are essentially similar triangles; let me call this y 1 let me call this x 1. And, let me have this length as 1 1 and this length as sum factor f (1 1). Because the triangles are similar it should be possible for you to work out the ratio x 1 over x is the same as1 over 1 plus a y 1 over y is a same as f and x square plus y square is equal to 1 1 square; do a bit of algebra. And you will get x 1 over 1 plus f the whole square plus y 1 over f the whole square equals 1 1 square precise ellipse. I make lot of mistakes so do not trust me; work this algebra all by yourself you will realize that I may have made a mistake.

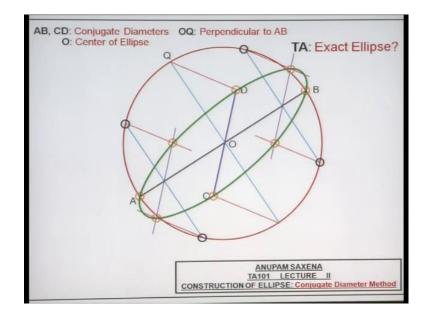
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Construction of ellipse the parallelogram method; given a parallelogram how do you construct an ellipse? Well, divide this parallelogram into a bunch of parts at the top I guess 8 parts here you know like so. And, also well but before that mark these points 1, 2, 3, 4, 5, 6, 7 and 8 and also divide this side of parallelogram into those many equal number of parts. Now, watch out for the way I am numbering them; I am numbering these points from here 0, 1, 2, 3, 4, 5, 6, 7, 8 and over here it is gone a be slightly different, it is gone a be from here to here. So, once have that then I will have 1 reference point over here and other reference point over there and watch out for the line segment I am constructing. Well, in fact I can have the same points on this side; watch out for the line segment I am constructing. So, I will imagine that this point and this point essentially will be lying on the ellipse; with this as reference I will have my line segment pass through point 1, with this as reference I will have my line segment. From here I will have line segment pass through this point 2 and from here I will have my line segment pass through this point 2 here.

So, the intersection of this line segment and this line segment over here will give me another point on the ellipse. So, it is always a good idea for you to you know have some reference associated with the points or the geometric entities at here using for construction. For example, from here I will draw line segment passing through 3, from here I am gone a be having a line segment passing through this point 3 here; intersection of these 2 line segments will be a point on the ellipse and I will continue like so all right. I can do the same thing and on this side of the parallelogram we have got just follow of the construction. And, what I could do is I could mirror these sets of points; mirror these sets of points over here. And, once have these points essentially all these points they have to be lying on the ellipse all right. Now, you try to figure with this is an exact or an approximate ellipse; also it is a possible for you to identify the conjugate diameters; try to figure it up?

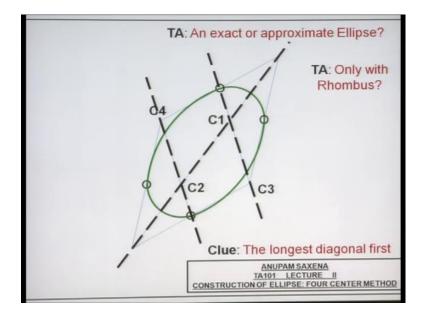
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Construction of ellipse another method by means of conjugate diameters. So, given 2 conjugate diameters; let me name them as A B and C D they intersect at point O; A B and C D are conjugate diameters. Let say the O is the center of the ellipse. Now, with O as the center and A B as the diameter I am gone a be drawing this outer circle in red. And, let me draw O Q such that O Q is perpendicular to A B; let me join D and Q. Let me divide this outer circle into say equal number of parts; let me also join point C with the end point over here. Now, you will see for I am using line segments with different colors; just to indicate that they are parallels. So, this line segment is parallel with this line segment. Now, watch out what I am gone a be doing with these points be careful. Now, I am gone a be drawing a line which is parallel to this line here O Q and likewise over here; again I am depicting these lines using color blue just to emphasize that these lines are parallel.

