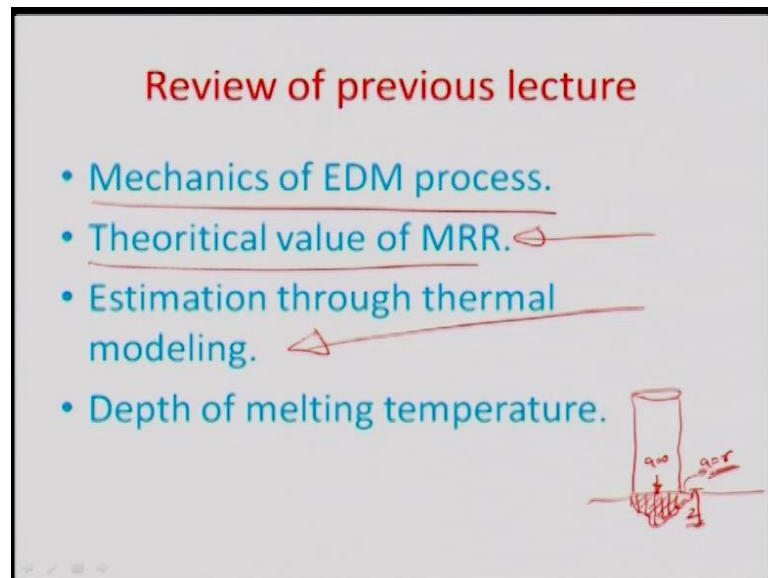


**Microsystem Fabrication with Advanced Manufacturing Techniques**  
**Prof. Shantanu Bhattacharya**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 23**

(Refer Slide Time: 00:24)



Hello welcome back to this lectured twenty three on microsystem fabrication by advanced manufacturing processes lets quickly recap what we did in the lat class. So, we studied about the mechanics of e d m process as just to for the sake of repetition like to just say that is all about the formulation of spark between the tool the electrode, and the cathode the two electrode, and the anode the anode being the work material to electrode being the cathode material, and there as, and there as always field in between which is created in a, because of a voltage gradient, and there is a dielectric medium in between, and it is all about breaking down the or reaching the breakdown potential of that medium.

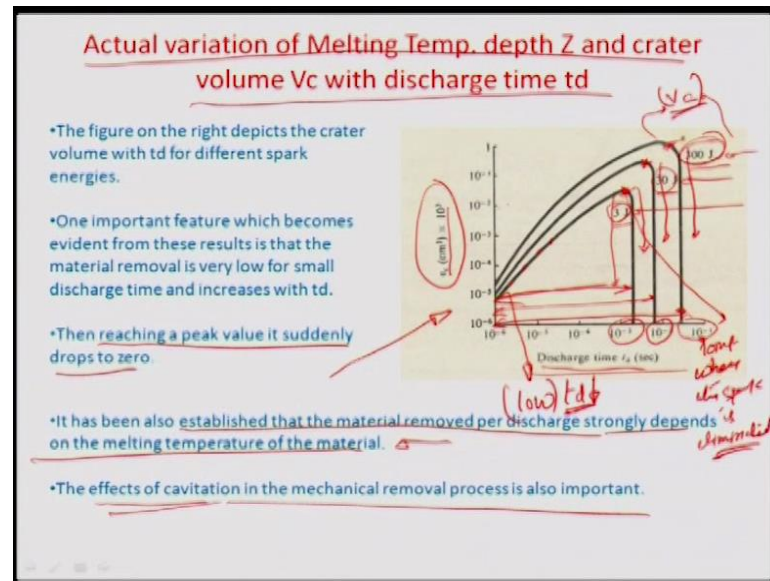
So, that there is a electron flux which comes from the tool side, and ands in to the anode side the work side, and in a way it basically leads to a is a strong electron pressure, and thus a thermal wave, and basically it creates a melt pool which also a increases the distance between the two electrodes at that point, and the feel goes down below the break down potential and. So, the spark gets activated some other place of closes the distance and. So, this way the spark goes around, and that creates a material to be removed.

