Microsystems Fabrication with Advanced Manufacturing Techniques
Prof. Shantanu Bhattacharya
Department of Mechanical Engineering
Indian Institute Of Technology, Kanpur

Lecture - 07

Hello and welcome back, I would today like to give a brief introduction on to the process USM ultrasonic machining as we left yesterday. And then, as we already have discussed before that the mechanical, you know, processes were mechanical energy is applied for material removal like USM or AJM, ultrasonic machining. Once, this whole fundamental process is clearly defined. Then, I would like to go for the applications part. The process, which should be in a subsequent lecture.

So, USM as we have already discussed is the short form the acronym of the process ultrasonic machining. As the name indicates, it basically involves the application of ultrasonic frequencies.

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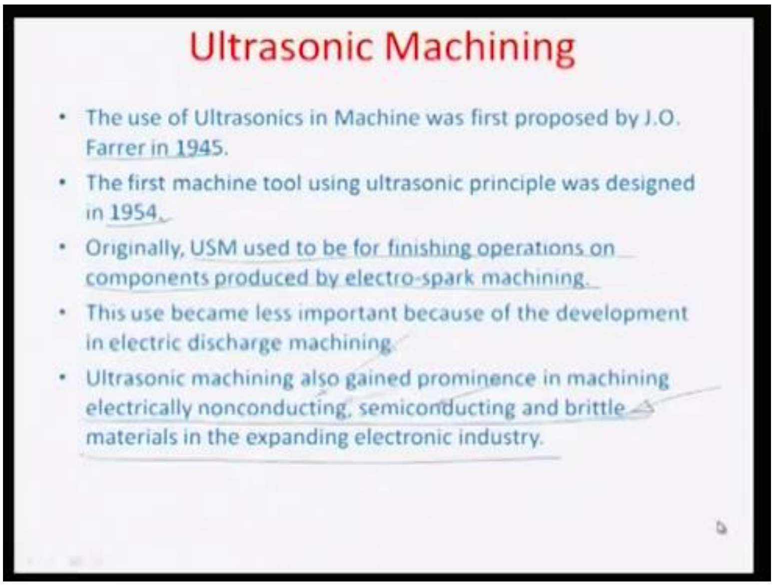

And as we have already discussed before, the process came in to existence. At least, it was proposed for the first time by J.O. Farrer in 1945. On the basis of which the first tool of the USM was designed in this principal, using the principal in 1954. So, we have already seen that, USM is applied mostly to electrically non-conducting or semiconducting surfaces. And particularly for brittle materials, where the main mechanism of material removal is brittle facture.

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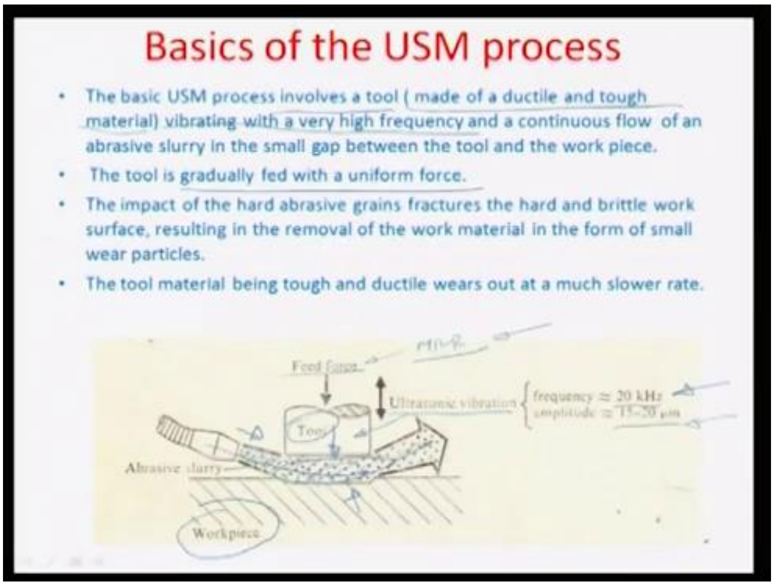

The USM basically is a process again which involves a vibrating tool head. And this was illustrated yesterday, very clearly. And this tool head is basically used over an abrasive slurry. You can see, this is the region of the slurry. This is the work piece. This right here is the tool. And so basically, small amplitudes of motion about 15 to 20 microns are given to this tool head. And it quizzes or compresses the abrasive particles, which are coming between the tool and the work piece.

And this happens, so at a very high frequency of about 20 kilo hertz in the ultrasonic range. And basically, the movement or the small amplitudes of the motion of the tool has a positive impact of plugging of the abrasive grain over the surface. And this is basically the principal mechanism of material removal in a USM process. So, we will try to explore different models for estimating, how material removal rate can be characterized or how it can be organized?

And the first model, which comes into you know, mind, when talking about particularly the material removal rate is, what we also know as the M.C. Shaw's theory or M.C. Shaw's model of USM.

