Micro system Fabrication with Advance Manufacturing Techniques

Prof. Shantanu Bhattacharya
Department Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 28

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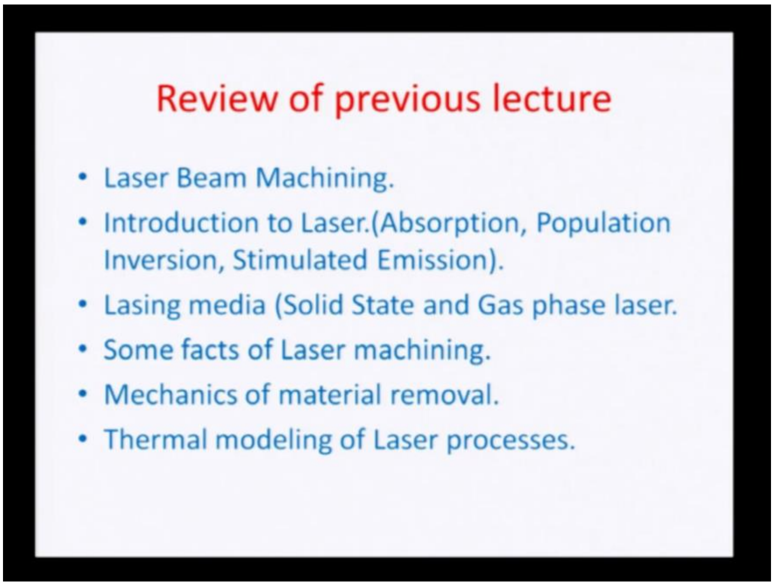

Hello, and welcome back to this twenty eighth lecture of micro system fabrication by advances manufacturing processes let us recap quickly what we did last lecture we talked about laser beam machining as we probably already know that it is basically the photon to matter interaction which converts which essentially converts into the physics of photon to photon conversion and... So, there is volatilize vibration induced by the high level of energy by the photons which causes local temperature raise, and it is a surface phenomena. So, the temperature raises to a certain extent, because of an extremely high intensity of a energy packed in.

So, that it goes into the vaporization state, and it creates a a sort of burst effect, and that is why the machining probably is much faster, and it is a surface phenomena which is different than the e beam machining where e beam typically particulates to an region or a small skin which is called the beam transparent layer. So, it is more towards the surface that the l b m is geared to we talked about the introductory concepts of laser laser beam light amplification by stimulated emission of radiation were in at the first stage there is an absorption there is an frequency which is sent in frequency of light which results in in very concertized manner an electron to go to a higher orbital from a lower state, and an avalanche of these processes are produced together which is known as population inversion.

Once this state is reached all the high energy atoms are when incident or or when when having the same frequency or same energy coming out of photon which is fresh, and strikes the inverted population system leads to essentially a higher energy, but more stimulated energy which is independent of the intensity, and this is called stimulated emission. So, there is huge coherent amount of energy which is lost suddenly, and this energy is tried to retain until there is a threshold which happens after which the energy starts escaping which is the laser beam. So, we also talked about various lasing media like solid state, and gas phase lasers where the medium changes from solid to gaseous in nature solid phase includes of course, ruby crystals other different kind of such media where the population inversion is possible with the certain frequency, and gas phase of course, can vary from helium to argon neon all these different kind of media for lasing action take place.

So, some facts of laser mediated machining was discussed the amount of uh the magnitude of intensity that is amount of power per unit area clubbed into a surface machining surface is very, very high in laser machining also the time that that makes the time of the machining extremely small duration of machining is very small in laser, and then of course,, because you can super focus thorough expensive optics the beam to a small spots there is tremendous increase in the resolution of the system these days lasers are used for laser beam based lithography where up to size of about two to one micron range is possible using laser beams.

Also discussed about the mechanics of the material removal where there is a photon to photon conversion, and starred working on the thermal model where we would assume a constant heat flux on a small area or a region on the surface where the beam on the beam incident on the sight of the surface, and with that we would try to make a model of the heat transfer based material properties of the wok material like thermal diffusivity density specific volume specific heat conductivity so on so forth

So, let us go back on that model, and try to extend this to forward a little further. So, we already have seen how the beam a can be modeled in a one dimensional heat using the principals of one dimensional heat conduction where the depth from the surface is considered to be z, and t being the time, and temperature with respect to different depths from the surfaces z as the function of time was given by equation.

