Microsystem Fabrication with Advanced Manufacturing Techniques
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Lecture - 8

Hello and welcome back to this eighth lecture on Microsystems fabrication by advanced manufacturing processes.

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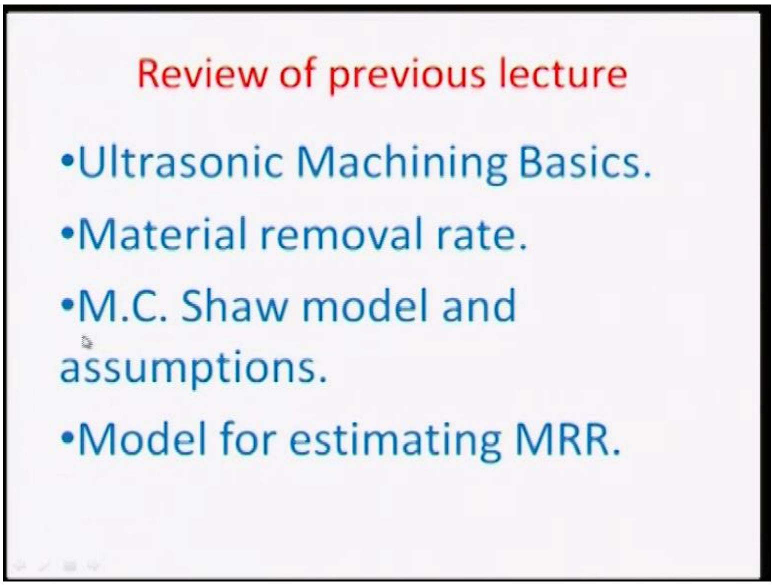

Review of previous lecture

- Ultrasonic Machining Basics.
- Material removal rate.
- M.C. Shaw model and assumptions.
- Model for estimating MRR.

Basically let us just do a quick recap of what we had finished last time. So, we had looked into the u s m or ultrasonic machining process, we had also tried to find out some estimation about the material removal rate, we also did investigate this M. C Shaws.

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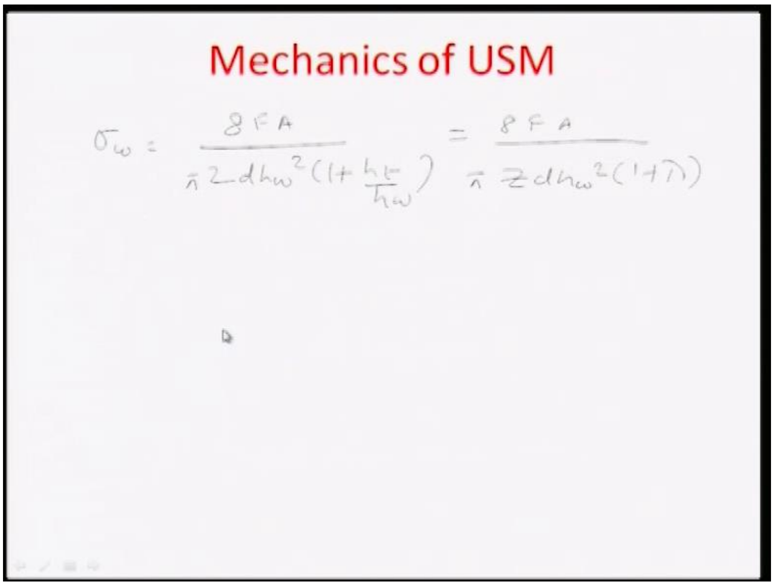

Mechanics of USM

$$\sigma_w = \frac{8FA}{\pi Z d h_w^2 \left(1 + \frac{h_t}{h_w}\right)} = \frac{8FA}{\pi Z d h_w^2 (1+\lambda)}$$

Model for predictions of material removal and did some assumptions in this model.

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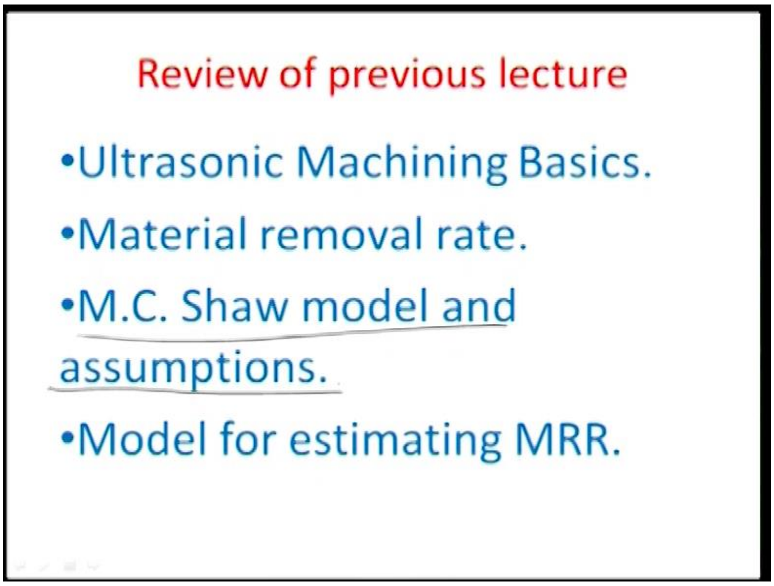

Review of previous lecture

- Ultrasonic Machining Basics.
- Material removal rate.
- M.C. Shaw model and assumptions.
- Model for estimating MRR.

We talked about that you know the grain; the abrasive grain should be treated as spherical and also it should be treated as if there are many numbers of grains between the tool head and the work piece. And also that the indentation created by this grain, would produce a crater on the surface and for all practical purposes, we should consider the amplitude of motion of the tool head to be constant so on so forth.

So, there were set of assumptions that we had made for predicting the MRR or material removal rate. Then, we started modeling to somehow to estimate what this MRR value would be. And in the process of doing that, we arrived at a formulation given here in this slide right here.

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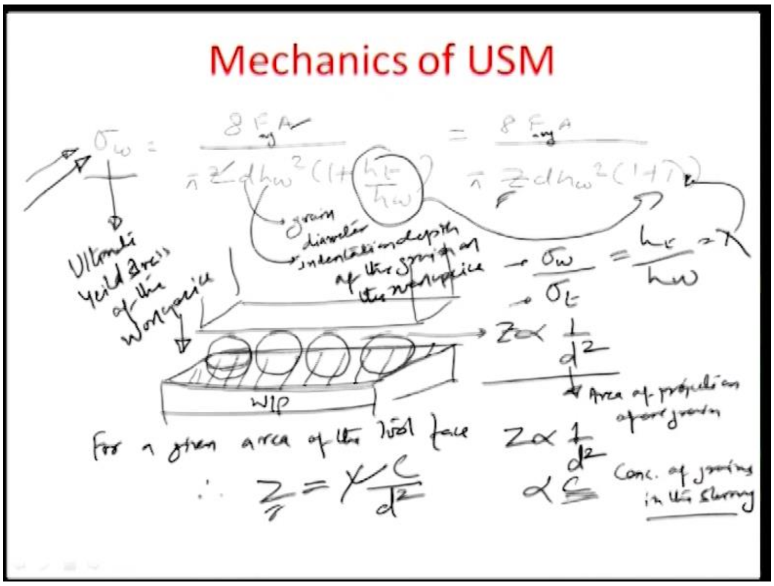

Mechanics of USM

We were talking about the ultimate yield is ultimate yield stress of a material of the work piece sigma w and we correlated this to the average force that the tool, the vibrating tool had would give on the grains; the amplitude of motion of the tool a and the number of grains or particles making impact per cycle, d right here was the grain diameter of the abrasive grains, h w was the indention depth of the grain on the work piece. And this parameter here known as lambda was basically the ratio of the ultimate flow stresses of the work piece in a tool, as we already have seen before in details that you know; the ultimate yield stress is really inversely proportional to the indentation depth.

So, we can assume that sigma w by sigma t; the ultimate yield stress of work piece to the tool is nothing, but the inverse ratio of the depth of the indentation of the tool to the work piece and this we considered as lambda here which comes in to this equation here. So, therefore, we have ways and means to predict the ultimate yield stress of the work material here. Let us call it ultimate yield or let us say flow stress yield stress of the work piece.

So, that is how we have a very well defined relation between the various parameters associated with the force given by the vibrating tool head to the area of the grain, which is really interfacing with the surface and the ultimate flow stress of the work material sigma w which is in question. Now, if we really try to see what the z value is; the grains per impact number of grains in contact between the work piece and the tool in 1 impact would be; so let us say that, if we assume that the number of grains acting is inversely proportional to the square of diameter of each grain which is obvious, because supposing there is an area like this on which you have these many grains, these are the grains on that area. And the average diameter of the grain is given by d as we have predicted before. So, this diameter here right here is d.

So, the amount of occupation of the grain area would definitely be a function of the overall area that is coming between the tool and the work piece. So, this is the tool; this is the vibrating tool head and the tool is coming down like this and the grains are coming in between the tool and the work piece. This is the work piece ok. And so the influence of the diameter of the grain, on the effective area of the work piece that can be machined which is showed by the shaded region is obvious.

So, therefore, if we assume that the number of grains acting let us say, these are z numbers in 1 impact between the tool and the work piece. So, if that is inversely proportional to the square of the diameter of these grains, which also is signified or signifying the sort of area or projection of 1 grain. So, it will not really be improper to assume in this kind of a relationship.

So, for a given area of the tool face, z is actually proportional to inversely proportional to the square of the grain diameter and also z would be proportional to the concentration of the grains in the slurry. So, if c is the concentration of the grains in the slurry or a concentration term, so more is the concentration, more would be the number of z’s, more would be the value of z; the number of particles between the tool and the shaded area here work piece. So, therefore, we can say that z is equal to some constant psi times of c by d square and we can actually substitute the value of z from here to here in this particular equation.

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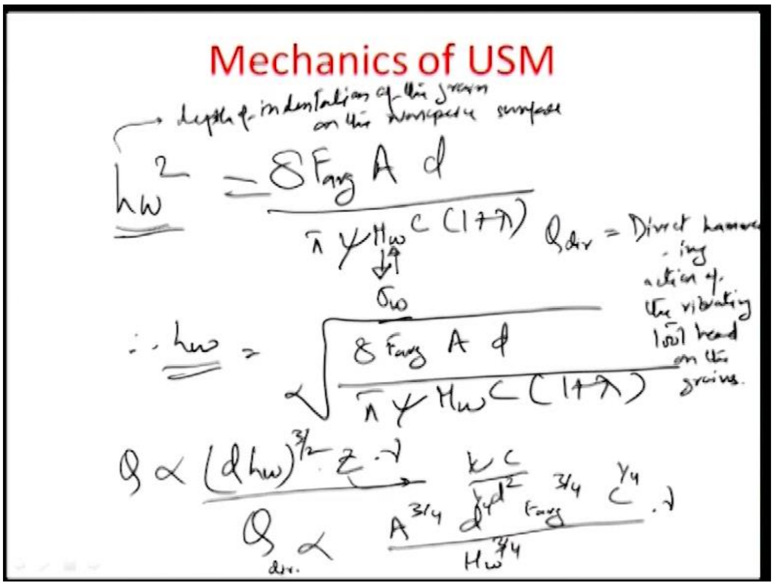

Mechanics of USM

depth of indentation of the grain on the workpiece surface

$$h_w^2 = \frac{8 F_{avg} A d}{\pi \psi H_w c (1+\lambda)}$$

$$H_w \Updownarrow \sigma_w$$

$$\therefore h_w = \propto \sqrt{\frac{8 F_{avg} A d}{\pi \psi H_w C (1+\lambda)}}$$

Q_{dir} = Direct hammering action of the vibrating tool head on the grains.

$$Q \propto (d h_w)^{3/2} \cdot z \cdot \nu \qquad z \rightarrow \frac{kc}{\psi d^2}$$

$$Q_{dir} \propto \frac{A^{3/4} d^{1/4} F_{avg}^{3/4} C^{1/4}}{H_w^{3/4}} \cdot \nu$$

So, the final form of the equation can come out to be. Square of h w is actually equal to 8 times of average force times of amplitude A times of grain diameter divided by pie times of psi and the flow stress of the material is nothing, but the hardness of the material. So, sigma w and h w are kind of inter convertible, so h w times of c times of 1 plus lambda. And where h w now; this h w as you know is the depth of indentation of the grain on the work piece surface. So, therefore, h w always becomes equal to the square root of 8 F average A d divided by pie psi h w c 1 plus lambda.