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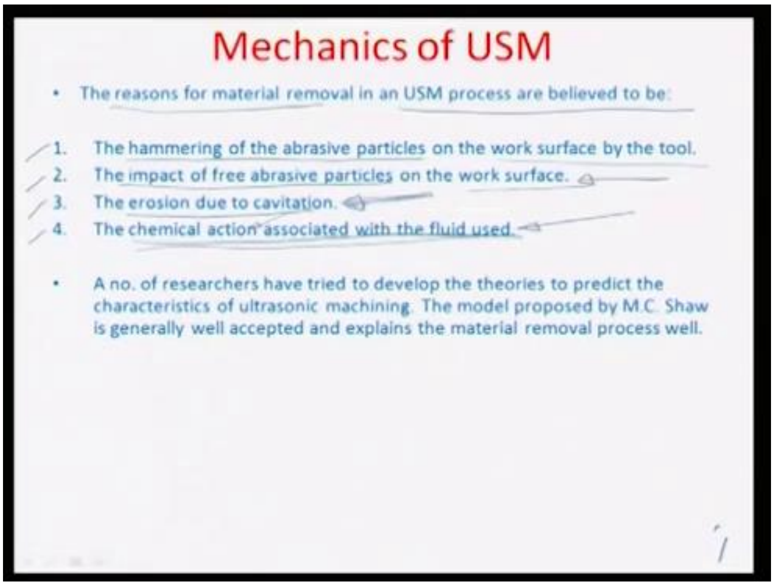

So, this right here of material removal in the USM process. And the principle reasons for material removal, a kind of illustrated here in to this four categories. And the first category is, because of the hammering, a direct hammering of the abrasive particles on the work surface by the tool. The second category is the impact of free abrasive particles on the work surface. So, what you mean by free abrasive grains here?

It is that, when basically this slurry comes between the tool and the work piece. There is a gap between the tool and the work piece, which is almost always existing. And also, because we are giving small amplitude of motion to the tool, on the position of the tool changes from very close on to the surface to farther away for the surface. And there is always the possibility of grains of small grain dia, say about close to 15 macrons or 20 microns to fly between the tool surface and the work piece surface.

Because, there is in one instant at least when the tool is farthest, in it farthest position away from the work peace, there is a lot of gap between where more than one abrasive particle can come. It can stick or impact the tool surface. And it can actually come back and in pinch the work piece surface, because of the impact of the tool surface or form the tool surface. And so, that mechanism also we need to consider or that aspect also we need to consider, while considering the material removal rate estimation.

So, that is the impact of free abrasive particles on the work surface, as well as stated here. And then, off course there is this effect of erosion. And that principally comes, because of cavitations. Now, I would like to explain, a little more on cavitations. So, what exactly is meant by cavitations is that, whenever there is a highly frequency object. Let us say, there is a may be a surface which is moving on which a fluid rest.

And it is moving at a certain frequency, very high frequency with respect to the fluid over it, where the fluid is resting on it. So, what typically would happen is that, the fluid because of very large inertial component may not be able to move with that higher frequency. And there is always going to be air gaps or gaps, where air can be a sort of pulled in or you know, some degassing can happen of the solution also or the material which is over this surface.

And there are some cavities which are formulated, because of the relative difference of the motion between the high inertial fluid and the high frequency surface. So, these cavities are basically nothing but, either low pressure zones which is pulling in air, where air can be out cast from the liquid. Or you know, in case there is an option of air to outcast sideways. This air can come in. And it can do this; you know sort of small inclusions within as bubbles within the fluid on the work surface.

These bubbles actually get formulated hugely at a certain frequency, which is also very high or 20 kilo hertz in the USM process also. And as formation of the bubbles would definitely lead to some kind of full forces, generating full forces where the materials which are actually fractured using this brittle fracture theory on the surface, would actually get pulled in by such a bubble. And it would actually promote fast dissolution of the daubery, which is formulated because of the USM process.

So, therefore erosion of the material due to cavitations or due to the formulation of the bubbles in a USM is also another principle mechanism of material removal. And then, finally there is chemical action associated with the fluid used. Assuming, if there is some kind of a softening of the material or some kind of you know, change in the material property by the fluid is that, is being used as a slurry for carrying the abrasive.

There is always a chemical action reaction which is involved on the surface, which would actually loose in the material, a little more. And that can help in the material removal process. So, these are the principally four mechanisms hammering, direct hammering of the abrasive particles, impact of free abrasive particles, erosion due to cavitation and chemical action associated with the fluid that is used.

So, we would like to now illustrate, a little bit on the numerically available predictions of theories or models, which are associated with the USM process. And while talking about the estimation of material removal rate in an USM, the most widely used theory was processed by M.C. Shaw and what is known as the Shaw model. And we will try to discuss the whole material removal rate on the basis of, the theory proposed by the Shaw's model.

So, let see some of the assumptions which were made by the Shaw's model. Here, illustrated here in the next slide.

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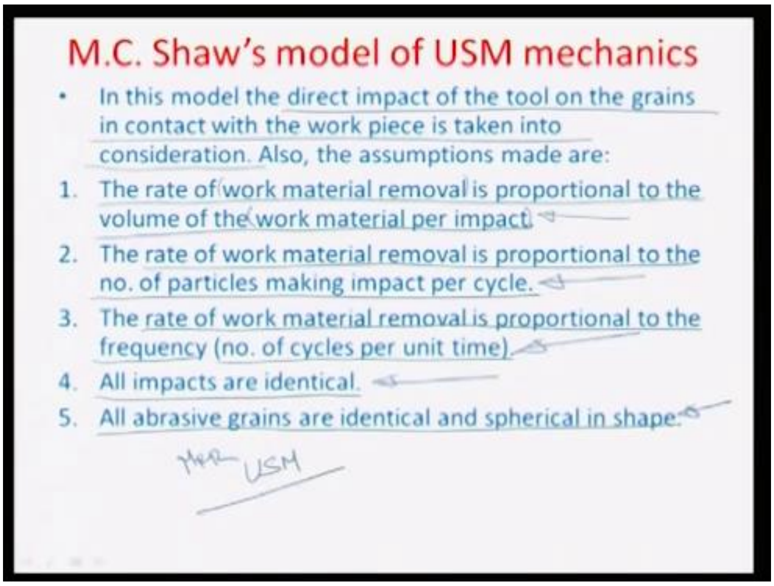


So, in this model the direct impact of the tool on the grains in contact with the work piece is taken into consideration. And as we see later on, when we do the numerical estimation of the MRR, the free impact or the impact of free grains would actually not be substantially contribute of the material removal rate. And whatever would be principally contributive would be the direct hammering action. As we see, just a little bit later.