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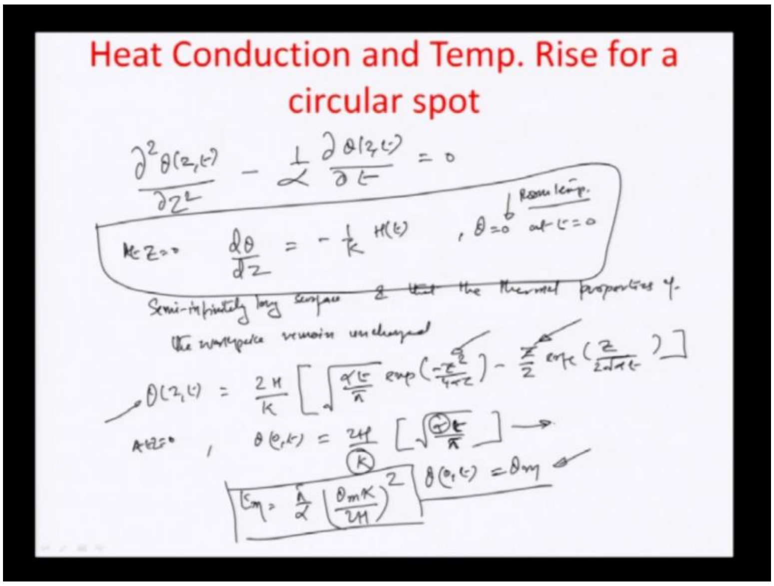

Represent here as dell square theta z t by dell z square minus one by alpha dell theta z t by dell t equal zero we assumed boundary conditions there at the surface of the work piece corresponding to z equal to zero there would be a constant temperature gradient with respect to the z direction the depth, and it was given by minus one by k h t, and we further assumed by that it was a semi infinitely long surface, and that the thermal properties of the work piece remain unchanged, and assume theta equal to zero at time t equal to zero now this is the actually the base line temperature which is actually the room temperature about twenty four degrees or so, but, then we assume that to the base line and. So, theta can be equated to zero at time t equal to zero. So, assuming that we have arrived at the solution using conventional p d methods where theta z t was represented by twice h by k root over alpha t by pi exponential minus z square by four alpha tau minus tau by two error function z by twice root alpha t.

If we tweak this solution to the boundary conditions which have been represented here we have at z equal to zero theta zero t simply represented as twice h by k root over alpha t by pi this goes off and. So, does this and so therefore, we are actually left with sorry I just need to rewrite this, this was the z we are actually left with a very simplistic expression theta equal to two where h by k where k is a thermal conductivity to square root of alpha t by pi t being the time alpha is the thermal diffusivity it is a ratio of the conductivity in volume specific key material, and if you want really machining to happen you have to assume that this theta zero t of the surface it is the melting point of the surface almost immediately. So, if the melting point of the surface is known we can tend to estimate the time of machining t m to be pi by alpha theta m k divided by twice h square.

So, that is how the time of machining is we tried to calculate in some cases what typically this time would be, and we obtain for normal system where we would try to machine tungsten surface with only a ten percent coupled power of the beam we obtain the time of almost about close to fifty three micro seconds five point three micro seconds which is actually a very small number in terms of time, and machining. So, this process is really is very fast for in comparison to some of the other processes that have been illustrated before.

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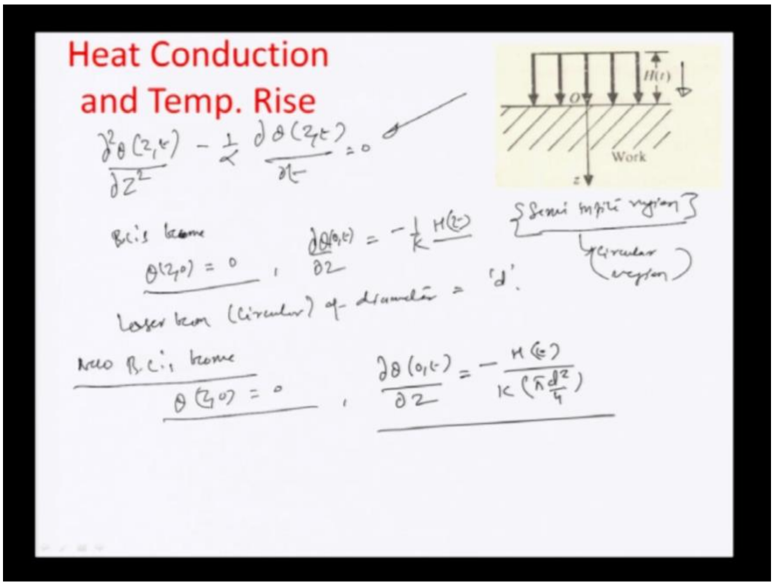

So, let us now work on slightly different problem, we already have from before the equation delta theta z t minus by d z two minus one by alpha delta theta z t by dell t equals to zero we know that the boundary conditions on the surface become theta z point of time zero zero, and dell theta by dell z that is the gradient of the temperature on the surface at the z direction on the surface corresponding to theta let us say zero at some point of time t is given by minus one by k h t h t being the heat flux, and which is actually constant, and continuous on the surface, and that is how a semi infinite region being exposed to a laser beam can be modeled we slightly change the connotation of the problem by converting this semi infinite region into a circular region which there mean by that the beam actually now has a diameter d.