If, you substitute this value of h w in the equation for q, you remember q earlier was actually determined to be proportional to d h w to the power of 3 by 2 times of value of z, times of the frequency nu. So, Q here can become of course, we can substitute all these values here. So, the Q finally, after substitution of h w and the value of z, which is actually square of inverse square of d times c times of psi, nu of course is the frequency we get. Q is proportional to amplitude to the power of 3 by 4, diameter d to the power of 1 by 4, the average force of the vibrating tool head to the power of 3 by 4 times of concentration to the power of 1 by 4 divided by the flow stress of the material or hardness of the material h w to the power 3 by 4 time of nu.

So, the rate of removal is through the direct hammering action of the grains due to the vibrating tools. So, this actually we can say as Q direct. Or in other words, Q direct is nothing, but the direct hammering action of the vibrating tool head on the grains thus creating a plowing action. So, as I told you there are 2 modalities of this material you know, removal 1 is of course, the direct hammering action of the vibrating tool head and the other, that is more then, that is not very important or not significant, although it is to be considered in model; is the impact that a free grain would have on the surface. Meaning thereby, if the gap between the tool and the work piece is very high and there is a possibility of the abrasive grain to freely flow between the tool head and the work piece.

So, the impact that the tool would give on the abrasive grain, would be converted as a sort of kinetic energy of the grain and this kinetic energy would come and impinge on the surface thereby removing the material from the work piece. So, that is separate mechanism. So, this Q that we have determined now, is really the direct hammering action, where you are squeezing the grain between the tool head and the work piece and you are giving a force average force F average between the grain and the on the grain by the tool, which is creating a direct plowing action.

So, that is what the first part is. Let us look at now the second part of the problem and the second problem is related to the kinetic energy of the grains. So, let us actually try to model that part.

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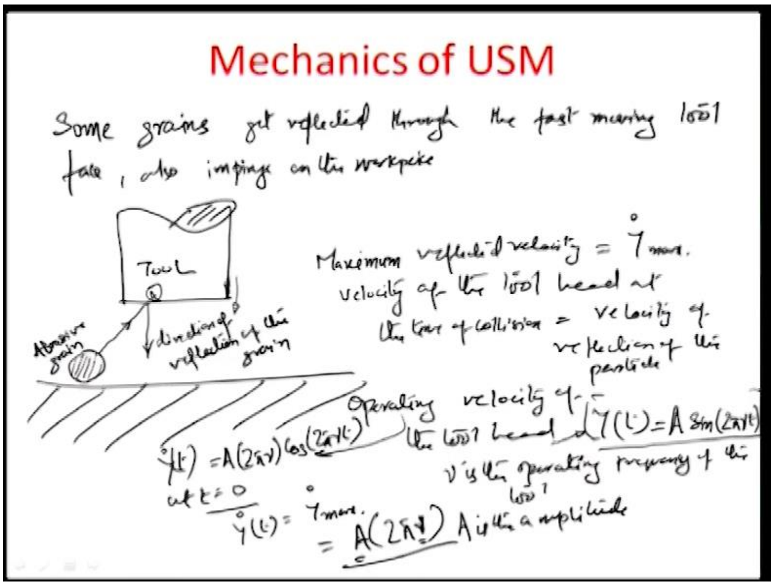

Mechanics of USM

Some grains get reflected through the fast moving tool face, also impinge on the workpeice

Maximum reflected velocity = $\dot{Y}_{max}$.

velocity of the tool head at the time of collision = velocity of reflection of the particle

Operating velocity of the tool head $Y(t) = A\sin(2\pi\nu t)$

$\dot{Y}(t) = A(2\pi\nu)\cos(2\pi\nu t)$

ν is the operating frequency of the tool

at $t = 0$

$\dot{Y}(t) = \dot{Y}_{max}$.

$= A(2\pi\nu)$ A is the amplitude

So, some grains get reflected through the fast moving tool face, also impinge on the work piece. So, we can estimate the depth of indentation in that case, by looking at the following. So, let us say this is the tool and this right here is the work piece and there is a grain there is an abrasive grain, which because of the motion imparted by the slurry goes, in strikes the tool in this particular direction coming out of it in this direction. And we will have to somehow predict, what the maximum reflected velocity is. So, this is the direction of reflection of the grain.

So, the maximum reflected velocity needs to be somehow determined in this particular case. So, that is actually, let us say Y dot maximum. So, as we know that, you know the grain velocity here of the abrasive grain, the initial velocity by which it is striking the tool, is of hardly any significance in comparison to the overall inertial component of the tool, because the tool first of all it is very heavy and number 2 is it is also vibrating at a certain velocity at ultrasonic frequency. Meaning thereby; that its velocity is also very high.

So, therefore, the velocity the initial velocity of the abrasive grain, as it strikes the tool surface here for example, in this position a here is not really of great significance. And we can say that, whatever is the velocity of the tool head is at that particular point of collision, at the time of collision would be equal to the velocity of reflection of the particle. So, it simply imparted, there is no specific inertial component associated with the abrasive grain, because of its small nature. It is few microns, as I told you abrasive grains could be in the range of 20 to 25 microns.

So, let us find out first, the operating velocity of the tool head as a function of time. So, Y t as you know, because it is a sort of simple harmonic motion imparted by the tool, so Y t can be written in, in form of an equation as the amplitude of motion A times of sin 2 pie nu t. nu is the operating frequency of the tool and A is the amplitude. And so therefore, that is what the equation of motion of the tool head would be.

So, the operating velocity of the tool head would be dependent on this equation of motion here. And so operating velocity can be written down as Y dot t; the first differential of Y with respect to time which is equal to A times of twice pie nu times, of cosine twice pie nu t. And as you know here, that at time t equal to let us say 0, which signifies probably the mean position of the particular tool, where the velocity is supposed to be the maximum.

So, this value Y dot t would be Y dot maximum, which is actually equal to A times of twice pie nu. A is the amplitude of motion, nu is the operating frequency and so A twice pie nu is basically the Y dot max or the velocity of motion.

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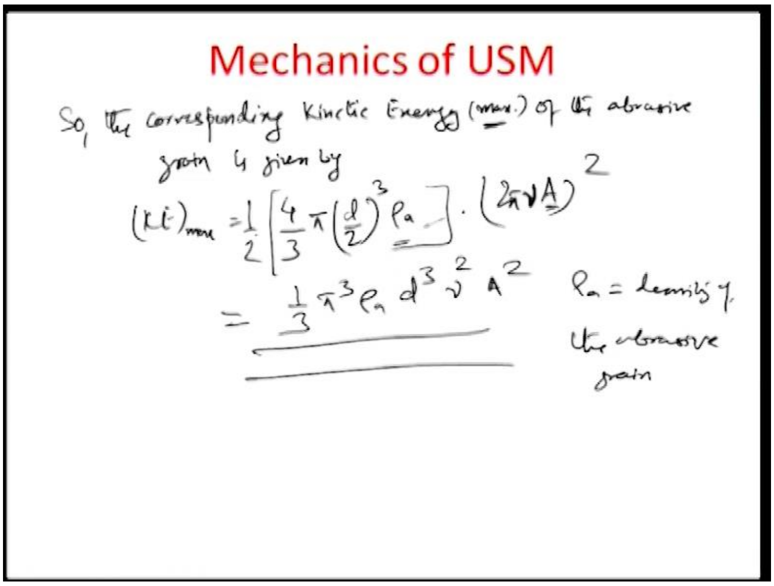

Mechanics of USM

So, the corresponding Kinetic Energy (max.) of the abrasive grain is given by

$$(KE)_{max} = \frac{1}{2}\left[\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \underline{\underline{\rho_a}}\right] \cdot (2\pi\nu \underline{\underline{A}})^2$$

$$= \underline{\underline{\frac{1}{3}\pi^3 \rho_a d^3 \nu^2 A^2}}$$

ρ_a = density of the abrasive grain

So, now we look into the aspect of the kinetic energy of the particular tool, once the maximum velocity of the grain is there. So, the Corresponding k e or kinetic energy, actually will be equal to the maximum kinetic energy, because its half m v square, v is the velocity of motion and v is equal to v max corresponding to the maximum velocity, the time when the tool is at the mean position.

So, therefore, the maximum kinetic energy of the abrasive grain, I already explained to you before, that it really is nothing, but the maximum velocity of the tool. The inertial component of its own self of the grain is so small, that we do not really treat that in this equation. And so therefore, the maximum kinetic energy of the abrasive grain is given by the term half m v square right and m here, because it is a spherical grain that we are assuming, with diameter d we can assume, it equal to be the volume of the grain which is 4 by 3 pie r q d by 2 whole cube times of the grain average grain density rho A.

So, this is actually the density of the abrasive material. This is not really the number of grains per unit volume, but it is a density of 1 grain per unit volume of that material. So, that basically is the mask component in the motion. So, its half m v square and v as you know, is 2 pie nu A square, where nu is basically the frequency, A is the amplitude of motion of the particular tool head and then, this is the characteristic property of the grain itself.

So, if we really try to solve this around, we get of term 1 by 3 cube of pie rho A d cube nu square A square, where rho A is the density of the abrasive grain. So, that is what the maximum kinetic energy in this particular case would be. So, basically now you want to really find out that, amount of energy which is really needed for indentation caused in the surface by a flying free flying abrasive grain, that comes and trikes on to the tool surface and impinges on to the work piece surface is the result of the reflected velocity.