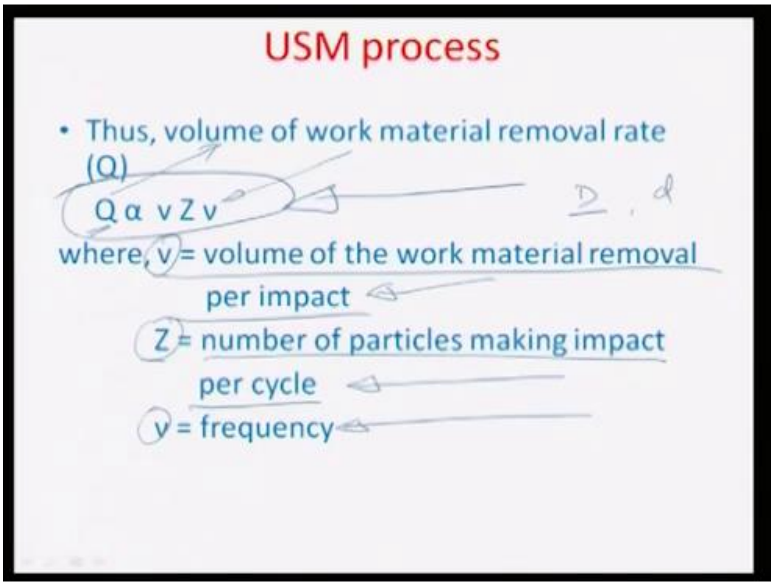
So, also some of the assumptions made of the following in the Shaw's model. Number 1, the rate of work material removal is proportional to the volume of the work material per impact, which makes sense that whatever you know. Material gets remove by one set of grains, coming in contact with the surface; because of the direct hammering action would be the work material per impact. And this somehow, would determined the overall work material removal rate.

At least, this should be proportional to the work material removal rate, because the other factor that it will depend on is the operating frequency of such plowing action, removing the work material. The rate of work material removal also is assumed to be proportional to the number of particles, making impact per cycle. If supposing, there is 1 abrasive grain and then subsequently in another case 5 abrasive grain, so the material removal is assumed to be 5 times the previous case.

For obvious reasons that, at one impact or one cycle, 5 grains are doing the plowing action, as supposed to one grain which was then earlier. And also the rate of work material removal is proportional to the frequency, as I already insisted before of the vibrating tool head, which is the number of cycles per unit time. And all these impacts are a kind of identical. And we assume that, there is no non idealized situations.

So, this situation that we are assuming is that, there is a tool which actually comes close to the surface. And there are certain grains, which are trapped between the tool and the worth piece. And you know, when the tool moves a part, there is always a cavitation or erosion of the material which has been brittle fractured. And the next set of an abrasive grain comes, when the tool again approaches. So, this is the idealized condition of operation of the USM.

So, all impacts are assumed, this way to be identical in the Shaw's model. And also, one more important factor is that, all abrasive grains are assumed to be identical and spherical in shape. So, these are the five assumptions that the Shaw's model makes. (Refer Slide Time: 11: 37)



And if you really go for a predictor estimation of the MRR or the Material Removal Rate of a USM process, this actually comes out to be of the form Q proportionate to v Z nu, where v is the volume of the work material removal per impact. And we will calculate this. What this volume is, in terms of indentation? Z here is the number of particles making impact per cycle. So, this basically at one impact, how many particles are coming between the tool head and the work piece surface?

Nu of course is the operational frequency. Meaning there by, it is a frequency of the vibrating tool head. And Q is the MRR or the volume of the work material removal or volume rate of the work material that is being moved. So, this is how the Shaw's model predicts the Q or the material removal rate acts. Now, let us actually look into the various terms here, like the v, the Z, the nu. And try to derive at least, an order of magnitude equation for different terms concerned in the USM process.

Like for example, may be the grain diameter or the diameter of the impact, so and so forth. And try to derive at this MRR to give some estimate of the, at least the order of magnitude of the various parameters, which are involved in the USM process for doing the MRR.

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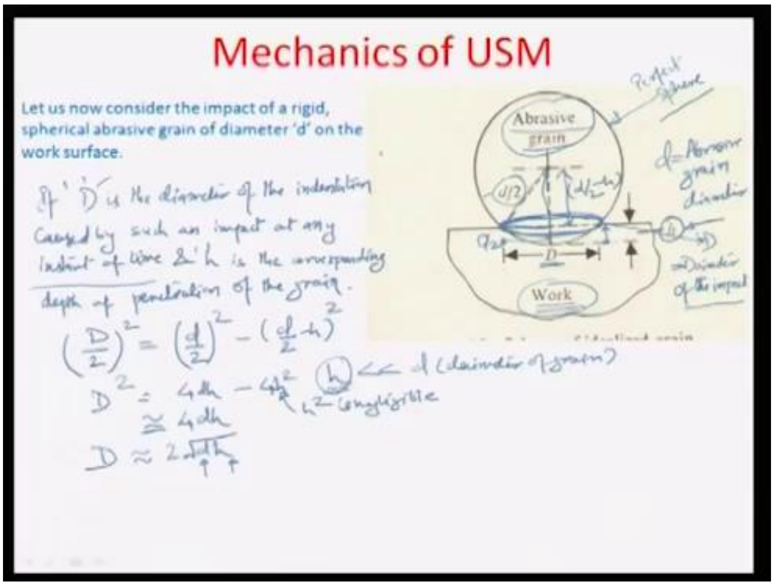

So, let us look at a little bit further by assuming that, in a USM process, now this is the work piece. And there is an abrasive grain, which is hitting the work piece. It is coming down, as if it is plowing the work piece. And we assumed that, there are two different diameters in questions. One is small d. As you can see here, the small d is nothing but, the diameter of the grain. So, it is the abrasive grain diameter.