So, we actually theoretically also estimated the material removal rate by zooming a cylindrical access a system were the spark a was like a cylinder which was a generating or which was the heat source a constant flux heat source on to a material of a infinite of infinite slab like nature, and there we assumed how a the heat transfer would takes place from  $r$  equal to zero to  $r$  equal to  $a$  which is a the the  $a$  the radius to the cylinder across which the spark would take place.

We made some presumptions including a zero hot a heat flow, and perfectly insulated a substrate a beyond the spark, and a that way we were able to estimate the depth of melting temperature which was the  $z$  value, and this depth of melting temperature would typically be a plot or an estimate of the creator which would eventually get formulated us where ever the temperature would reach the melting point there would be a melt pool which is generated, and this was helpful in estimation of a the  $m r r$  of the material may a rate through thermal modeling.

So, for example, if supposing there is a a cylinder cylindrical nature spark like this, and a there is a area formulated on the work piece where there is the zone of melt. So, this really is the  $z$  value the depth of melting temperature value, and there are various such depth across the a different point a of this spark assuming the gray dust temperature to have reached at the center of the spark corresponding to equal to zero, and a the radius having the minimum  $z$  value at equal to  $r$  the radius of the the spark, and that is how the depth of melting temperature was predict.

(Refer Slide Time: 03:34)



So, today we will look a little further into the various modalities associated with a how the actual variation of the melting temperature depth happens with respect to the creator volume, and also the discharge time.

So, a this graph right here illustrates how the volume of the greater a varies with respect to discharge time it is very, very hobbies that as the total amount of pulse power a the total amount of spark power is increased a to about hundred folds there is a gradual increase in the way that the crater  $V_c$  would be formulated. So, crater volume is more if energy pumped is more. So, that is. In fact, a understandable, and if you look at some other peculiarities in this graph one important aspect is that the volume removal a a  $V_c$  a volume of the crater is really very low at low discharge time.

So, if this value of  $t_d$  is low on the low discharge times the amount of volume removed as your seeing is very low, which is for hobbies reasons that if the a spark is not of enough duration the amount of a transaction time that it would take place for the heat energy to flow through the material will also will us, and, because of that the amount of depth that the melting temperature can cover which is also a function of the heat energy which it is able to release a spark is able to release on the surface that will also does, because of that you would not giving at enough amount of a a time at a certain hold point, and even before the melting temperature depth could reach its highest percolation threshold you have close to the spark energy. So, therefore, a the  $V_c$  value is low for

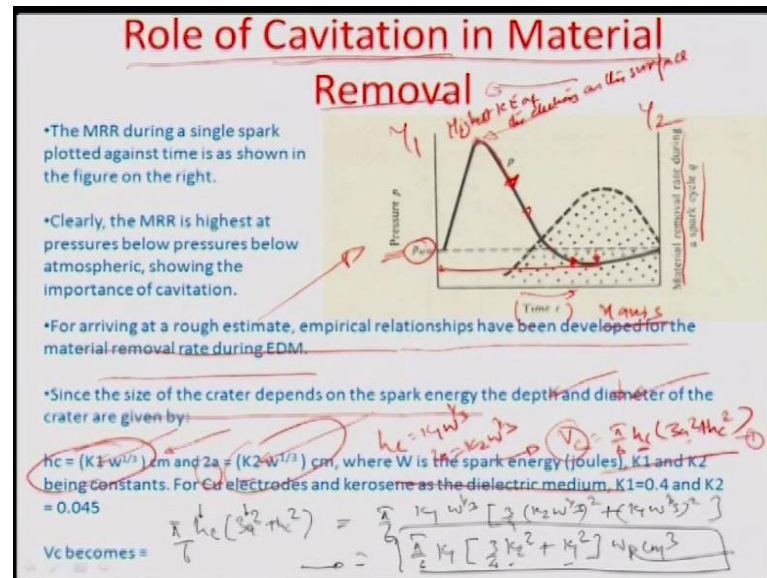
lower discharge time.

Then it suddenly peaks up. So, it reaches a peak value, and finally, drops down to zero as you can see here from these three places indicated by the spark. So, this area is really a sudden drop in the  $v_c$  which is of again obvious reasons that once the material is formulated as a liquid pool there is a tendency of washing away of the flux, and there is a sudden deformation of the spark, because now as the melt is reached, and the material has melted the amount of a potential gradient which is available  $dV$  by  $dx$  that it feels this is over than the breakdown field. So, the spark disappears, and whatever volume was initially formulated is typically the volume which is remaining as this thing goes away. So, therefore this actually indicates the point where the spark is eliminated at a certain energy value ok.