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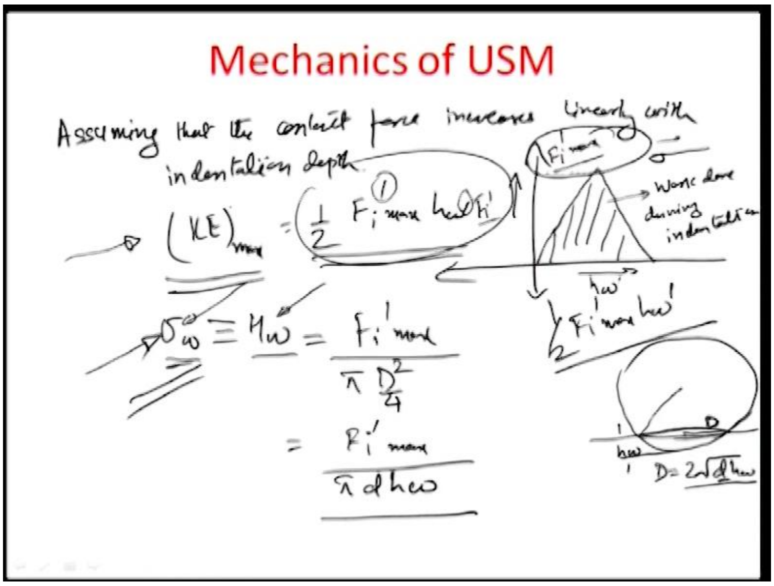

Mechanics of USM

Assuming that the contact force increases linearly with indentation depth

$$(KE)_{max} = \frac{1}{2} F'_{i\,max} h'_w \quad ①$$

$$\sigma_w \equiv H_w = \frac{F'_{i\,max}}{\pi \frac{D^2}{4}} = \frac{F'_{i\,max}}{\pi d h_w}$$

$$D = 2\sqrt{d h_w}$$

So, assuming that, during the indentation caused by such an impinging grain, the contact force increases linearly with the indentation depth, the K max whatever has been imparted on to the grain surface of the free flying abrasive grain by the tool surface, should really be equal to half F i dash max h w dash. If you remember the plot here cut, so in this graph here; let us say we assume that F i dash is linearly varying with respect to the depth of indentation h w dash. Mind you, we are using different subscripts here because; you know just to differentiate it from the case of direct impact, where F i average and h w over the 2 subscripts which were used there.

So, this is the linearly varying model, meaning thereby; that when the force is 0 at the beginning and when the grain has not yet stuck on the surface and then, the force slowly increases, because the grain gives you know all its momentum, all its energy to the surface and also faces the reverse force from the surface. And then, after a while, after the full indentation has been realized, the amount of force at that point can be treated as; F i dash maximum and then, you can assume that the grain slowly releases contact, meaning thereby; the rate fly is off the work piece surface and it goes all the way to force equal to 0.

So, the area under this curve here showed by the shaded area, is really the work done. The amount of work done because of which the indentation has happened, so during indentation. An area is actually given here by half F i dash maximum h w dash. And so we equate that to the maximum kinetic energy of the grain that has been obtained before. So, therefore, you know we can easily find out, so sigma w, which is actually equal to the also the hardness of the work piece.

So, these are all flow stresses, is related to the maximum force at that instant of point when the indentation had gone maximum, so F i dash max per unit area. So at that time, if we assume that you know the total grain dia, which has been projected on to the work piece surfaces is D and D is as you already know, twice root of d h w, where h w is this depth of indentation.

So, if you remember the first exercise on USM that we had done this modeling, that how about a grain with the diameter d impacting on a surface producing a depth h w. So, there was a relationship between this D here; the projector diameter of the grain on the surface and the grain dia. So, therefore, force per unit area that you get out of this equation, where F goes to F i max F i dash max; the maximum force of the grain on the surface, per unit the area at that time, which we assumed to be pie D square by 4.

So, we assume this area to be pie D square by 4. In other words, you can have this as F i dash max divided by pie d h w. So, that is what has to be equated to the hardness of the surface or the flow stress of the surface, for the condition that the grain would actually produce some deformation on the surface. And we already know from the previous equation, that this K e max can be related to this F i dash max. We would like to now formulate an equation for that.

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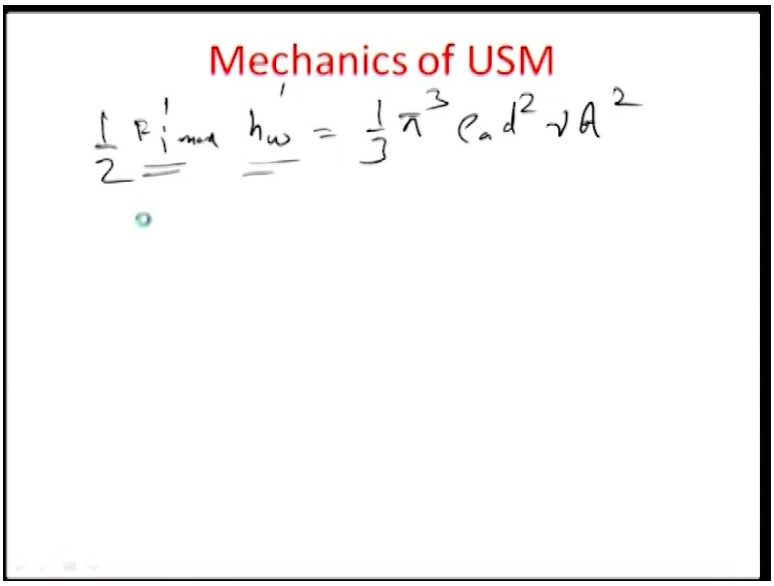

Mechanics of USM

$$\frac{1}{2} F'_{i\,max}\, h'_w = \frac{1}{3}\pi^3 \rho_a d^2 \nu A^2$$

So, half F i dash max times of h w dash, where F i dash max is the maximum force at maximum indentation h w dash. This can be equated equal to this kinetic energy maximum which had come from the last derivation: 1 by 3 pie cube rho A square of d nu A square and therefore, also from the equation that you have derived earlier.

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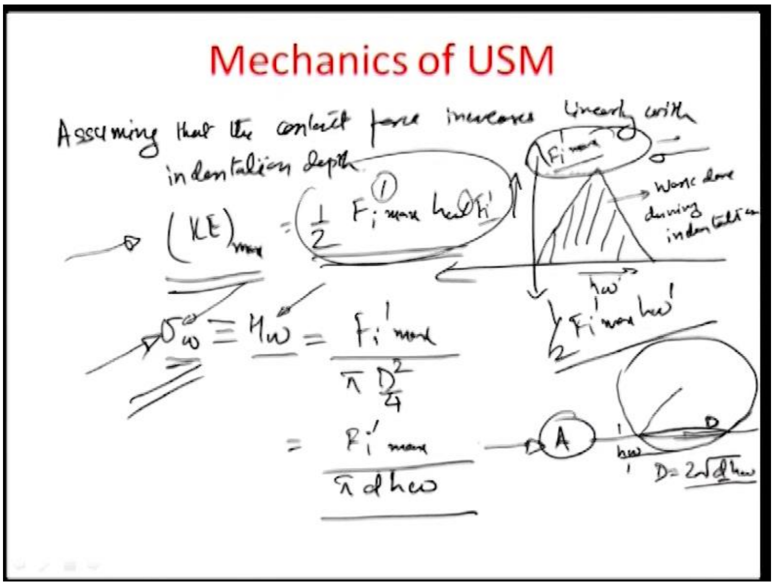

Mechanics of USM

Assuming that the contact force increases linearly with indentation depth

$$(KE)_{max} = \frac{1}{2} F'_{i\,max}\, h'_w \quad ①$$

$$\sigma_w \equiv H_w = \frac{F'_{i\,max}}{\pi \frac{D^2}{4}} = \frac{F'_{i\,max}}{\pi d h_w} \quad (A)$$

Here in this particular, instance let us call it equation A here.

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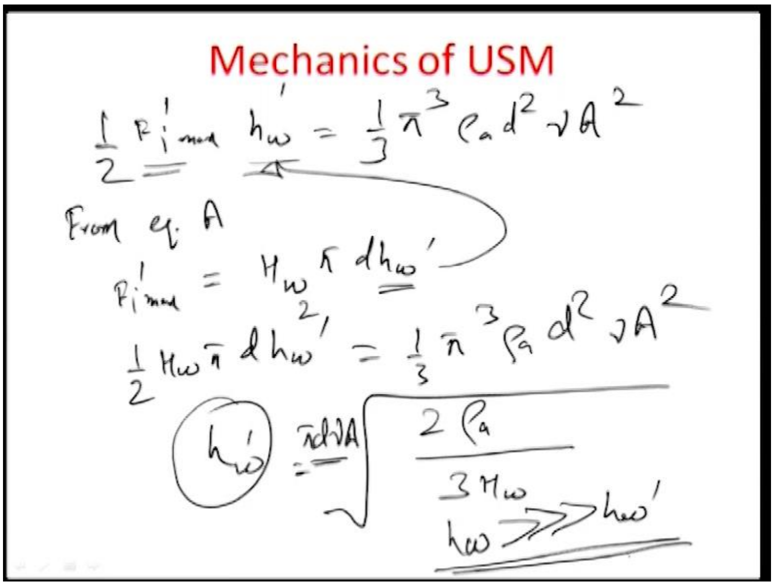

Mechanics of USM

$$\frac{1}{2} F'_{i\,max}\, h'_w = \frac{1}{3}\pi^3 \rho_a d^2 \nu A^2$$

From eq. A

$$F'_{i\,max} = H_w \pi d h'_w$$

$$\frac{1}{2} H_w \pi d h'^{2}_w = \frac{1}{3}\pi^3 \rho_a d^2 \nu A^2$$

$$h'_w = \pi d \nu A \sqrt{\frac{2\rho_a}{3H_w}}$$

$$h_w >> h'_w$$

From this equation A, you already know that F i dash max can equated equal to h w pie d smallest w dash, where this is the maximum indentation depth of a freely flowing abrasive grain on the surface. So, thus if we substitute this in this particular equation for F i dash max, we get a formulation half H w pie d h w dash square. half F i max dash times of H w dash is actually equal to 1 by 3 cube of pie rho A square of d nu A square and that way you can actually have h w dash as the under root of twice rho A abrasive grain density by 3 H w times of pie d nu A.

So, comparing this h w dash that you are obtained, with the earlier h w that was for case of a you know hammered grain or a direct impacted grain, we will find out that h w is very high in comparison to h w dash. You can compare both parallely. So, if you may just recall in the earlier case, the h w dash came out to be.

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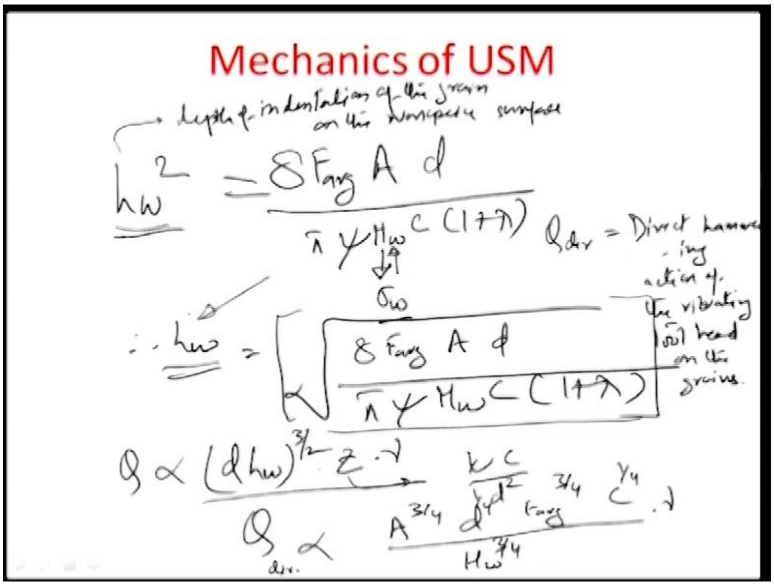

Mechanics of USM

depth of indentation of the grain on the workpiece surface

$$h_w^2 = \frac{8 F_{avg} A d}{\pi \psi H_w C (1+\lambda)}$$

$H_w \downarrow\uparrow \sigma_w$

Q_{dir} = Direct hammering action of the vibrating tool head on the grains.

$$\therefore h_w = \left[\propto \sqrt{\frac{8 F_{avg} A d}{\pi \psi H_w C (1+\lambda)}}\right]$$

$$Q \propto (d h_w)^{3/2} Z \cdot \nu \rightarrow \frac{k C}{\psi d^2}$$

$$Q_{dir} \propto \frac{A^{3/4} d^{1/4} F_{avg}^{3/4} C^{1/4}}{H_w^{3/4}} \cdot \nu$$

This whole 8 4s average A d by pie psi h w c 1 by 1 plus lambda. So, it really included a lot of terms, magnitude wise this h w dash coming from the direct hammering action, is always very high, in comparison to the h w dash that you obtained by the free flowing action of the grain. So, therefore, really the maximum material removal rate we conclude here.