And there is another term capital D, which is actually the diameter of the impact. So, this right here is a circular region which is the effected region, because of the plowing action of the grain. And we assume, another parameters D capital D, which is actually the diameter of the impact. Or in other words, it is the diameter of the creator which is formulated, because of the plowing of this abrasive grain on this work piece surface.

We assumed here by the Shaw's theory, that this grain here is a complete or a perfect spherical shape at structure. So, we assumed the grain to be a perfect sphere. Now, if we really find out the D to be here or if we defined the D to be the diameter of indentation, so if the D is the diameter of the indentation caused by such an impact at any instant, let us say. Instant of time and this small h here, as you can see is the depth of penetration of the grain.

So, at that particular instant of time, h is the corresponding depth of penetration of the grain. So, we have a geometrical relationship between all these, the capital D, the small d and the h. And it can be written down as capital D by 2 square. It is actually equal to small d by 2 square minus small d by 2 minus h square, just by using Pythagoras theorem. You know that, this height here is small d by 2 minus h. This is capital D by 2.

This size here is capital D by 2. And so therefore, this is the right angle rectangle. And we just simply applying Pythagoras theorem here. So, it becomes d by 2 square minus d by 2 minus h whole square. So, calculating it further, square of the D can be represented as 4 d h minus 4 h square. Typically, we do assume in USM process, that the grain penetration depth h is very, very small in comparison to the diameter of the grain d.

So, grain diameters can be something like typically about 20 to 25 microns. And in a USM process, typically this h could vary between anywhere between 1 and 5 microns. So, it is quite small in comparison to d. And so therefore, this equations can actually be approximated as 4 d h. We can neglect the h square term, square is negligible. And therefore, a relationship exists between the impact dia D and the grain dia.

As D is approximately equal to twice root of small d times of h, where small d is a grain dia, h is the penetration depth of the grain, in question. So, if you assume that the volume of the material dislarged is sort of proportional to Q of the D. So, as you know that this actually is the diameter of penetration of the grain. And if you assume that, this grain is a sort of hemispherical or the penetration is actually, it is a sort of hemispherical.

Then, we can safely say that, the penetration diameter D, the cube of the penetration diameter is estimating the volume of material removal. Even otherwise, if supposing h is too small and we cannot equalize capital D to small d the grain diameter, in that case also if the capital D signifies the impact diameter or the diameter where the grain has penetrated. At least, the cube of this dimension should be able to estimate.

In some proportion should be able to estimate the overall volume of the material that has been, work material that has been fractured, because of this impact.

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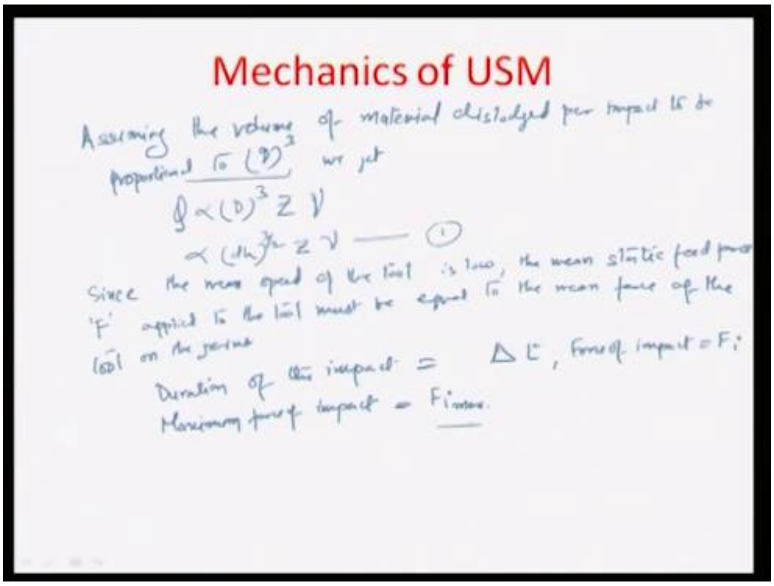

So, let us assume that and let say that the volume. So, we assume here, the volume of material dislarged per impact to be proportional to the cube of this diameter. So, we get the Q from the Shaw's theory earlier is proportional to volume of the work material removal per impact, which is actually cube of this D. For obvious reasons discussed before, times of the number of grains or a number of particles coming to, deliver the impact per cycle, and so the cyclic frequency nu.

So, here we can substitute the value of D has d h under root. So, this becomes d h to the power 3 by 2 times of Z times of the tool frequency nu. Let us make this equation as equation 1. Now, let us also understand the impact of the force on such a grain diameter. Because, typically the grain diameter or the grain is been pushed, using a certain force by the tool.

So, for bringing home a good model, it should be able to somehow correlate the force per unit area coming on the impacted area of the work piece to be equal to the flows trust of the work material. So, it is a cut of threshold which when reached, would actually promulgate deformation. And sometimes, fracture as in this particular case. So, the ultimate flows trust of the material, needs to be at least reached.