So, you have a laser beam circular of diameter d, and you want to actually try, and see how you model this equation for a circular laser beam which is more realistic, and more closer to the real world situation. So, here the boundary conditions are to be slightly tweet, because of that, and the new boundary conditions become theta z equal to at at any point of time is at at a point of time t is equal to zero is zero, because laser beam is supposed to just get start radiating the surface at time t equal to zero therefore, the temperature is still the room temperature the base line temperature on the other difference which would have in this particular case is that the gradient of temperature at the surface for corresponding to all different points of time t equals to zero now really becomes the function of good diameter, and we can consider this to be minus h t by k pi d square by four.

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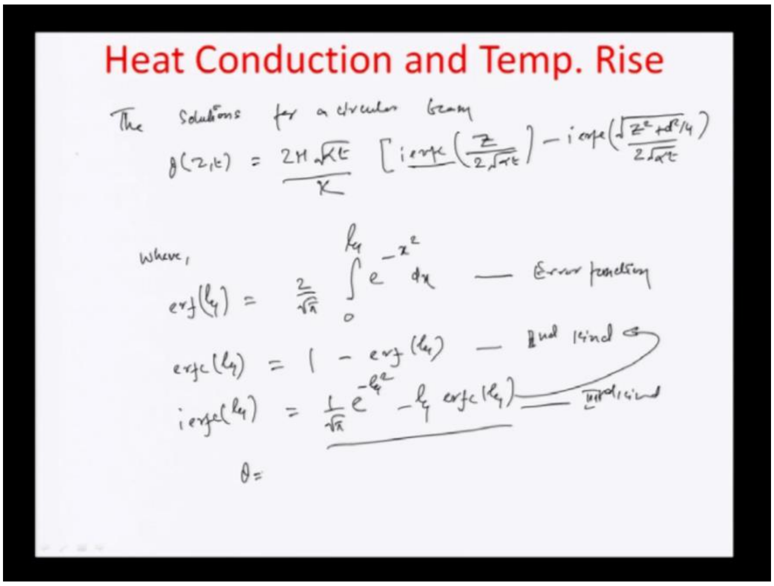

So, if we assume this to be the new boundary conditions the solution that would emerge to this equation for a circular beam they come equal to theta z t equals to twice h root of alpha t by k error function of the third kind z by twice root of alpha t minus same again times root of square of z plus d square by four divided by twice root of alpha t.

Just worth mentioning that this that this three different error functions of different kinds would be represented as the basic error function of variable zeta is the numerical integration two by root pi integral vary between zero one zeta e to the power of minus square of x d x the second kind is basically a variation of this error function, then just write it algebraically as one minus error function zeta this is only for simplicity sake that we are assuming this, and there is another representation of the same error function, and we call it error function third kind or third type it is one by root pi e three power of minus zeta square minus zeta times of error function of zeta in that of second kind.

So, we call this third kind. So, that is how we have defined these different values if you may recall from the earlier slide as well we had if the heat flux were a step function meaning there by that we assume that the heat starts at point of zero at point of time t zero equals to constant heat flux h, and continuous there in for all points of time. So, it is like a step function in that case the solution that came out involved this second kind, and it is basically nothing, but one minus error function, and just for simplicity sake for algebraic representation.

We are trying to represent error function in various ways. So, that you can short in the notational represent of the whole formulation that has been widened. So, here the same thing is done with third kind which is again is slightly complex form of what we have in the error function to of second type. So, in a nut shell if you were to really find out the value of theta is zero t corresponding z equals to zero, and for all points of time t; that means, the temperature variation on the surface with the exposure to beam starting from point of time t equals to zero onwards. So, you just amount to putting a value of z to be zero in this particular expression.

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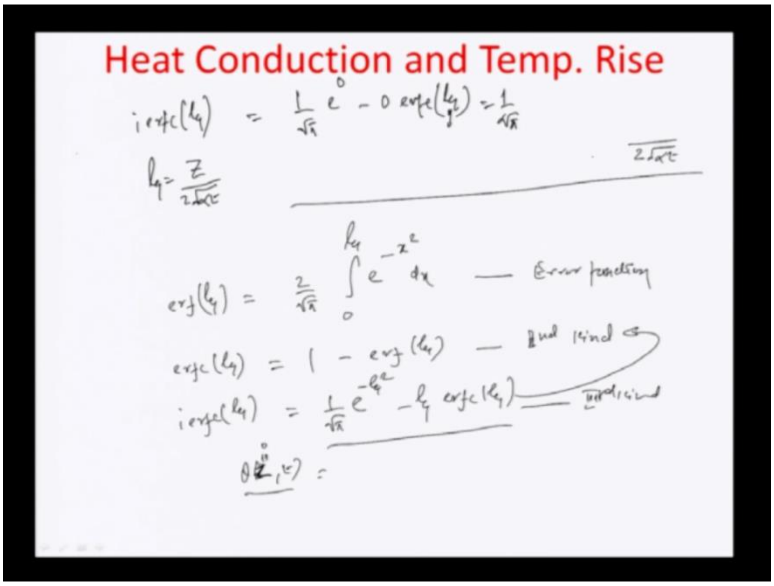

Here I am trying to find out how it would behave with respect to the given value. So, the value of the error function of the third kind ierfc zeta for zeta which is equal to again z by twice root of alpha t as we have assumed in earlier in this question here right here that is what the zeta value is. So, this becomes equal to very simply just a one by root of pi into the e power of zero minus zero error function of zeta equal to zero. So, it is simply one by root pi.