What is very important to be seen is that this value  $a$  is almost varying with the amount of pulse energy. So, the pulse energy was three joules the criticality threshold here where the spark would be eliminated would reach faster. So, the discharge time would be a let say in this case about close to ten to the power of minus three or. So, in this case it is about of ten to the power of minus two which is a thirty joules, and if the spark energy is even higher the discharge time is again further higher ten to the power minus one and. So, that is how  $a$  this threshold is reached.

It has also been established that the material removal per discharge strongly depends on the melting temperature of the material. So, if there were a high melting point material this  $v_c$  value would automatically be lower, and vice versa, and also a one important factor is cavitation which I will just talk on the next line, but there are effects of cavitation in mechanical removal processes, and this extremely important, because cavitation is a principle a mechanism of removal of material in EDM has will be seen in the next line.

(Refer Slide Time: 07:48)



So, let us look at the role of cavitation in material removal in the EDM process. As we refer to this particular figure, here it is between pressure of the zone of the spark which is plotted on the y-axis, and the material removal rate during a spark cycle which is plotted on the y-axis, with time scale on the x-axis.

And a very interesting thing comes into picture. So, if we look at the way that the pressure is varying and on the top of the material and the way that MRR is varying, you can really see that the MRR is highest when the pressure that the medium would generate on the surface in question is the lowest. So, if the MRR is high, which is indicated by the peak here, high MRR value, and this range the pressure at this point is the lowest as you can see here.

So, it is even below than that atmosphere. So, what is really going on if you really, and this is a plot, this is a real plot of the pressure versus the MRR happening on the tool surface. So, as you know that there is always a tendency of the electrons emanated from one of the electrodes that is cathodes in the scale to give a sort of pressure wave or a view of compression on to the anode surface which is the work piece in this particular case. So, there is always an electron pressure which remains from coming from the medium emanating from the medium, and coming on to the anode surface of the work piece surface.

Now, supposing this pressure is there, then there is of course you know a compression

in a rarefactions kind of longitudinal wave a that we are talking about and. So, therefore, a there may be one instance of time when the when the pressure is high, and the next instance of time when there is a lower pressure or rarefaction is reaching a a the medium. So, it is like bust of electrons that is coming in that is how was spark is really composed of. So, if you assume that, then there is tendency of the pressure to go up which is being shown here with respect to time. So, a the pressure is highest here meaning there by that the electron pressure is highest here, and this pressure is really in terms of kinetic energy that the electrons are providing on to the materials surface. So, there is highest kinetic energy as you see the graph here in this particular case the highest kinetic energy of the electrons on the surface.

So, a off course this a pressure wave, then changes as a the electron pressure has gone or it has eliminated, and a the pressure slowly falls down. So, it comes to some atmospheric pressure value, and then even goes below, because there is a tendency of the flu it to move pas creating a velocity had and. So, typically the pressure head that would be available is lower in this particular case.

So, here you can see the pressure going down, then the atmospheric pressure. So, this is the case where there is a tendency of material which has been formulated earlier in the volt pool to be attracted towards that low pressure zone which is created. So, the pressure is falling below atmospheric pressure, and there is a tendency of the material the melt pool to go into that low pressure zone. So, therefore, the you can say that the is swept away a more efficiently at that point a were the pressure falls down the atmospheric pressure, and that is the reason why m r r is the highest at low pressure. So, this effect itself is known as cavitation, because of difference in pressure a there is a transport of the liquid pool of material which is there on work piece surface more towards the low pressure zone which has been suddenly created, because of that rarefaction in the at the medium which has come.

So, if you consider a series of this you know rarefaction, and compressions coming there may be really points whether is oscillating pressure, and then the pressures whenever they hit below the atmospheric pressure is really a the the most efficient debri removal point for a the the system. So, therefore, m r r a is just out of face. So, if the pressure is lowest the m r r is the highest there.