It can be more than that. The impact force can be more than that, per unit area. But, then at least it should be the flows trust of the material, for the material to start coming out or start getting fracture. So, the assumptions would behind the model really is to somehow be able to protect from the applied force. What amount of force per unit area is coming on one grain, as a matter of factor of many grains?

And that has to be somehow, equated to the ultimate flows trust of the material that you are releasing or removing of the surface. So, let us look at it. And see, what this estimated is? So, since the mean speed of the tool is low. We can assume that, the mean static force F applied to the tool must be equal to the mean force of the tool of the grain. There is hardly an inertial component of the grain also, which is involved.

The grain itself is too small for involving any inertial component of the grain, as such in this theory. So, therefore we say that, since the mean speed of the tool is low. The mean static force of feed of the grain, let us call it F here. Applied to the tool must be equal to the main force of the tool on the grains. Now, on the duration of such an impact is assumed to be some small value delta t, let us say. And the maximum value of the impact force can be something like F i max.

The nature of variation of F with time can be plotted and seen. And the behavior can be estimated as, how the force would vary from the stage that, the grain is coming in contact with the tool to the stage that the grain is getting plowed, because of continuous hammering of the tool to the stage, when the material gets dislarged and the grain becomes a free grain to move along with the slurry.

So, this whole event or sequence of events can be recorded, as a force applied by a reaction force applied by the grain on the tool, in just the opposite or the reverse to the action that is happening on the work peace. And plot that F, the force of the tool with respect to time. So, let us say that, the duration of the impact is some value, delta t small.And the maximum, let say the force of impact is represented by the terminal F i.

And the maximum force of impact is represented again by a F i max, maximum value of F i at a certain incidents of time.

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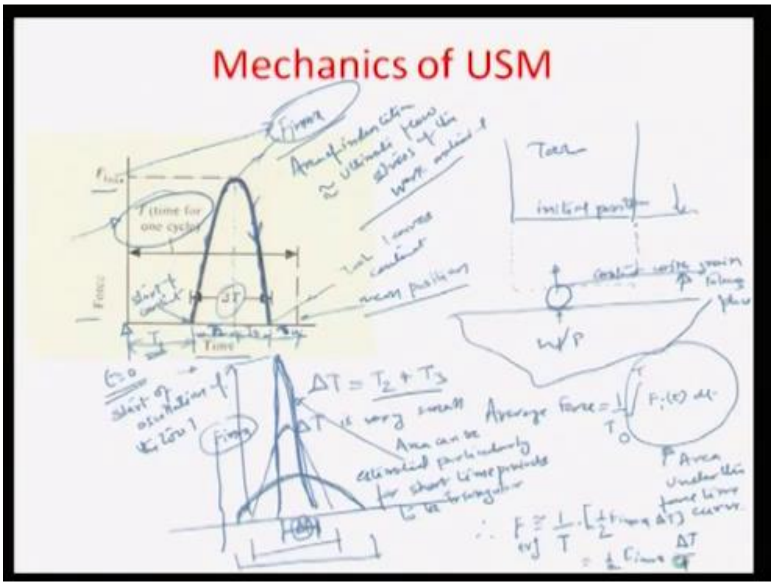

So, if we really look at the variation of the force F i with respect to time, it looks something like this. So, as you can see here, is a several stages in the plot, where the whole grain is being hammered and crushed between the tool and the work piece surface. You start with time t equal to 0 here, for example. So, this is the case when the grain the abrasive grain is free. There is no direct contact of the tool of the grain.

And let us say, that is where the tool has started towards was late. So, this is the start of oscillation of the tool. So, the tool does not find out a grain, unless it moves closer to the surface by a certain distance. And this time here, let say for example, this can be T 1 is a spent in coming in contact with the grain, which is already in contact with the work surface from the other side. So, this is the tool, let say. And this is the surface that, we are trying to machine.

And there is a grain, which is resting somewhere here, an Abrasive grain which is somewhere here. So, the time T 1 is the really spend by the tool in covering, this distance here. From the initial position of the tool to the position where contact with grain takes place, which is already in contact with the work piece surface. So, that is represented by time T 1. Now, the tool has come and it has started to do the plowing action.

The tool is now some where here. And it has started to just about to do the plowing action. And therefore, there is always reverse reaction force, which is available from the grain on to the tool, which is being plotted as a force in the force time diagram, here. And therefore you know, really there is no deformation which happens, until a critical value of stress is reached at this level here, where it can start to penetrate inside the work piece at this particular interface.

So, the force is continuously increasing, all the way to F i max. And the F i max, per unit area of indentation is really the ultimate flow stress of the material. So, F i max per unit area of indentation can be equated safely to the ultimate flow stress of the work material. So, that is about the time. Let us say a time T 2, which is spent in reaching from direct contact which starts here. Start of contact, all the way to F i max.

Corresponding to this point here, which is the force needed for causing the material to flow. And then, off course once the material has started flowing on, the grains start going inside. The force on the grain comes down. It comes down and the tool, really you know after this point does not face much force, because the material is in the flows state. So, the force comes down. And then, after a while the tool leaves contract.

So, the force is zero. So, it comes down here and some where the tool leaves contact. And from here to here, so that happens. Let us say, after time T 3 and after T 3 is passed, then the tool leaves contact. So, the tool leaves contact from this point and goes back to its original position or the mean position, because it is an oscillating tool. So, this is the whole time cycle for the tool. And the force verses time cloth plot.

Now, if we assume this characteristic, because typically impacts are over very short durations. The delta T, that is the time needed for the whole impact and flow process, which is also equal to T 2 plus T 3. As in this case, you can see is very very small. So, delta T is very small. And it is obvious to assume that in this kind of a situation, if the delta T were large or the delta T is very small, slowly you know the different time values as you can see here.