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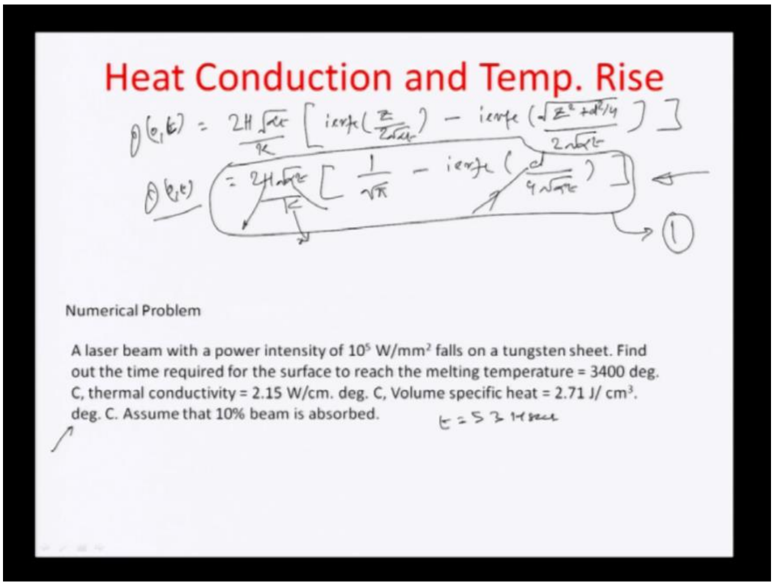

So, if we represent this value or we substitute this value in the equation for the temperature on the surface theta zero for all point of time theta zero t ewe get this essentially lets just fresh side the whole expression twice h alpha t by k, and the error function of third kind z by twice root of alpha t minus the error function of the third kind again root of z square plus d square by four by twice root of alpha t corresponding to z equal to zero. So, this further becomes equal to one by root pi we just derived it the last step minus ierfc the error function of the third kind ierfc, and this goes a way here and. So, it is basically d by four root of alpha t. So, that is essentially have the temperature variation on the surface of the machine piece. So, the function of the time can be represented as. So, if you know the beam diameter in this particular case d is the beam diameter, and of the different material properties of the material like k alpha. So, on. So, forth, and also are aware of the coupled heat flux which in this case also is assumed to be like a step function.

So, starting at time t equal to zero you have a finite heat flux h which translates overall point of time in space. So, all point of time and. So, therefore, the theta the surface temperature as a function of time can really be equated to the melting point a melting temperature of the work piece material, and you can have the good estimate of the time of machining based on looking at the various values obtained in this various formulation here let us call it equation one. So, let us just now slightly change this problem we have an earlier problem here where we find out assuming surface or the work piece to be a semi finite region we found out that the time of machining in this case was a very small value about a fifty three micro seconds.

Now, the same problem, if we just change the beam from interacting with the semi finite region of the work piece to circular beam a diameter d how the whole time would get modified.

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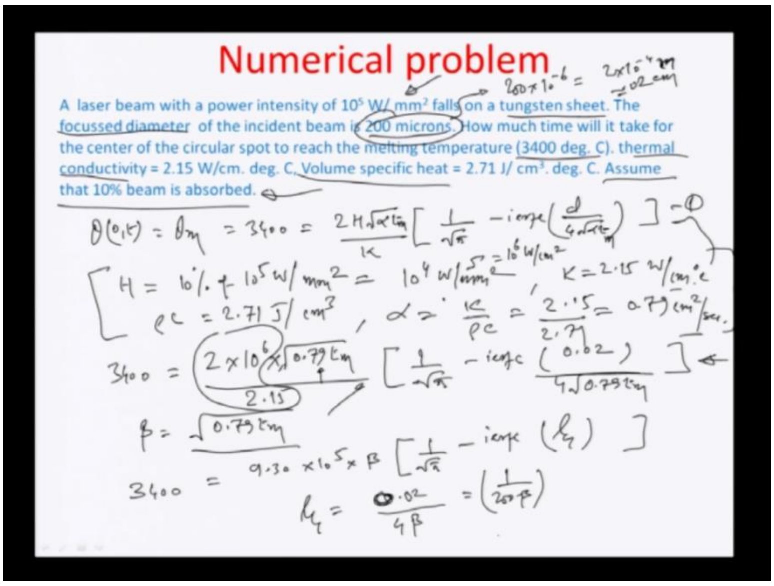