So, let us now try to arrive at a rough estimate of an empirical nature for material removal rates during EDM operation, and initially once some experiments have been performed as various people do, there are some things which we can find out intuitively one is that the size of the crater would depend on the spark energy as you can see here in this graph below where as a spark energy is increased the size of crater increases which is demonstrative demonstrated by the fact that more energy delivered means more melting.

And so it depends on the depth, and the diameter the spark energy, and the size of the crater really depends on the depth as well as the diameter you remember how we calculated the volume value to be equal to  $\pi r^2 h$  at a time of three square plus  $\pi r^2 h$  square this was what we derived last time as the crater volume the volume of that small crater of height  $h$ , and radius  $r$ . So, the volume depends on the depth as well as the diameter  $r$  being the radius in the diameter. So, empirically it has been found out that the way that the  $h$  the crater height as well as the radius of the crater  $r$  were is really dependent on the spark pulse power or the spark power or the spark energy.

So, if I say  $w$  is the spark energy in joules empirically it has been found out that this  $h$  the depth of the crater the depth melting temperature that varies as the  $w$  to the power of one by three. So, it is a cube root of  $w$  with which the  $h$  the depth the crater is proportional, and similarly. So, is the case with diameter the only difference in this case is that the constant of proportion are different in one case it is  $k_1$  other  $k_2$ . So, this constant proportionalities are two different values.

So,  $h$  is equal to  $k_1 w^{1/3}$ , and  $r$  is equal to  $k_2 w^{1/3}$ , and these cases  $k_1$ , and  $k_2$  they are typically a constant for a fixed of electrodes on a fixed solution, and typically for copper electrodes, and kerosene as the dielectric medium the  $k_1$  value is treated as point four  $k_2$  its point zero four five it changes to some other like iron, and the medium may be something like let's say some other silicon oil the  $k_1$   $k_2$  as well. So, it is really a property which depends on what materials are being machined what material being used for the electrode, and what material dielectric through it.

And therefore, if you just put these two formulations, and empirically obtained  $h$ , and  $r$  into the spark volume estimate which is represented here by equation one we

see, then becomes equal to  $\pi$  by six times of  $k_1$  time of three by four  $k_2$  square plus  $k_1$  square  $w$  to the power of or  $w$  a centimeter cube ok.

So, that is how you would characterize  $v_c$  a just substituting these two values. So, a as you know this is equal to  $\pi$  by six times of  $h_c$  time of three  $s$  square plus  $h_c$  square, and we just substitute the values obtained in both the cases. So, you have  $\pi$  by six times of  $k_1 w$  to the power of one by three which is the  $h_c$  value time of three by four a  $s$   $h_k$  two times of  $w$  to the power of one by three square, and plus  $a$  instead of  $h_c$  you can have  $k_1 w$  to the power of one by three square.

So, in other word this can be represented as  $\pi$  by six  $k_1$  time of three by four square of  $k_2$  plus  $k_1$  square, and we get  $w$  outside the bracket centimeter cube. So, that is how the volume can be estimated empirically out of a the various powers that the system is using  $a$  in joules  $w$ . So, this is the spark power or spark energy, and the various constant is  $k_1$ , and  $k_2$  depending on what is the medium what are the... So on and so for. it of the  $mrr$  can also be arrived that based on what is the melting temperature of the material in question. So, if  $\theta_m$  a let say some melting temperature empirically there may have to be a relationship between the material removal rate, and that particular temperature.

(Refer Slide Time: 17:54)

### Role of Melting Temperature of the work material

- A rough estimate of MRR can also be had from the melting point of the work-piece material.
- $Q = 4 \times 10^4 \theta_m^{-1.23} \text{ mm}^3 / \text{amp-min.}$
- Where,  $Q$  = material removal rate ( $\text{mm}^3 / \text{amp-min}$ )
- $\theta_m$  = melting temperature (deg. C)
- In this relation we have assumed the average sparking condition.
- The MRR also depends strongly on the circulation of the dielectric fluid.
- Without a forced circulation the wear particles continuously melt and reunite with the electrode.