It promulgates the curve to go from a, a round shape all the way to a, shape of a triangle. So, if this time gap is very very small, delta T. Then, it can you know, the force is pretty high, because it is an impact force which creates at the stress. So, it will not be very much ironies to assume, that the major of variation of F i is triangular, particularly when this delta T is very very small. So, therefore T, if we look at the average force, it can really be a time average force.

Meaning there by, if this is full, the time for the vibrating tool had time period for the vibrating tool head is T, capital T. Then, the average force can be written down as integral 0 to T, F i as a function of time, let us say dt divided by capital T. And this typically is nothing but, the area under the curve, the force time curve. Area under the force time curve was illustrated here, already. The area can be estimated, particularly for short time periods to be triangular.

Therefore, the total force here can be approximated by 1 by time period T, times of area of this triangle here. One who's height is F i max and who's base is delta T. So, obvious altitude is the total area of a triangle. So, we write half F i max times of delta T. And this can be given a subscript F average. So, we see here that, the average force really is actually equal to half of F i max delta T by time T.

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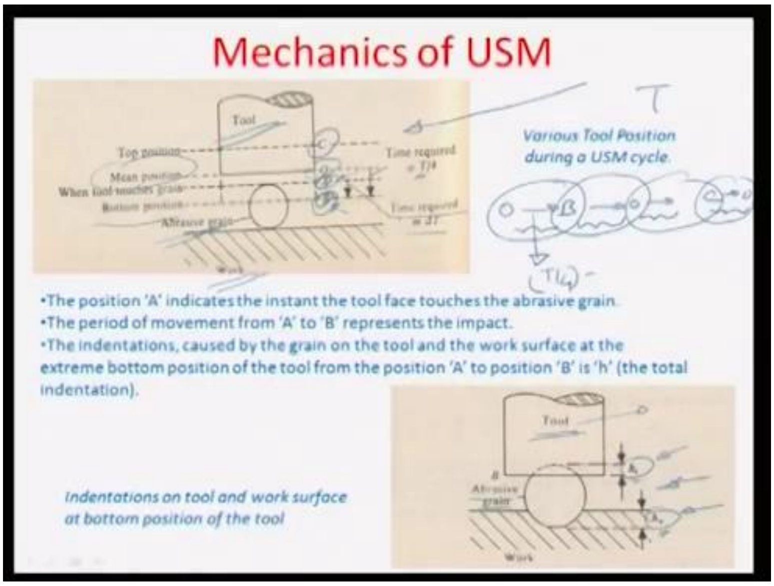

So, the motion of the tool for the various positions, tool positions along the whole cycle is indicated in this particular figure here. And this, we have already discussed. But, we have just like to diagrammatically repeat the whole statements, we made earlier for the just for the sake of repetition. This is the tool here. The work piece and there is an abrasive grain in between. And the tool goes between positions C, which is the top most position, the other extremity of the tool farther away from the surface.

And nearer to the work piece, the position which is also the bottom position, that is B. But, the mean position of the tool is illustrated by the point O, here. So, typically the force really starts to get executed from the point A, goes all the way up to B. And then, after the tool releases the contact, it is a zero force. And the tool comes back. And that is how, the plot of the force and time had been arrived at in the last slide.

The total time delta T for the force to grow from 0 to F i max, all the way to zero is nothing but, the force starting from the position A up to the position B. Also it is a important to assume, that if the total time period of the tool for this whole oscillation, where it goes from O to B, back to O to C and back to O. So, there are in total 4 times needed for translations between O and B, B and O, O and C and C and O.

On other words, for one of these if the total time period of oscillation of T, the total time needed for only one of these motions or one sector of these motions is T by 4. I would also like to illustrate, what happens when the abrasive grain gets starts getting crush between the tool and the work piece, as I illustrated hear. So, we can see here that, there is a certain depth of indentation on the work piece surface.

And certain other depth of indentation on the tool surface given h by w and h t has been indicated here. And these are the top and the bottom, you know surfaces of the tool in the work piece respectively. In case the grain is engraved on to both. And there is an engraftment in the tool side as well, because the tool also has an ultimate flows stress. And all though by design, we make the flows stress to be of the tool to be higher than the flows stress of the work piece.

But, still you know during the process of the impact etcetera, there is a general tendency because of very small area of the grain interface with respect to the tool. The pressure can really, the force ((Refer Time: 37:44)) area can really go to the level of the flows stress of the tool material also. And therefore, there has to be an indentation on the tools side and there has to be an indentation on the work piece side. So, coming back to this theory again, with these assumptions we move forward.

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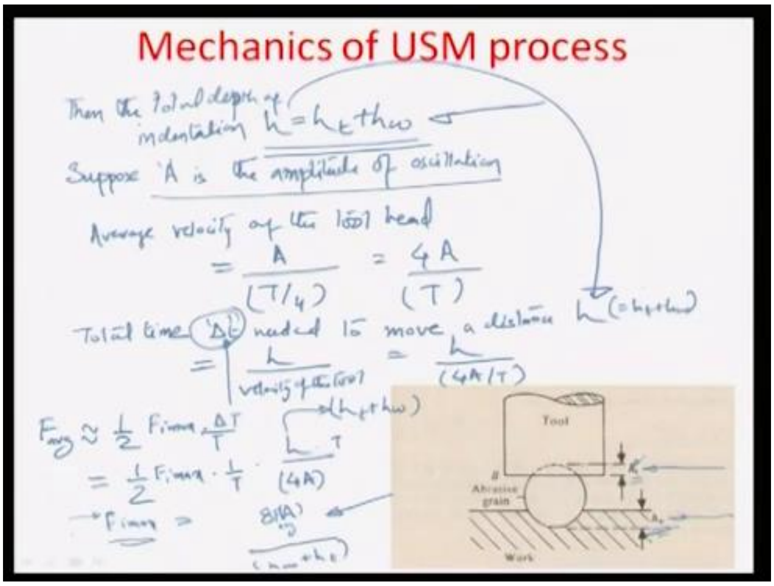

And therefore, one thing is very clear that, because there is a depth of indentation both on the tools side and the work piece side. In this case, for sake of convenience we are representing it by h t and h w. So, the total depth of indentation, h as represented by h t plus h w, where obviously h w is the work piece indentation and h t is the tool indentation, from the tools side. Supposing, A is the amplitude of oscillations, actually it predicts the motion, the philosophy of the tool.