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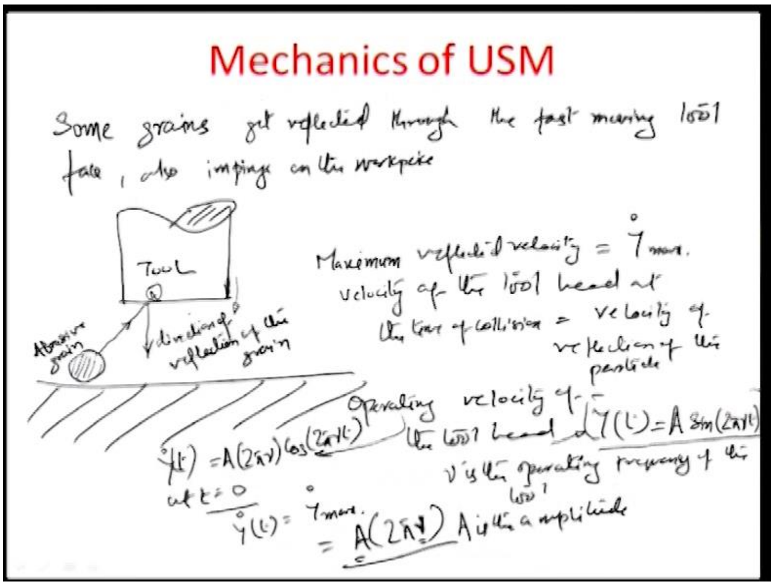

Mechanics of USM

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Maximum reflected velocity = $\dot{Y}_{max}$.

velocity of the tool head at the time of collision = velocity of reflection of the particle

Operating velocity of the tool head $Y(t) = A \sin(2\pi\nu t)$

$\dot{Y}(t) = A(2\pi\nu)\cos(2\pi\nu t)$

ν is the operating frequency of the tool

at $t = 0$

$\dot{Y}(t) = \dot{Y}_{max}$

$= A(2\pi\nu)$ A is the amplitude

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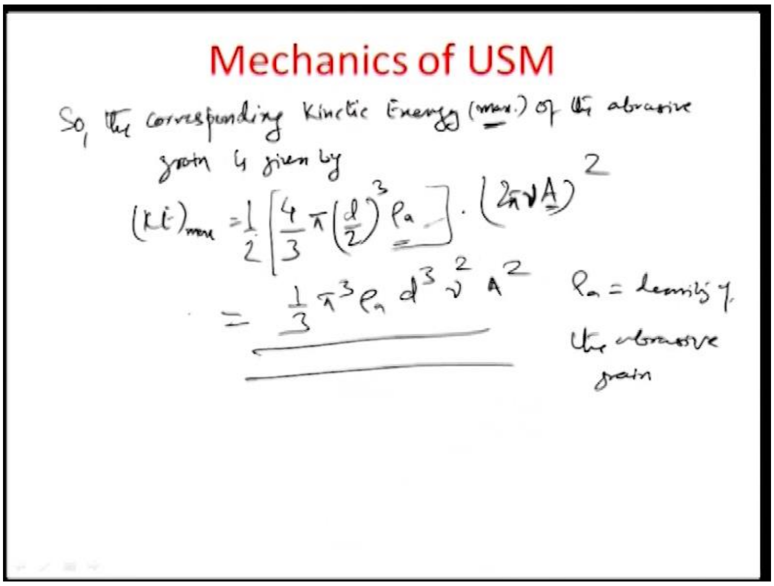

Mechanics of USM

So, the corresponding Kinetic Energy (max.) of the abrasive grain is given by

$$(KE)_{max} = \frac{1}{2}\left[\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \rho_a\right]\cdot\left(2\pi\nu A\right)^2$$

$$= \underline{\underline{\frac{1}{3}\pi^3 \rho_a d^3 \nu^2 A^2}}$$

ρ_a = density of the abrasive grain

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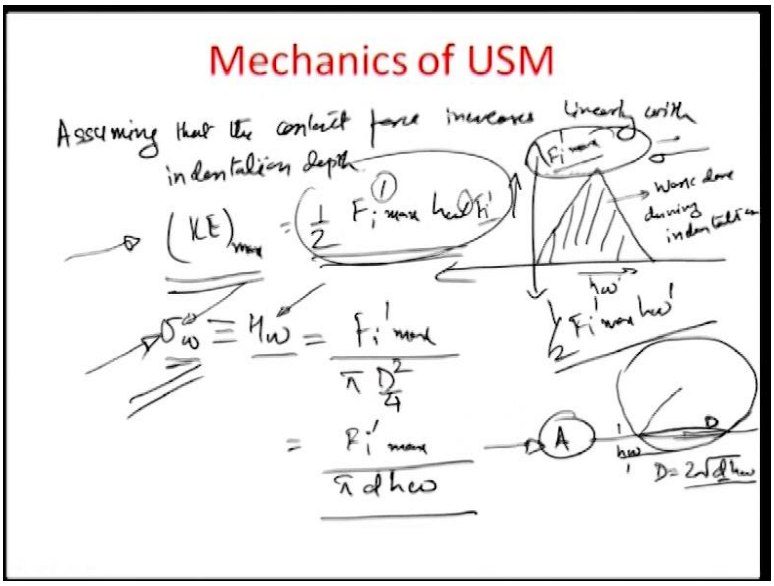

Mechanics of USM

Assuming that the contact force increases linearly with indentation depth

$$(KE)_{max} = \frac{1}{2} F'_{i\,max} h'_w$$

$$\sigma_w \equiv H_w = \frac{F'_{i\,max}}{\pi \frac{D^2}{4}}$$

$$= \frac{F'_{i\,max}}{\pi d h_w}$$

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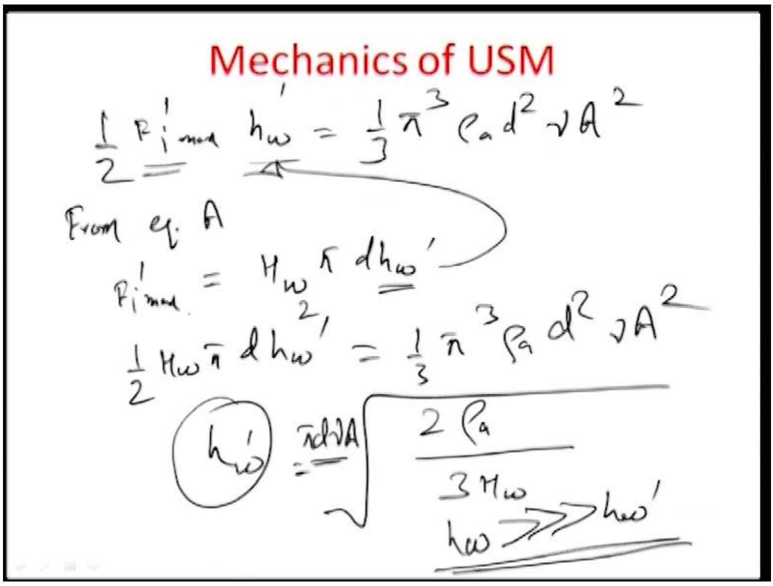

Mechanics of USM

$$\frac{1}{2} P'_{i\,max}\, h'_w = \frac{1}{3}\pi^3 \rho_a d^2 \nu A^2$$

From eq. A

$$P'_{i\,max} = H_w \pi d h'_w$$

$$\frac{1}{2} H_w \pi d h'^2_w = \frac{1}{3}\pi^3 \rho_a d^2 \nu A^2$$

$$\boxed{h'_w} = \pi d \nu A \sqrt{\frac{2\rho_a}{3H_w}}$$

$$h_w \gg h'_w$$

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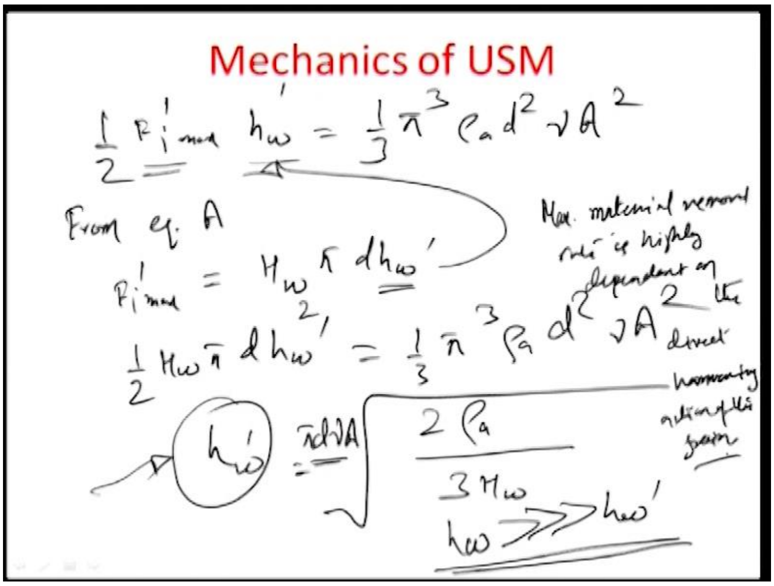

Mechanics of USM

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From eq. A

$$P'_{i\,max} = H_w \pi d h'_w$$

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$$\boxed{h'_w} = \pi d \nu A \sqrt{\frac{2\rho_a}{3H_w}}$$

$$h_w \gg h'_w$$

Max. material removal rate is highly dependent on the direct hammering action of the grain

So, the maximum material removal rate is highly dependent on the direct hammering action of the grain. So, is dependent on the direct hammering action of the grain. So, it can be concluded, that most of the material is really removed by the direct hammering. And very less amount of material comes out, because of the free flowing impact, which is really not relevant to mention here also. And from the earlier relationship, we already have seen.