Let us have a clear look at it. So, now, we have tricked the problem slightly. So, you have a focus the beam of diameter up to two hundred microns, and the remaining conditions are being same work piece tungsten sheet power intensity is about the same ten to the power of five watt per milli meter square assume about ten percent of absorption reaming ninety percent is reflected on the surface, and then all other properties like thermal conductivity volume specific heat are given for tungsten sheet. So, it is merely the same problem with assuming this particular case the beam as mode of semi finite beam, but it is actually a circular beam of diameter two hundred microns let us see how the difference would come in terms of machining time for both the cases.

So, let us write down the complete equation for the temperature theta zero t equals it m in this particular case as we know it is thirty four hundred degree celsius, and this kind is equated twice h root of alpha t by k times one by root of pi minus ierfc all functional third kind d by four root of alpha t. If we plug in the various values here for example, h, and in this particular case is ten percent of ten to the power of five watts per mili meter square amount of power which is coupled comes out to the ten to the power of four watts per milli meter square, and we have alpha in this particular case as two point one five watt per centimeter degrees celsius volume specific heat row c two point seven one joule per centimeter cube making, I am sorry this is k thermal conductivity making the volume making the thermal diffusivity alpha the ratio of k by row c two point one five two point seven one this comes out to be by a zero point seven nine centimeter square per seconds.

So, with all these parameters from these questions we kind of plug in these in these to the equation one here, and try to obtain the calculate the value of the time of machine t which is involved at several places here in this equation as you can see. So, with that thirty four hundred equals twice times of…

If you just prefer converting this into watts per centimeter square this comes out to be ten to the power of six watts per centimeter square, because everything else has to be consistent the units have to be all in and. So, to a reasonable extent, and we are left with the two times of ten to the power of six times of root of zero point seven nine centimeter square per second times t m the value of the machining time divide by k with just two point one five degree celsius times of one by root of pi minus the error function of the third kind of the whole term. So, let us convert this two hundred microns into centimeters. So, this comes out to be two hundred ten to the power of minus six that is about two ten to the power of minus four unit or about point zero two centimeters. So, this can divided by the four root of again zero point seven nine t m. So, let us call this value beta root of zero point seven nine t m, and we tried to calculate the value for beta from this particular equation here right here.

So, the first thing we need to do is to put the beta value in right at this equation here, and we can re rewrite this equation has thirty four hundred equals this whole thing can be calculated, and written down as nine point three zero times ten to the power of five times of beta, and this is the beta value, and times one by root of pi minus the error functional third kind of this whole term here which we call zeta here. So, there forces you can rightly see here that zeta is basically nothing, but zero point zero two I am sorry let us just right this thing here. So, zero point zero two by four times of beta which makes it equal to one by two hundred beta. So, that is how zeta can be classified.

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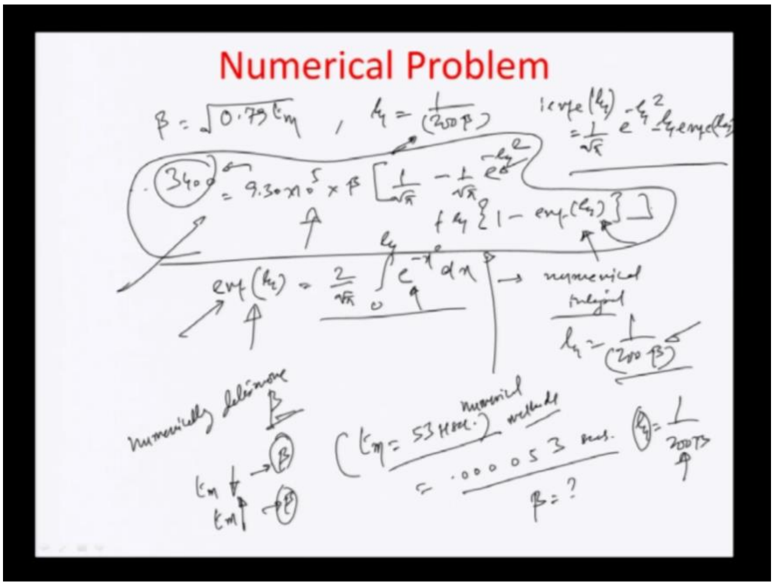

So, in summary beta has been estimated as zero point seven nine time of machine t m in zeta the parameter here one by two hundred beta, and we can write down this equation if as we all know that error function of the third kind of the parameter zeta can be represented as one by root of pi into the power of minus zeta square minus zeta error function of zeta k.