By just looking at, how much time is needed for the tool to move between the various sectors? ((Refer Time: 39:27)) This for the sake of repetition, I would like to bring your attention to this last slide again. So, as I told you the whole path motion of the tool is represented by a motion from O to B, B back to O, O to C and C back to O. And the amplitude to motion really is this one sectoral movement, let us call it A.

That is how you defined amplitude in case of simple harmonic motions. So, the total velocity of this tool is, assuming A to be the amplitude of oscillation of the tool. So, the average velocity of the tool had. It is basically, the amplitude A by the time period in 1 quarter cycle. That is how the difficult definition of amplitude comes into picture. So, it is 4 times of amplitude of the tool divided by the total time period of motion of the oscillating tool have.

So, if supposing we have already seen that, the total depth which is moved, because of indentation on the tool side and the work piece side is equal to h. How much total time will be needed to cover that particular depth h? It should be equal to the depth h divided by the velocity of the tool head. The velocity has already been estimated as 4 A divided by T. And therefore, the total time which again is actually delta T.

That is needed to move a distance h, which is actually equal to h t plus h w. The total indentation should be nothing but, h t divided by the velocity of the tool. Velocity has already been calculated as 4 A by T. So, it is h t divided by 4 A by time period T. So, the average force which has been earlier predicted as half F i max times of delta T, which is same as this delta T. The amount of time needed for the total indentation to proceed divided by T, can be actually now represented as half F i max times of 1 by T, times of the value of this delta T, which is h divided by 4 A times of capital T.

And this h again as you know is nothing but, the indentation of the tool side and the indentation on the work piece side. So, here we are able to generate an expression for this maximum force on the tool head, which is actually equal to 4 times the amplitude of motion of the tool, times of 2 here. So, it is basically which is basically 8 times F, average times of amplitude of motion A, divided by h w plus h t.

So, that how you calculate the maximum force coming on the tool head? And then, try to do something to equate this force per unit area of the indentation to the ultimate flows stress of the material.

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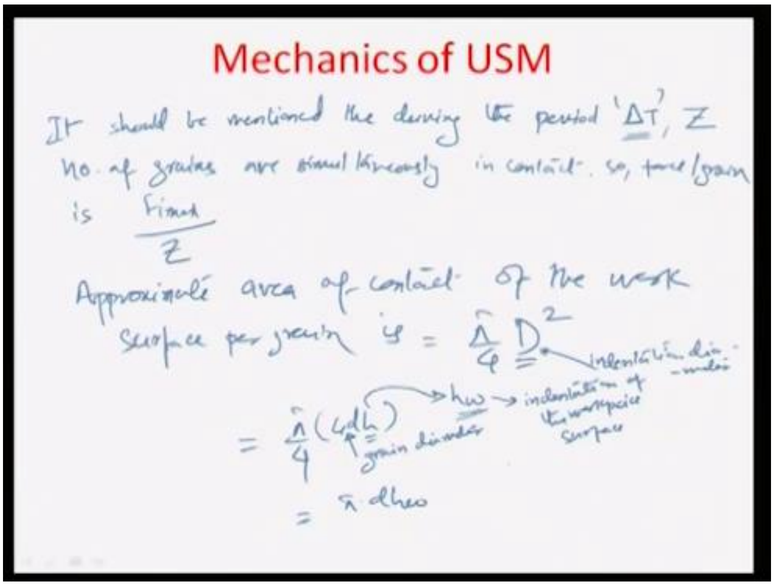

So, let us actually try and do that by looking at, what is the force at least per grain, which comes between the several grains which are there, it mean I said the tool and the work piece surface. So, it should be mentioned that, during the period delta small t or delta capital T where the indentation is happening. It is not a single grain, which is doing this indentation. So, if you assume that, there are z grains which are they are between the tool and the work piece surface.

At that small instant of time, delta t where the indentation is happening and they are all placed at equal height with respect to the surface. And the tool is perfectly flat surface.

So, therefore all these grains are together going same distances, in terms of its plowing actions on to the work piece. So, in that case if suppose Z numbers of grains are simultaneously in contact. So, force per grain is represented as F i max by Z for obvious reasons.

And if you suppose estimate that, the approximate area of contact of the work surface, per grain is pie by 4 capital D square. Remember D was actually the indentation diameter. So, this can also be estimated as pi by 4 times of 4 d h, where this small d is the grain diameter and this h here is the indentation of the work piece surface. Remind you, this h is really not equal to h t plus h w, because while considering the geometry of the penetration of the grain on to the work piece, only the indentation on the work piece side was being considered and not the grain.

So, this h in the equation for the relationship between the indentation diameter, the grain diameter and the travers h. They are the, h actually is the depth of indentation on the work piece surface, so h w. So, that is how we can relate all these. And then, we can mention this to be equal to pi d h w.