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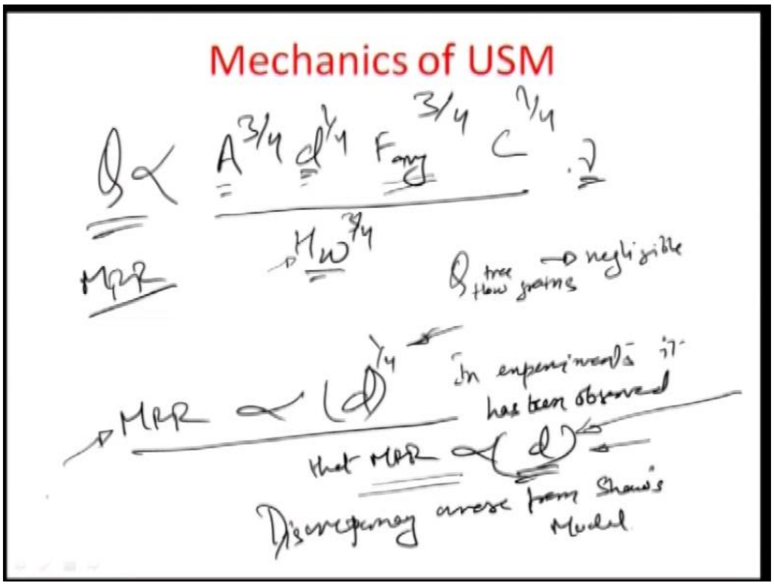

Mechanics of USM

$$Q \propto \frac{A^{3/4}\, d^{1/4}\, F_{avg}^{3/4}\, C^{1/4}}{H_w^{3/4}} \cdot \nu$$

MRR

$Q_{\text{free flow grains}} \rightarrow$ negligible

$MRR \propto (d)^{1/4}$

In experiments it has been observed that $MRR \propto (d)$

Discrepancy arose from Shaw's Model

The MRR Q is proportional to A 3 by 4 grain diameter d to the power 1 by 4 average force to the power of average tool force to the power 3 4 times of concentration to the power of 1 by 4 divided by hardness to the power of hardness to the work piece to the power of 3 by 4 times of nu, where nu is the operating frequency, A is the amplitude, d is the on the grain diameter, this is the average force and h w is the hardness of the work piece.

As I already discussed that the Q, because of free flowing grains, the MRR because of free flowing grains is negligible. Therefore, this really is the MRR value and therefore, we say to say that MRR is proportional to the d power 1 by 4, where d is the grain dia. Unfortunately that is not, so but because an experiment, it has been observed that the material removal rate is proportional to the first power of d and not d to the power of 1 by 4 here.

So, this was the discrepancy that, you know arose from the shaws model, because of which, some explanation needed to be given, so that somehow this experimental data which comes out to be proportional to d, could be easily fit inside the you know the data which has been theoretically derived by the shaws model. So, therefore, shaw actually pride to find out in reality, what goes on or what happens. So, this discrepancy was addressed by shaw finally, by looking at the overall shape of the grain.

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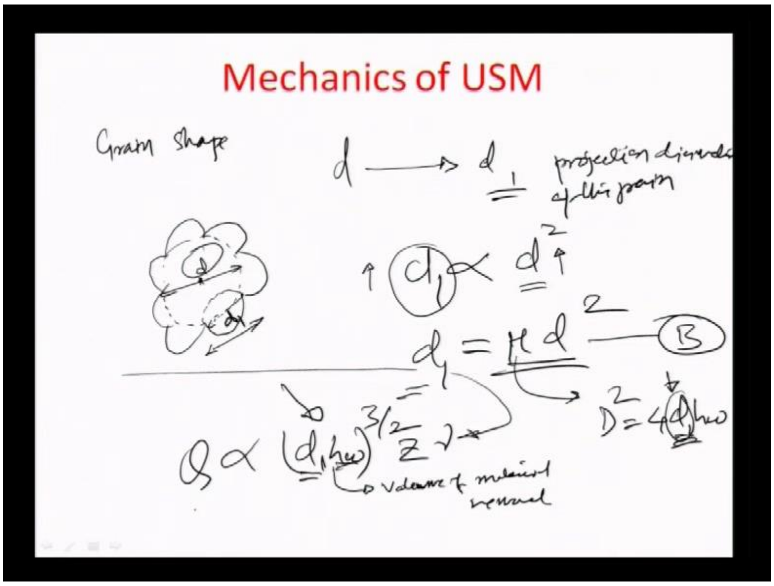

Mechanics of USM

So, shaw actually looked at the grain shape under a microscope and found out that the grain actually is not a spherical grain, but a sort of flowery structure on the surface something like this. And what really was impacting the work piece surface, was not this overall average spherical grain diameter d has been illustrated in many times in the model, but this small diameter here, of 1 such you can say this can be as ferulate d 1.

So, essentially this is the diameter, which would affect the material removal process and it would in turn indent on the surface. The surface area also would be determined by d 1 and not d. So, he very closely monitored, if there exists a correlation between the grain diameter d and this small we can call it the projection diameter of the grain d 1 dash d 1. And what interestingly he found out is, that yes there exists correlation, where you know these 2 things can be very well correlated as d 1 in the projection diameter being proportional to the square of the average grain diameter cap d; meaning thereby, that if this diameter increases, d 1 almost increases as a square of this diameter.

Therefore, it say to assume that d 1 is actually equal to a constant nu times square of d and this mu can vary between close to 1 or somewhere less than 1. And that way you can have a very nice formulation between d 1 and square of d. So, if you actually use this, let us call this equation B in the theory of Q and Q you already know is proportional to this now it is d 1 h w to the power of 3 by 2 times of z, times of nu. Remember this is the volume of material removal and this d 1 h w to the power of 3 by 2 is now indicative of the new projection grain diameter, which is actually the diameter causing the indent or the impact on the surface, times of the depth of indentation h w, which does not remain which remains almost same to the power of 3 by 2.

And this is correlated by that formulation: square of d is actually equal to 4 times of d 1 h w. So, we are taking the modified diameter of this projection which is actually causing the indentation and trying to find the relationship between this diameter and the overall diameter here the grain diameter d.

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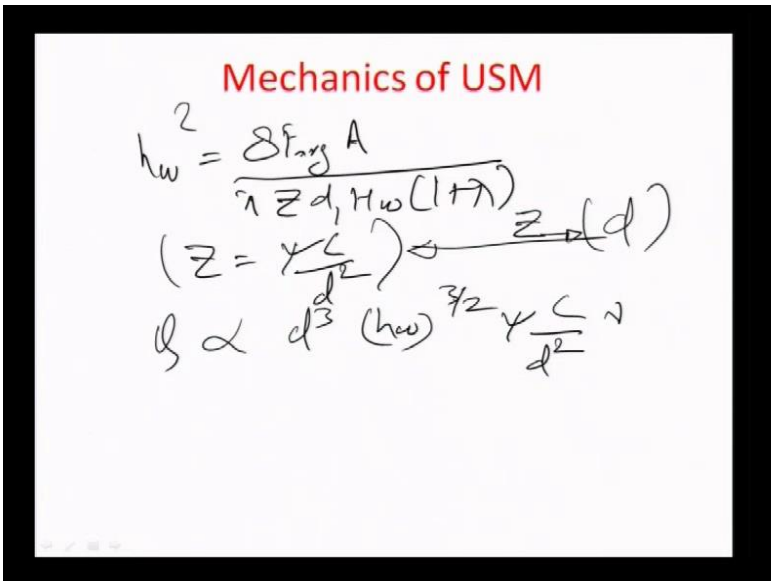

Mechanics of USM

$$h_w^2 = \frac{8 F_{avg} A}{\pi Z d_1 H_w (1+\lambda)}$$

$$\left(Z = \psi \frac{C}{d^2}\right) \longleftarrow Z(d)$$

$$Q \propto d^3 (h_w)^{3/2} \psi \frac{C}{d^2} \nu$$

So, if you put this expression d 1 into, you know this particular expression here, you get that of course h w, as i already told you for a hammering case hammering grain case, can be correlated by this relationship 8 F average times of amplitude A divided by pie z d 1 H w 1 plus lambda. And we already know that z is actually proportional to the concentration and inversely proportional to the square of the grain diameter.

So, if we put all these together on the equation for Q, the q equation becomes equal to cube of d times of h w to the power of 3 by 2 times of psi c by d square nu. 1 thing which is interesting to observe here is, that the z value is still is dependent on the average grain diameter value for obvious reasons, that the number of particles which are making impact on the surface between, let us say a fixed tool area and a fixed working surface, is really determined by the area of an area of projection of an average overall grain.

The area of projection of an average over all grain is nothing, but proportional to the square of the diameter the average diameter of the grain, not the diameter of the projection, projections can be many on a grain surface. So, that is why the z value does not alter. The z value is still inversely proportional to the square of the diameter, because that is ultimately the determinant of what would be the grain to grain spacing. The average diameter of the grain is the determinant of the grain to grain spacing between the tool surface and the work piece surface assuming a fixed tool area.

So, therefore, this expression here becomes conveniently changed and Q becomes conveniently proportional to the grain diameter the average grain diameter d, which is in consonance with of course, the experimental observation. And therefore, this gives you the total prediction of shaw theory towards the different parameters involved in the material removal rate of USM process.

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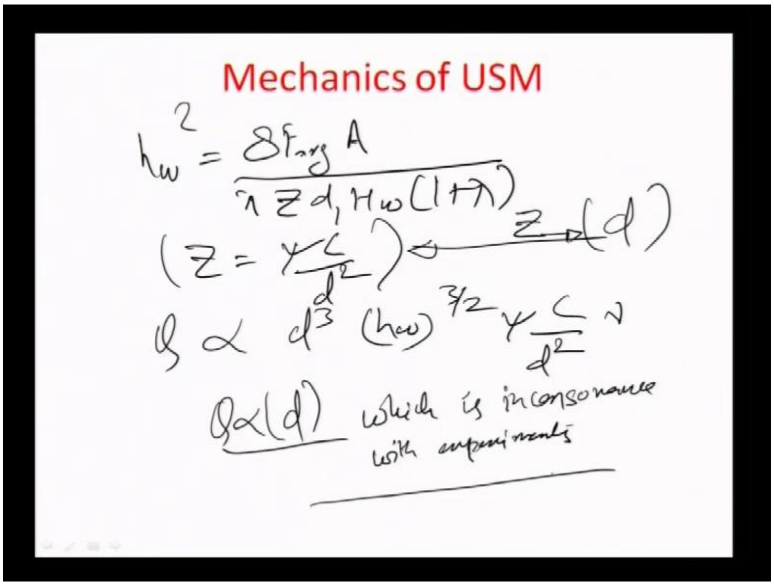

Mechanics of USM

$$h_w^2 = \frac{8 F_{avg} A}{\pi Z d_1 H_w (1+\lambda)}$$

$$\left(Z = \frac{\psi C}{d^2} \right) \leftarrow Z \propto (d)$$

$$Q \propto d^3 (h_w)^{3/2} \psi \frac{C}{d^2} \nu$$

$\underline{Q \propto (d)}$ which is inconsonance with experiments

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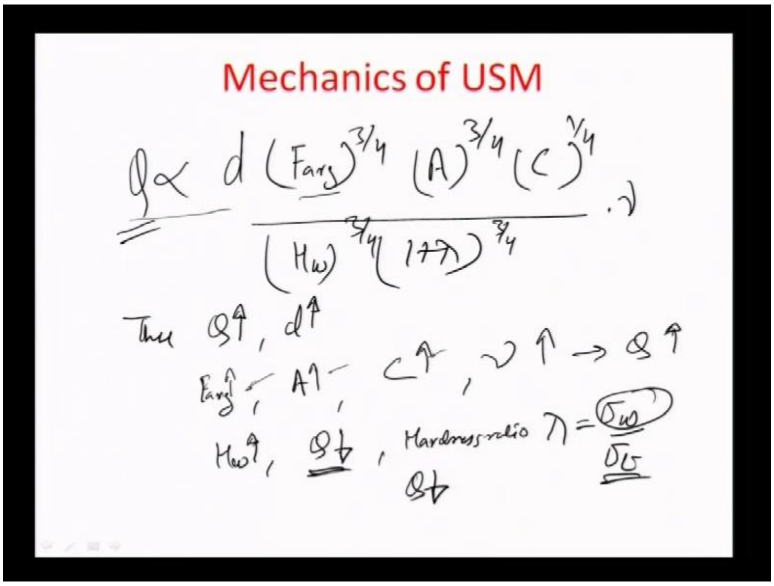