So, here the zeta value is of course, is one by two hundred zeta. So, we can have this represented as three thirty four hundred equals nine point three ten to the power of five times of beta times of one by root of pi minus one by root of pi exponential to power of the minus zeta square plus zeta times of one minus error function of zeta that is how we can represent thirty four hundred, and that we already know that the error function of zeta is actually twice by root pi integral zero to zeta into the power of minus x square d x this is a numerical integral. So, you have standard tables calculating the error function as a area under the curve of e to the power of x square with respect to the x, and for different zeta values zeta ranges were in between zero, and different zeta values real values of zeta you have different error function values.

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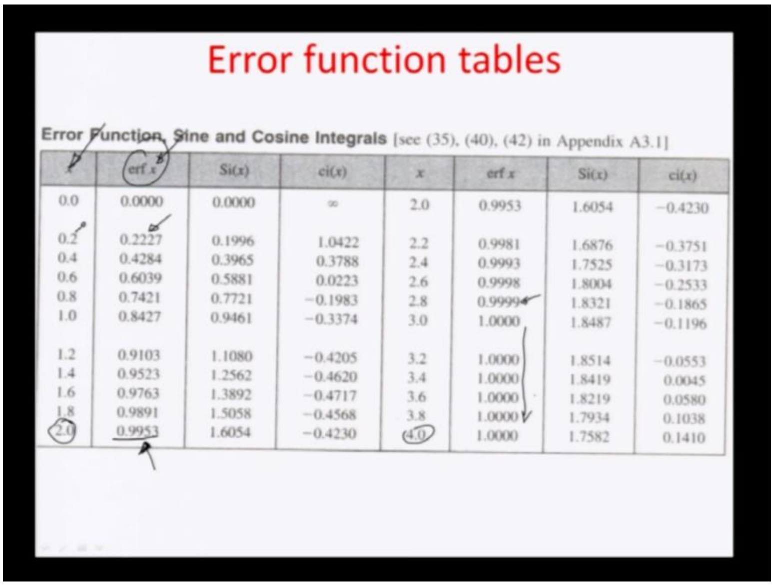

Error function tables

Error Function, Sine and Cosine Integrals [see (35), (40), (42) in Appendix A3.1]

x	erf x	Si(x)	ci(x)	x	erf x	Si(x)	ci(x)
0.0	0.0000	0.0000	∞	2.0	0.9953	1.6054	−0.4230
0.2	0.2227	0.1996	1.0422	2.2	0.9981	1.6876	−0.3751
0.4	0.4284	0.3965	0.3788	2.4	0.9993	1.7525	−0.3173
0.6	0.6039	0.5881	0.0223	2.6	0.9998	1.8004	−0.2533
0.8	0.7421	0.7721	−0.1983	2.8	0.9999	1.8321	−0.1865
1.0	0.8427	0.9461	−0.3374	3.0	1.0000	1.8487	−0.1196
1.2	0.9103	1.1080	−0.4205	3.2	1.0000	1.8514	−0.0553
1.4	0.9523	1.2562	−0.4620	3.4	1.0000	1.8419	0.0045
1.6	0.9763	1.3892	−0.4717	3.6	1.0000	1.8219	0.0580
1.8	0.9891	1.5058	−0.4568	3.8	1.0000	1.7934	0.1038
2.0	0.9953	1.6054	−0.4230	4.0	1.0000	1.7582	0.1410

This is represented here in this particular table as you can see for a certain question x here this represents the error function of x and. So, this is essentially the zeta where equal to two zero point two the area equal to the curve twice root of pi of the area into the curve which is actually the the error function is point two two two seven and. So, you take those all reverted by two for the curve two have to complete area of by one. So, actually to four. So, the essentially the variation from two to four is very less as you can see its almost going to point nine nine five three.

Here, and then goes to about to point nine nine nine nine at two point eight x value, then after that following this whole region is about the unity. So, it converges the value converges about four the error function. So, in this way we can try to estimate by using a software the various values of zeta equals to one by two hundred beta by plugging the beta value I am trying to see what the error function comes out to be equal to plug this back here, and numerically try to determine what beta can be…

So, the equation can really be solved by numerical methods in a creative manner of course, you start with the value of beta corresponding to the semi infinite region time which was obtained fifty three micro seconds is point zero zero zero zero five three seconds I am putting this t m value that the corresponding beta value plug this beta to find out the zeta value, and the first or first equation that can come out from the zeta is by plugging, and playing with the zeta here this is the error function you can calculate what e to the power of minus zeta square of unit of piput it back here, and try to find out how close this equation, and comes to thirty four hundred, then you can actually vary the t m to a slightly lower value, and try to again estimate beta vary the t m to to a higher value, and try to estimate the beta, and see what is the trend here is it going closer to thirty four hundred or far away from thirty four hundred this equation.

So, based on that you actually figure out a good beta value for which this whole expression right hand side would be equal to the left hand side in this particular case using the table, and the numerical integration value.