Now, from here I am gone a be drawing a line parallel to C D and with one of the point over here I am gone a be drawing a line segment parallel to D Q. And, likewise from here as well I am gone a be lying rather I am gone be drawing a line segment, I am not lying i am going to be drawing a line segment parallel to D Q from here. So, of course this point and this point will be a part of the ellipse; likewise I will draw a line segment parallel to C D. Such that it passes through the intersection between this blue line and line A B; from here I will draw a line parallel to O D; from here I will draw line parallel to again O D. And, of course this red line this red line over here they would be intersecting with the purple line. So, these 2 points again will be belonging to the ellipse that I am seeking. So, once again point here, here intersection between the purples and the reds that is essentially give me points on the ellipse; intersection between the purples and the reds. So, this is how I draw an ellipse using conjugate diameters; is this an exact ellipse go figure?

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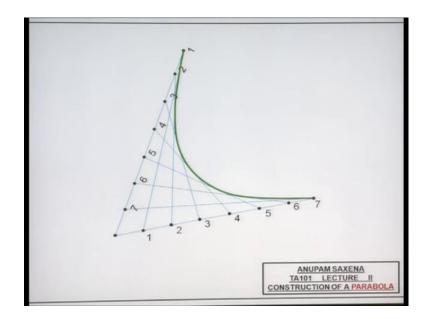
One of the final methods of constructing an ellipse the 4 center method. So, given a rhombus once again given a rhombus these 4 lengths are the same just that the angles not 90 degrees otherwise it have been a square; identify the centers of each of the line segments. Now, the first thing you want do is identify and draw the longest diagonal. So, of course this is not the longest diagonal but see other one is the longest diagonal. Now, look at this point go to the opposite vertex join this point with this vertex and likewise

look at this point look at the opposite vertex join this point with this vertex. So, you will essentially have this as one of the centers, this as one of the centers and 2 more centers.

So let me call this center as C 1 with C 1 as centered and this is radius draw an arc. So, of course this arc is gone a be having these 2 points lying on it; with this point as centered and this as radius draw another arc; there will be having these 2 midpoints of the respective edges of the rhombus on it. The third center or rather the fourth center would be here, the third center would be here. So, with this as centered this as radius draw the third arc and with C 3 as centered and this is radius draw the fourth arc looks like an ellipse; but is this an exact ellipse? I will give you clue; well if you think about this is an actually this is actually an arc think about this, this also is an arc rather or likewise these 2 are also arcs. So, of course I mean this cannot be an exact ellipse it has to be approximate again. So, to be able to construct an ellipse using the 4 center method; step one identify the longest diagonal but before that make sure you look at the midpoint of the 4sides of the rhombus ok.

Now, there would be a vertex on the this side of the longest diagonal and on the this side of the longest diagonal. So, one vertex over here, one vertex over here; identify those vertices and join the corresponding vertices with the corresponding midpoints of the edges of the rhombus. So, from here to here and then from here to here. So, you will essentially have 4 centers center C 1, center C 2, center C 3 and center C 4 and follow the procedure; as I said this is or this has to be an approximate ellipse. Because all these four geometric entities or arcs; they cannot be a part of the ellipse. And, of course this as i said the clue you have to identify the longest diagonal first. But this method only works with rhombus it is it not work with parallelogram for examples. For example, if this length and this length you know they have to be the same and if they are different from these 2 lengths then this method will not work; incidentally this arc has to be tangent to these 2 line segments; this part it has to be tangent to these 2 line segments. And, that would not be the case for any generic parallelogram.

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Construction of a parabola all right. So, given 1 line segment divide that into equal number of parts let us say 1, 2, 3, 4, 5, 6, 7 an other line segment divide them also into equal number of parts just make sure that you follow the numbering. So, for this line segment I am numbering from this point over here; which is the intersection between this line segment and this line segment. For the other line segment I am numbering the other way around. So, I am starting from one of the end points of this line segment and join 1 to 1, 2 to 2, 3 to 3 keep going. And, try to figure an arc that is tangent to all these line segments; once you get that which shows this green curve which is the parabola that you seeking; keep thinking and analyzing until next time.