Mechanics of USM

$$\underline{\underline{Q}} \propto \frac{d\,(\underline{F_{avg}})^{3/4}\,(A)^{3/4}\,(C)^{1/4}}{(H_w)^{3/4}\,(1+\lambda)^{3/4}} \cdot \nu$$

Thus $Q\uparrow$, $d\uparrow$,

$F_{avg}\uparrow$, $A\uparrow$, $C\uparrow$, $\nu\uparrow \rightarrow Q\uparrow$

$H_w\uparrow$, $\underline{Q\downarrow}$, Hardness ratio $\lambda = \frac{\boxed{\sigma_w}}{\underline{\underline{\sigma_t}}}$

$Q\downarrow$

So, what I am now trying to, what I am now I will try to do, is basically try to evaluate some of the characteristics, typical characteristics of how Q will vary with what parameter. So, let us actually write this all thing down here. So, Q as you know now is proportional to d times, of the average force F average to the power of 3 by 4 times, of amplitude of motion of the tool to the power of 3 by 4 times, of c concentration to the power of 1 by 4 divided by the hardness to the power of 3 by 4 1 plus lambda to the power of 3 by 4 times, of nu, where nu is the average frequency.

And thus as you know that Q would increase, if the grain diameter would increase; obviously, because there is a direct proportionality between the 2. And if, supposing all these other parameters like: the force average, the amplitude of motion, the concentration of the grain and the average frequency. If they have increases, they would significantly impact the Q. So, the Q increases because of them. And if the hardness of the work piece is more, then of course, the Q falls down. So, Q is inversely proportional to it.

And also so is true about the hardness ratio. And hardness ratio as you have earlier defined, is very well defined as the relationship between what the work piece hardness or flow stresses with respect to the tool hardness or stress. So, if the work piece hardness is more, it is obvious that the Q or the material removal rate would fall down. So, that is in a nutshell, what the predictions of shaw theory actually show. And experimentally they have been many times verified by various people that, these trends are actually true.

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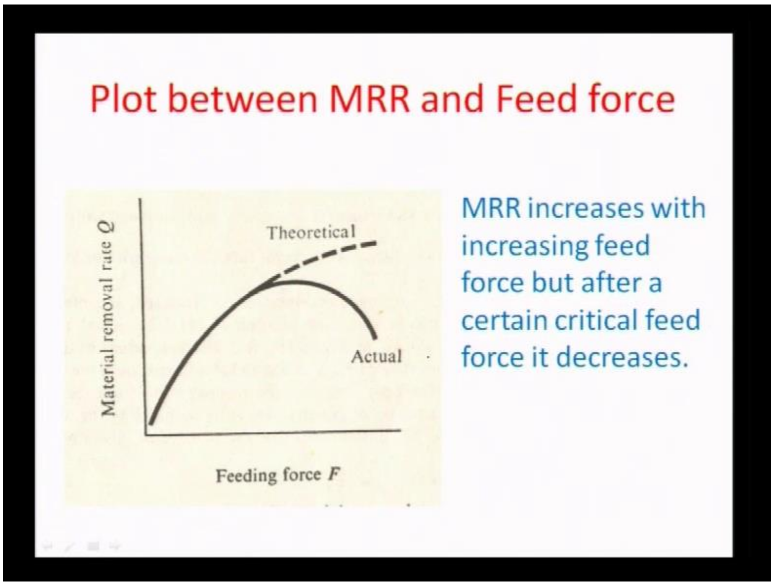

Plot between MRR and Feed force

MRR increases with increasing feed force but after a certain critical feed force it decreases.

So, we would now like to go ahead and look at some of the experimental trends of different aspects of the shaw theory and how actually and theoretically predicted values, would differ above a certain limit of 1 parameter may be. So, 1 case is the MRR plot of with respect to the feed force or the average force.

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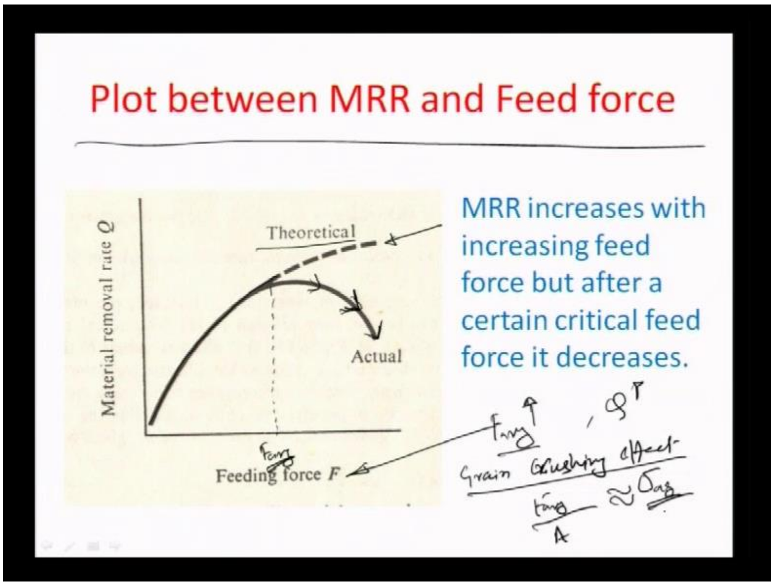

Plot between MRR and Feed force

MRR increases with increasing feed force but after a certain critical feed force it decreases.

$F_{avg} \uparrow$, $Q \uparrow$

Grain crushing effect

$\frac{F_{avg}}{A} \approx \sigma_{avg}$

So, this actually is a plot between the average feeding force F average as you saw. And that is obvious to assume that if average is more, then Q is more. So, theoretically predicted trends would look something like this, which is represented by this dotted line here as if, if the F average keeps on increasing, then the Q should increase. But then, what is interesting here is that above a certain limit of the feed force, let us say above a certain limit of the F average force, there is a depreciation of the material removal rate. And the material removal rate comes down up to the certain critical feed force. And that happens, because of a very important effect which practically, you know almost always happens into in this USM systems which is also known as the grain crushing effect.

So, if the feed force is hired and hired to a value that, this F average per unit area of the grain actually equals to the ultimate flow stress of the grain itself; abrasive grain itself. So, therefore, there is a possibility that the grain itself would get broken into pieces and there is a crushing effect.

So, the number of active grains which are now available, at that critical feed force, would simply you know go down, so because they are themselves getting crushed. And therefore, the material is almost always reasonable to assume that, because of this crushing effect of the grains etcetera, the number of available complete grains which come between let us say, the tool head and the work piece are lessoned. And, so would be the material removal rate.

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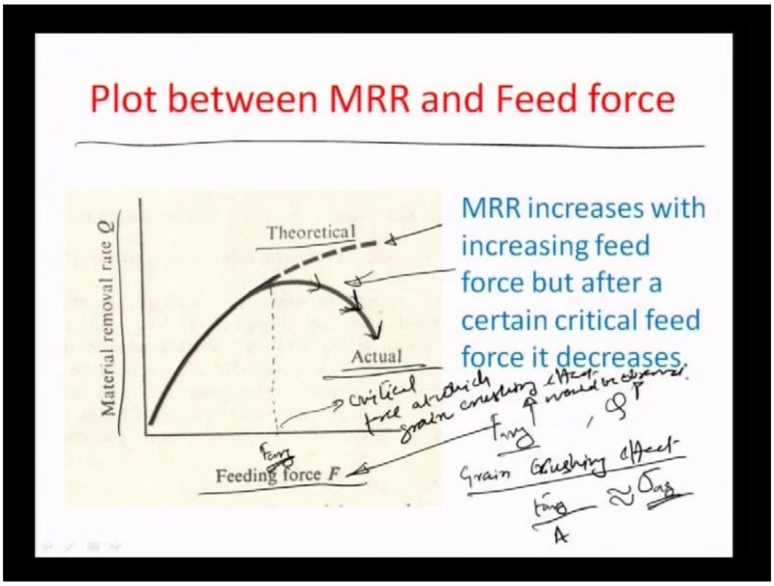
Plot between MRR and Feed force

MRR increases with increasing feed force but after a certain critical feed force it decreases.

critical force at which grain crushing effect would be observed.

F_{avg} ↑, Q ↑

Grain Crushing effect

$\frac{F_{avg}}{A} \approx \sigma_{ag}$

And therefore, the actual trend of the material removal rate is shown by this particular illustration. So, this really is a critical force, above which the grain crushing would start to take place, critical force at which grain crushing effect would be observed. So, that is in a nutshell, what would happen to the trend of material removal rate with respect to the feed force.

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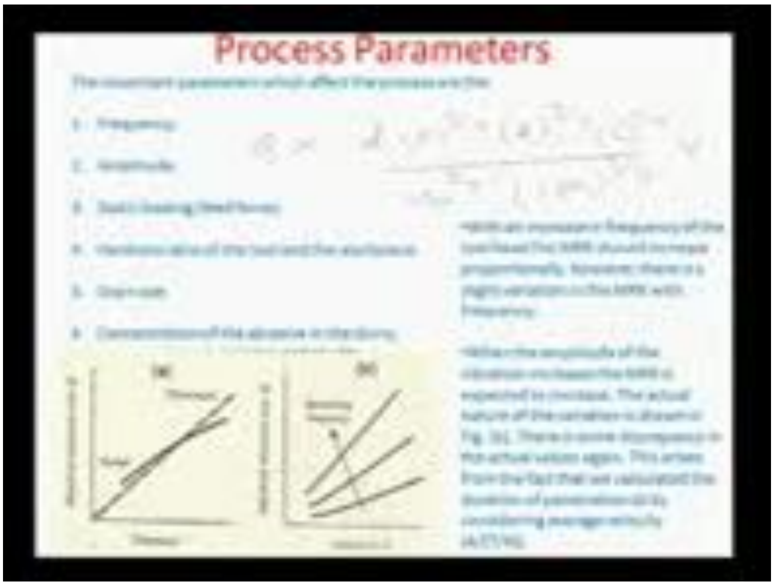

Process Parameters

So, there are some other interesting factors to be discussed. For example, as I have already pointed out that, with the frequency goes high, the material removal rate would go high. So, is visible in this particular trend here. Of course, you know the actual very slightly from the theoretical, although theoretical shows almost a direct relationship, linear relationship with increased frequency. But the actual is slightly different, because of reasons associated with the inertia of the slurry and the inertia also the tool head.

So, is the case with the amplitude; as you may recall that, amplitude if it increases, here A is proportional to 3 by 4. So, therefore, A to the power 3 by 4 is proportional to the Q the MRR material removal rate. So, therefore, any increase in amplitude would also record and increase in the Q value which is 2 here. As you see in 1 of the cases, for certain frequency, let us say nu 1, for you know an increase in the amplitude, there is a recorded increase in the material removal rate.