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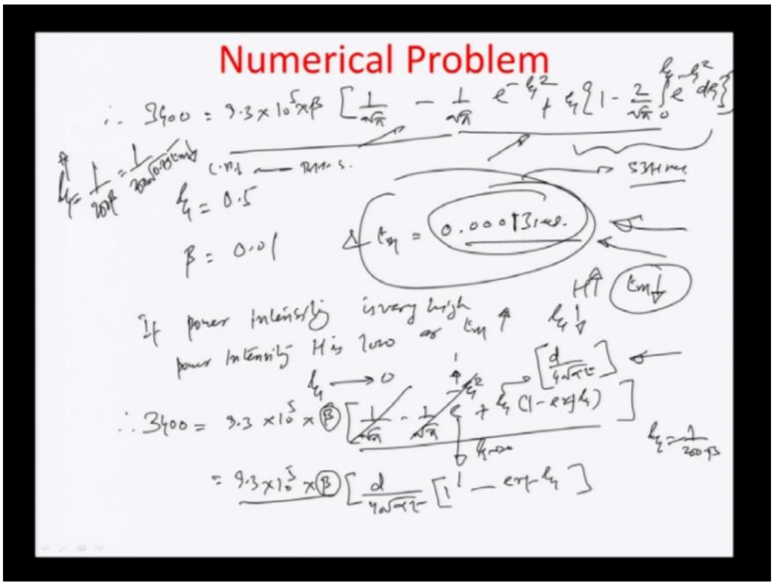

The solution of this equation thirty four hundred equals nine point three ten to the power of five times beta by one by root of pi minus one by root of pi e to the power of minus zeta square plus zeta times of one minus twice by root of pi integrate zero to zeta e to the power of minus zeta square d zeta this comes out to be corresponding to a zeta value of zero point five a beta of zero zero on, and time of machining zero point zero zero zero seven three seconds. So, it is a one three seconds.

So, the time of machining in this particular case as you can see point zero zero zero one three very small when comparison to the very very large when comparison to time of machining which was earlier for a semi infinite region for all these regions you are trying too reduce the beam area from a semi infinite attraction with the work piece to almost small value of the diameter d equal to two hundred microns. So, therefore, more time would be needed for this machining to happen, because of heat losses across the beam boundary to the remaining part of the solid. So, we will just see the effect of power intensity.

So, if like suppose the power intensity in this particular is h is very high of course,, because of a higher h the t m. So, that is, and if the t m reduces, then you have this value of zeta here which has been estimated to be one hundred one by two hundred beta root of zero point seven nine t m. So, as the t m reduces zeta value goes up. So, some changes should happen in this particular equation based on that the zeta is either reducing or increasing. So, a suppose in there is a case when the power intensity of the beam is low or time of machining is high, and subsequently zeta is falling down here zeta goes down.

So; obviously, if we look at this part of the equation here this equation or this part of the equation with a smaller value of zeta should typically go down for example, if zeta where approaching zero, then the lets just write down the equation once more thirty four hundred equals nine point three ten to the power of five times of beta times of one by root of pi minus one by root of pi minus e to the power of minus zeta square, and zeta plus zeta times of one minus error function of zeta that a what this term is corresponding to in this particular case. So, if zeta is going to zero, then this term goes to one, and effectively we are having a zeta value which is d by four root of alpha t as we all have already decided before, and off course its small, but, then we just want to find out, because there is a smaller term which is here which is also beta let us find out the overall effect on this equation, because of that.

So, one thing is goes to one corresponding to zeta ten into zero this to terms one by pie cancel with each other, and we are left with nine point three into ten to the power of five time of beta time of zeta, and lets us write the value of zeta here which is d by four route of alpha t on the surface corresponding to z is equal to zero times of one minus the error function of the value zeta. zeta of course is one by two hundred beta as you have seen before. So, therefore, if you just put this whole expression back in place, and try to see how this equation change.

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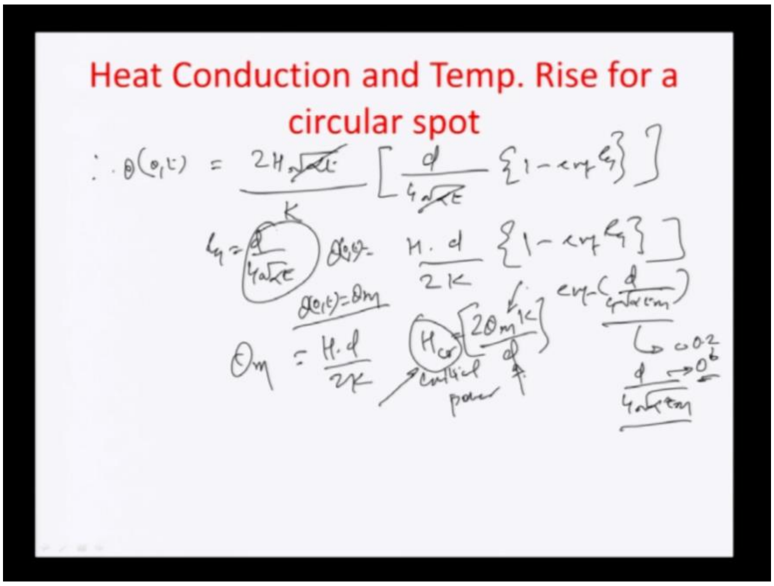

How we have zeta on the surface corresponding to z equal to zero at any function as a function of time t twice a h route of alpha t this is the beta value mind you divided by k times of now we have b by four root alpha t times of again one minus error function of zeta, and zeta is d by four root alpha t.