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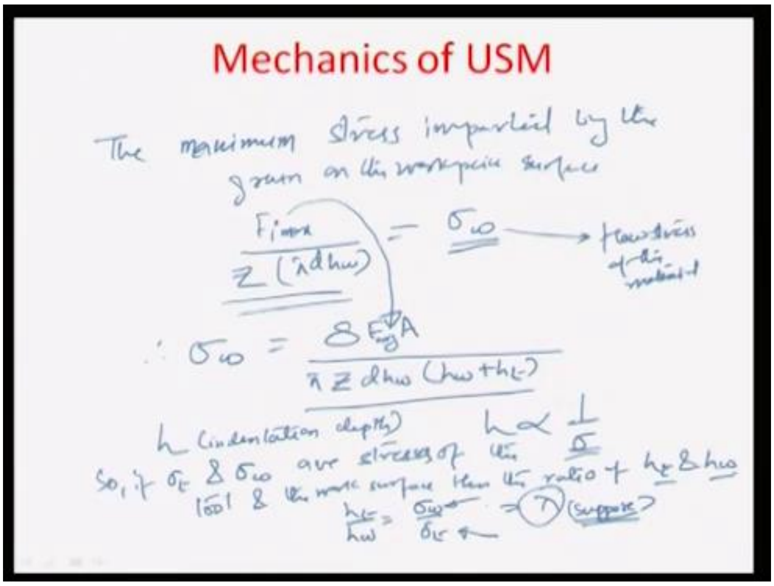

Simultaneously, the maximum stress imparted by the grain on the work piece surface is given by the total force per grain, which is available per unit area, which is pi d h w from the previous slide. So, this stress should be actually equal to the flows stress of the materials sigma w. So, this is the flow stress of the material. For the fracture to happen, the brittle fracture to happen or for the material to get deformed and the greater would have start to formulate, in that situation.

So, therefore, the sigma w here is represented by the earlier equation as 8 times of force average times of amplitude A divided by pi z d h w times of h w plus h t, from the previous equation. So, we just simply substituting the value of F i max in this particular equation for sigma w. So, it is quite reasonable to assume that, the depth of the penetration is inversely proportional to the flows stress of the material, as long as the load in the indenting sphere diameter remains the same.

And we can always say that, if h b is the indentation depth. This h can be inversely proportional to the flow stress of the material, meaning there by that type of material has a higher flows stress. For a certain force level, it could have lower depth of indentation and vice versa. So, if sigma t and sigma w are the flows stresses of the tool and the work surface, then the ratio of h t there is a depth of the indentation on the tools side.

And h w, that is the depth of the indentation on the work piece side can be represented by the ratio sigma w by sigma t, where this is basically the ultimate flows stress of the work material. And this, here right here is the ultimate flows stress of the tool material. Let us suppose, that this ratio is equal to some constant, lamda. Particularly that we are not changing the tool material of the work piece material. So, it is really a, it is a material property and that ratio is represented by this factor, lamda here.

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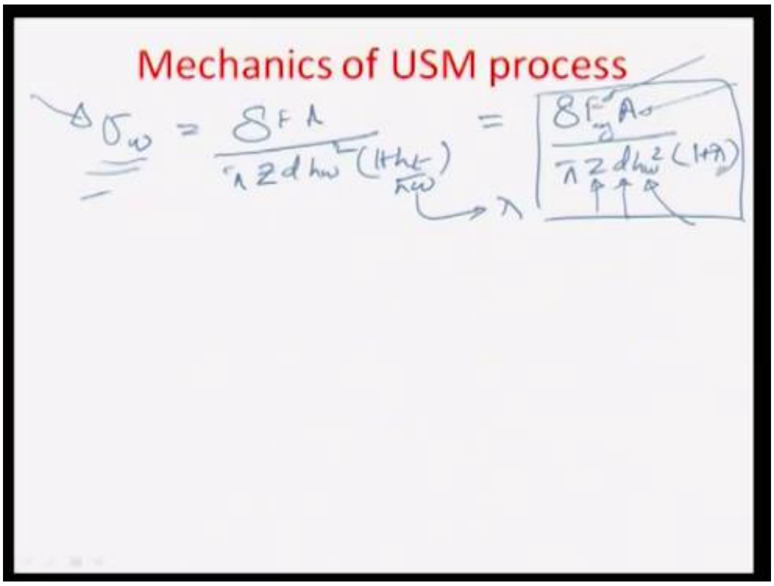

So, therefore the maximum the sigma w, the ultimate hill stress which is develop can slightly be modified as 8 F A divided by pi Z t h w square times of 1 plus h t by h w, which actually signifying this constant lamda. So, we can write this down as 8 F A divided by pi Z t h w square by 1 plus lamda. So, as of now we will continue this in the next lecture.

But, as of now we have come to know that, there is a way that you can actually relate the ultimate flow stress of the work material with respect to the force, the average force on the tool, the amplitude of motion of the tool, the number of grains per impact or impacting the surface all at one go between the tool and the work piece. The grain diameter and the penetration depth on the work pieces square, times of this material property which is the ratio between the ultimate flows stress of the work and the tool with respect to each other.

So, with this we come to the end of today’s lecture. And we will continue this in the next class. And try to find out, how we can put a value for Z, in terms of numbers per unit volume of the particles, that would be a composition of the… That would be indicate you, of the composition of the slurry, the abrasive slurry. And then, try to find out what is the MRR, based on all these different parameters or at least in order of magnitude basis. Then, we can also estimate certain plots and trends from the actual experimental methods to this theoretical model and try to ascertain whether, they are in ((Refer Time: 53:23)) or they are in consonance with each other.

Thank you.