If, supposing the nu the operating frequency keeps on varying between let us say, nu 1 to nu 3, where nu 3 is greater than nu 1, you can see that there is a double effect. So, 1 is the effect because of amplitude and another is an overall increase because the frequency domain in which you are operating. And mind you frequency is proportional to the MRR here is also increasing.

So, as you increase the frequency, the overall material removal rate with different you know, for different frequencies the amplitude, there would be have a linear increase. So, we have already studied this aspect: the feed force, where you saw that there is a grain crushing effect which is there. And some other trends that can be useful are that related to the, what would happen for example, with increasing amplitude and feed force.

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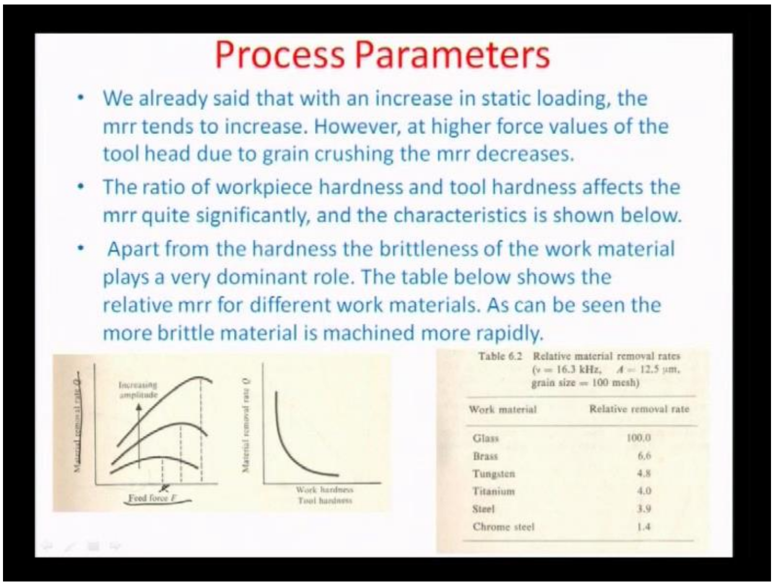

Process Parameters

- We already said that with an increase in static loading, the mrr tends to increase. However, at higher force values of the tool head due to grain crushing the mrr decreases.
- The ratio of workpiece hardness and tool hardness affects the mrr quite significantly, and the characteristics is shown below.
- Apart from the hardness the brittleness of the work material plays a very dominant role. The table below shows the relative mrr for different work materials. As can be seen the more brittle material is machined more rapidly.

Table 6.2 Relative material removal rates (v = 16.3 kHz, A = 12.5 μm, grain size = 100 mesh)

Work material	Relative removal rate
Glass	100.0
Brass	6.6
Tungsten	4.8
Titanium	4.0
Steel	3.9
Chrome steel	1.4

And, so this actually is illustrated by this particular figure here. So, with an increasing amplitude, if the feed force is higher, for every feed force there is a crushing critical limit. For example: if the feed force is at lower amplitude, meaning there by that, the gap between the overall gap between the tool surface and the work piece surface is lower. So, at a certain critical feed force value here, the grain crushing would happen and this would keep on increasing. So, the critical limit of the feed force goes on increasing, as you can see here at which grain crushing begins.

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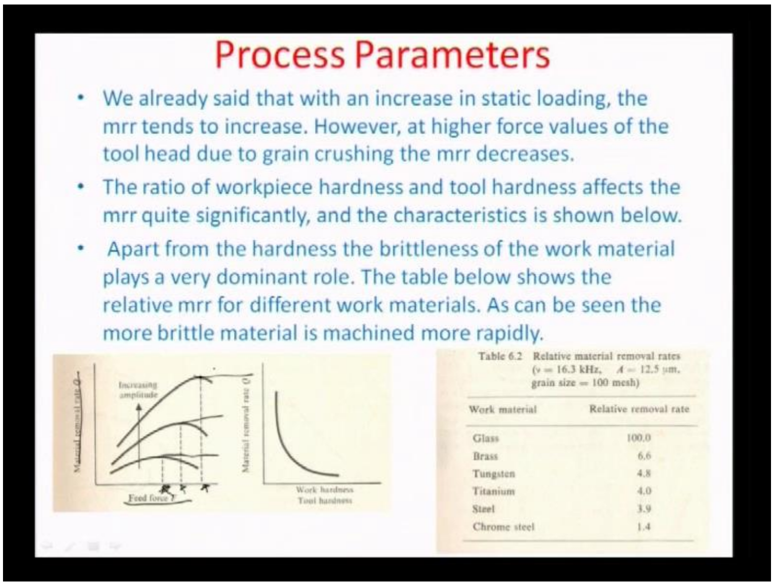

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For example: at lower amplitude it begins much earlier and at higher amplitude it begins later. And that is probably obvious, because the gap in this case between the tool and the work piece is more and, so you know it is important to see. If the gap is more, then the critical feed force which would be needed for having this grain crushing effect, would actually be higher, because the tool has a higher relaxation time for going from the surface, all the way towards its other extremity, amplitude of motion is more.

So, if you have more relaxation time, then there is a possibility of crushing to happen, at a higher feed force in comparison to, if you have less relaxation time in case of a lower amplitude.

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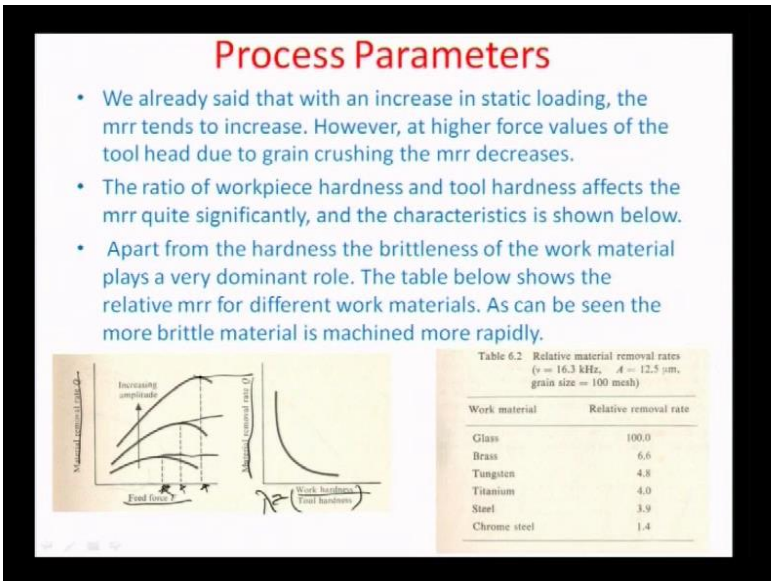

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Also important is that, if the lambda value that is the work hardness to the tool hardness as I had illustrated before is increased. There is a reduction in the material removal rate which comes; obviously.

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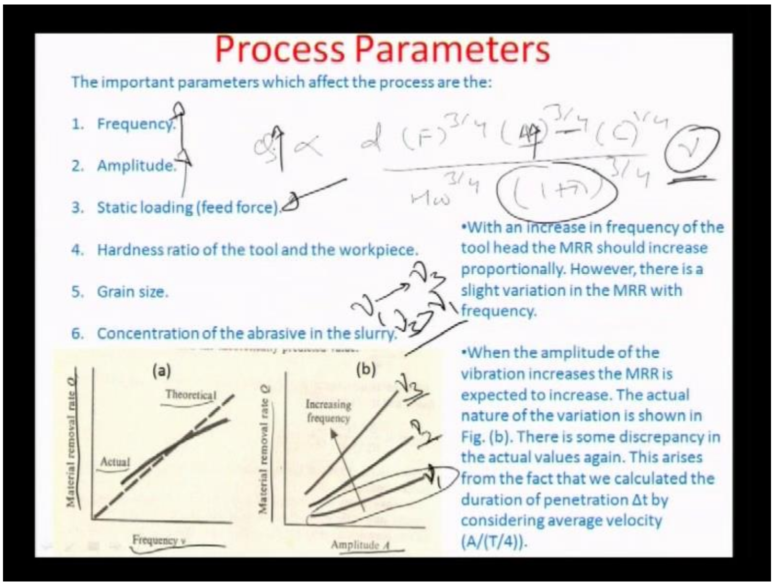

Process Parameters

The important parameters which affect the process are the:

1. Frequency.
2. Amplitude.
3. Static loading (feed force).
4. Hardness ratio of the tool and the workpiece.
5. Grain size.
6. Concentration of the abrasive in the slurry.

$$Q \propto \frac{d (F)^{3/4} (A)^{3/4} (C)^{1/4}}{H_w^{3/4} \boxed{(1+\lambda)}^{3/4}} \boxed{\nu}$$

- With an increase in frequency of the tool head the MRR should increase proportionally. However, there is a slight variation in the MRR with frequency.
- When the amplitude of the vibration increases the MRR is expected to increase. The actual nature of the variation is shown in Fig. (b). There is some discrepancy in the actual values again. This arises from the fact that we calculated the duration of penetration Δt by considering average velocity (A/(T/4)).

Because of this equation here, as you know 1 by lambda to the power of 3 by 4 is inversely proportional to the mean material removal rate Q.

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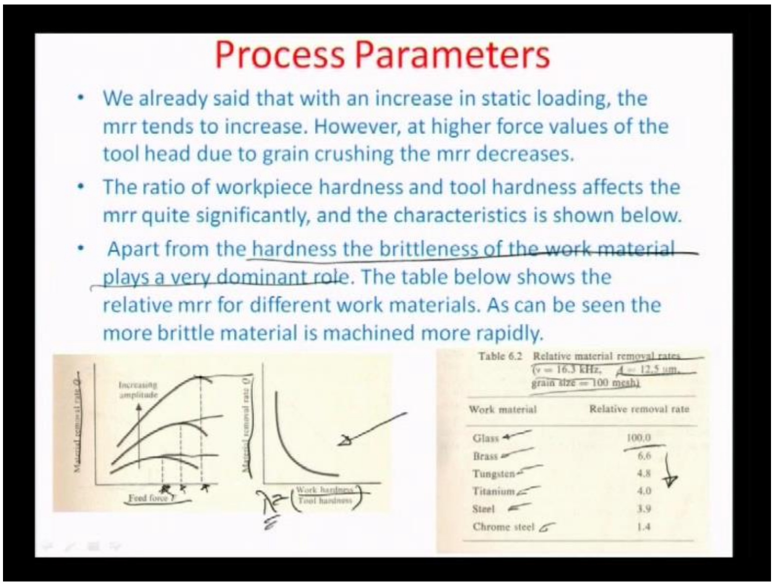

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And therefore, it is good to assume that if lambda increases, the material removal rate would fall down. And these are some of the relative material removal rates for a frequency of let us say, 16.3 kilo hertz of the vibrating tool head and amplitude of 12.5 micro meters of the vibrating tool head and a grain size of 100 mesh. So, you can see that for different work materials like: more brittle materials; glass, the material removal rate is very high, which effectively means that the work hardness by tool hardness is lower in this particular case. And if it is more, the tile in nature as is going slowly on a higher and higher scale; you can see the MRR is reducing because of change of material here.