As well we know this root alpha t goes way, and we left with h d by twice k one minus error function of zeta that is equal to the surface temperature as the function of time, if supposing the again value of this error function d by root of four alpha t m this tends to zero, and if you may look into the table, you will find reason for that that is as extents to zero the error function ten to zero.

So, we are talking about typically zeta value between point two, and point I mean zero point zero. So, in that event the expression here would change to the simple formulation that total amount of power which is needed. So, that on the minimum possible melting temperature is fit upon let say for an example this is equal to theta m. So, h d by two k is typically theta m, and in other words the power which is needed which is called the critical power; that means, power enough for the melting temperature to hit upon is represented simply by twice k theta m by d d is the beam diameter, and these are the simplistic assumption for a machinist where you can assume that you know the d value is in microns, and it it goes to an extent, and this whole value by four is an argument here d by four root of t m kind of tends to d between zero point two, and zero something like that. So, in that even the critical power which is needed for a temperature to go a melting point of the work piece is represented by this h c r equal to twice theta m k by d. So, having said that we can actually solve a small numerical problem where we define or we we try to find out this value of the critical power for a laser machining system.

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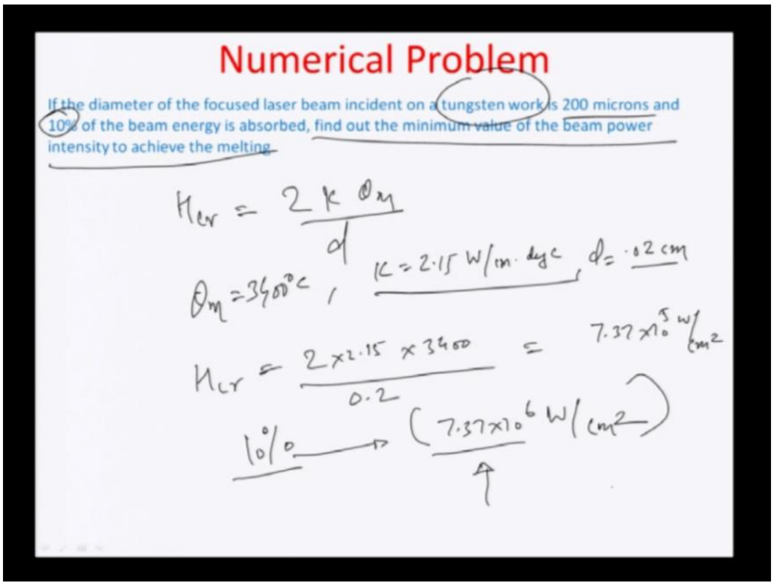

Here let us say for example, if we already know that the diameter to be two hundred microns of the beam, and we assume to same tungsten work piece meaning there by that the other properties of thermal conductivity, and the thermal diffusivity remain similar to what we have taken earlier, and we assume that ten percent of the beam is absorbed ninety percent is reflected. So, we need to find out that critical value of the beam power, and let us look at how what kind of power values would be hitting upon in this particular case. So, we already know that h c r here critical represented.

Here as twice k theta m by d theta m as we know is the melting temperature of tungsten about thirty four hundred celsius we already know the k value as defined earlier to be two point one five watts per centimeter degree celsius thermal connectivity of tungsten work piece. So, in this particular case the beam diameter of course, is two hundred microns, let us put it in centimeters as point zero two centimeters. So, we get the critical power h c r to be equal to twice two point one five times of thirty four hundred by zero point two that comes out to be seven point three seven ten to the power of five watt per centimeter square we already know that ten percent of the beam power is the only power which is coupled to the system. So, we are left with seven point three seven ten to the power of six watt per centimeter square as the incident power for just about reaching the melting point of the tungsten on the surface on by, because of the beam matter interaction.

So, I think we have come to the end of today's lecture, and in the next session I would like to actually do this problem for steady state drilling operation where a through hole is generated within a work piece, and how the cylindrical periphery of the drilled hole results in the heat transfer from the center which is at the highest temperature to outwards and. So, therefore, the time of machining, and some criticalities would change in that model. So, following that we would look into some of the application areas for all these different machining processes for micro system designs, and fabrication.

Thank you.