So, of course, the hardness and the brittleness both of the work material, place a very dominant role in this process. And therefore, particularly in MEMS applications or micro systems applications, when we talk about silicon micro machining or when we talk about glass micro machining and they are very brittle in nature. So, the paradigm is really very high material removal rate, which has to be well controlled so that, you can actually have a small channel imprinted through a masking technology. That will probably show at the end of all this fundamental process analysis of the mechanical kind.

So, that is what how this processes would be applied to fabrication of Microsystems Technology. So, basically there are certain other aspects, which I would also like to point out here.

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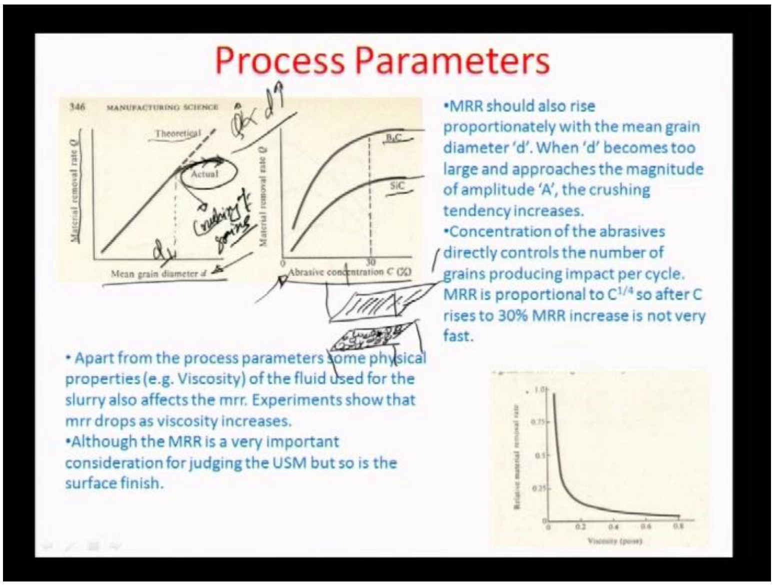

Process Parameters

- MRR should also rise proportionately with the mean grain diameter 'd'. When 'd' becomes too large and approaches the magnitude of amplitude 'A', the crushing tendency increases.
- Concentration of the abrasives directly controls the number of grains producing impact per cycle. MRR is proportional to $C^{1/4}$ so after C rises to 30% MRR increase is not very fast.
- Apart from the process parameters some physical properties (e.g. Viscosity) of the fluid used for the slurry also affects the mrr. Experiments show that mrr drops as viscosity increases.
- Although the MRR is a very important consideration for judging the USM but so is the surface finish.

For example: lets ay if we talk about how the variation of material removal rate with the mean grain diameter. It is obvious to assume that, as the grain diameter increases, the material removal rate theoretically, should be proportional increasing, as you already have mentioned earlier that, Q is proportional to the mean grain diameter d. But again the important aspect of grain crushing comes here, because of the grain is too high in diameter, there is the tendency of the tool to crush, a start crushing the grains as you can see here, so crushing of grains. And the moment this crushing phenomenon happens, as you know the MRR goes down.

So, the actual value of a MRR for a higher diameter grain greater than, let us say critical diameter given here, would be not following the theoretical trend, it would actually start coming down. And so it is a true with concentration. So, for example, you know if you keep on loading the grains in the slurry at a higher and higher concentration, for 2 different materials it has been proposed here let us say, for boron carbide with the different hardness and grain hardness and silicon carbide with the different relatively lower grain hardness.

You can see that, with the increase in the abrasive concentration, the MRR kind of plate hoes and that is, because you can always between the tool and the work piece. Supposing this is the tool surface and this other is the work piece surface and there are lot of grains on it. So, you can only pack this area available of the tool to its fullest capacity. For example: if you load more number of grains, this density of the grains per unit area of the tool work piece surface, would keep on increase up to only a certain limited value, beyond which any further grains cannot be accommodated.

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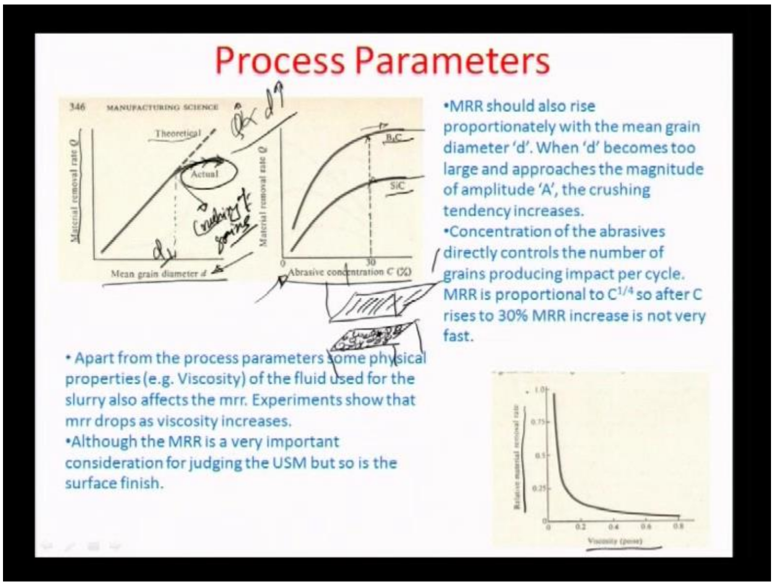

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- Apart from the process parameters some physical properties (e.g. Viscosity) of the fluid used for the slurry also affects the mrr. Experiments show that mrr drops as viscosity increases.
- Although the MRR is a very important consideration for judging the USM but so is the surface finish.

So, even if the concentration is increased beyond that any further, you do not see much you know material removal, because the amount of grains which are at probably the critical concentration here, are fully packed into this area. So, therefore, there is a plat hoeing action of the MRR with the increase in concentration beyond a certain critical concentration.

So, is the case with viscosity; a very important term for the slurry, particular when you already know that, at the very beginning I had mentioned that, the MRR in a USM is really dependent on, how or what the constitution of the slurry would be made up of abrasive particles in a fluid medium. So, if the viscosity of the slurry is more, meaning thereby; that the inter layer shear between the fluid carrying the particles are more, there is a tendency that, you know it will have a creepy motion. Or just like molasses, it will move very slowly and because of that, all the material which comes out essentially because of indentation etcetera would not be easily dissolvable in such a situation.

So, the division gradients that need to be established should be very high for the debry material, which is formulated, because of the indentation in the brittle fracture. Do not get carried away very easily in that case.

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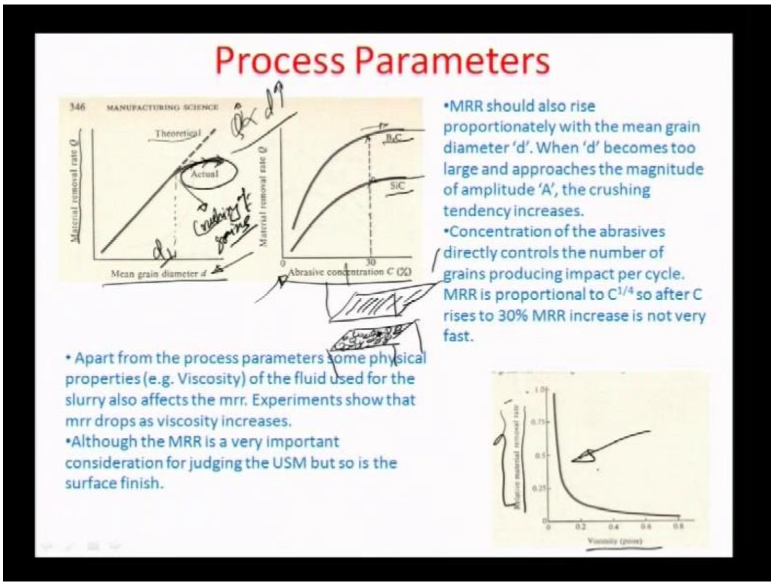

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- Although the MRR is a very important consideration for judging the USM but so is the surface finish.

So, therefore, with an increasing viscosity as you are seeing, there is a relative reduction in the material removal rate, as can be illustrated from this trend here. It is very important to know that, if the viscosity is higher, the removal of the material debry that would happen would be kind of at a lower rate.

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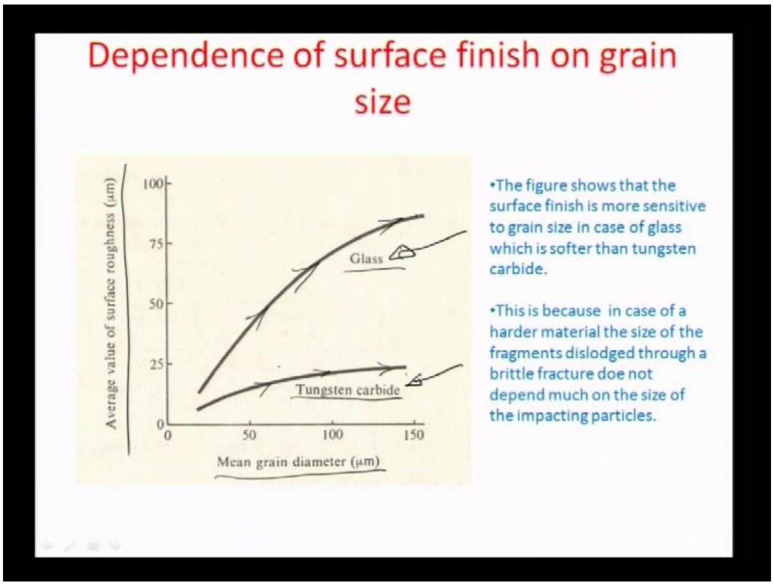

Dependence of surface finish on grain size

- The figure shows that the surface finish is more sensitive to grain size in case of glass which is softer than tungsten carbide.

- This is because in case of a harder material the size of the fragments dislodged through a brittle fracture doe not depend much on the size of the impacting particles.

So, that is in a nutshell what some of the trends operating trends would be. Another interesting factor is what happens you know for a brittle and a harder material. For example: in this case you can compare 2 such materials of average surface roughness values in microns between tungsten carbide and glass as you can see here. And with the mean grain diameter increased of course, there would be a critical grain diameter beyond which there would be grain crushing which takes place.

But what is important here to see that, if the brittle, if the surface is more brittle, then the surface roughness value which would eventually arrive at would be higher in comparison to harder material. But obvious reasons that, a brittle material would be more amenable to brittle fracture and greater chunks of pieces or materials would come out and they would form in turn larger craters and because of the larger craters the overall average roughness of the surface would be higher.

So, these are some of the dependencies of the various parameters associated with the AGM process. And what I would like to next do to. Today, we are of course, at the end of the lecture. But we would try to design some USM problems and predicatively ascertain; what is the material removal rate which would emanate from such a design. So probably the next class, whatever theory we have learned by the M. C. Shaws model of material removal, where we saw that the prominence of the direct impact or the direct hammering is much more in comparison to the free flowing grains and the way that, that removes material.

We would like to now design some problems in a manner so that, we can estimate the material removal rate. So, you have a ballpark idea of what are the rates that we are talking about. And in terms of the specific energy that is needed through this process, is opposed to some of the other comparative processes. We will try to compare. And then of course, once we had done with all that designing and the very important aspect of tool design would be taken into picture. And finally, we would like to apply these to Microsystems fabrication technology.

Thank you.