So, that relationship has also been defined in literature as if  $\theta_m$  were the melting temperature  $a$  in degree celsius of the material there the amount of material removal rate



in terms of a mm cube per charge of a the material of a millimeter cube per per amount of charge of the material is given by this expression here four ten to the power of four theta m to the power of minus one point two three. If therefore, indicates are a theta m for material is higher a the material removal rate should be lower, and vice versa there is on minus sign on this a as superscript a minus one point two three. So, a we off course have a assume to ka come at this particular a you know formula we have assume that a on the an average sparking condition a meaning there by that a we see the spark to be dancing all round a, and a the total removal rate q is a an average effect, because of such spark going around, it is not really a questioning a single spark, and the way it behaves its multiple sparks over the whole surface, and how the average material removal rate queue may be by that a multiple sparks formulating a different points of time with in the electrode.

So, the m r r also typically depends on the circulation of the dielectric fluid. So, if the kerosene oil or silicon oil which is there in side is flowed at a higher rate there will off course be a more velocity had, and lesser pressure had task forcing more material to go into this fluid and. So, m r r would go up, and without a forced circulation the wear particles a you know they continuously kind of melt, and reunite with electrode which also results in a problems.

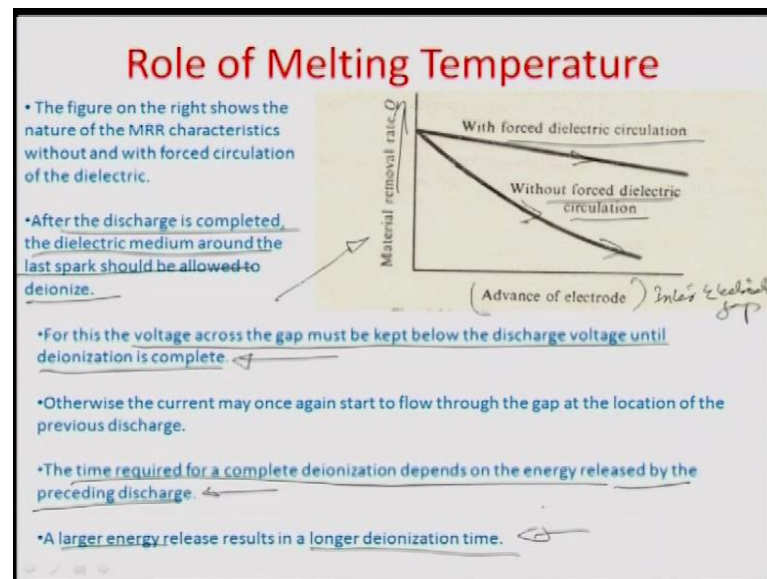
So, therefore, you are melting the same material again, and again, because there is no force circulation. So, circulation ensure that whatever debri is a formulated is being nicely packaged into the flowing stream, and goes out along with that stream a if if if the particles were there in that zone in that region they would really have a chance on the spark has the spark time had the discharge time as ended a to again a, because temperature the heat transfer at that scale is very high a the particle may as well unite back, and deposit as debri again. So, typically you would be again, and again remove in the same material layer a thus the m r r would come down in that particular manner.

So, two things we have understood today that the point where the m r r is the highest is really the point when the pressure is below that atmospheric pressure, and number two is that there is a huge utility for a the material a to be removed higher with higher flowing streams of the dielectric fluid.

So, velocity of circulation is a very critical par meter forget remaining the m r r a in this

particular a method or of of machine.

(Refer Slide Time: 20:50)



So. In fact, if you look at this trend here it actually mentions that with, and without forced dielectric circulation what are the . So, the material removal rate without forced as your seeing is you know it really goes down as the electrode advanced towards each other. So, this is the inter electrode gap.

And we are talking about i g. So, the material removal rate is going down fast you know a without the circulation, but it is kind of rested id does not go that fast if there is forced a electric circulation.

So, a one good point a that also can be is that a after discharge is complete let say the you spark settles down the dielectric medium around the the last spark should be allowed to be nice fully. So, the iron column which is formulated should be a able to get, and that would be faster if supposing there is a force dielectric circulation otherwise there is a tendency of the spark to get reformulated in every time at the same zone, then the overall a rate of removal would reduce, because of that, because already there is a melt pool which has been formulated in the zone were the spark their before.

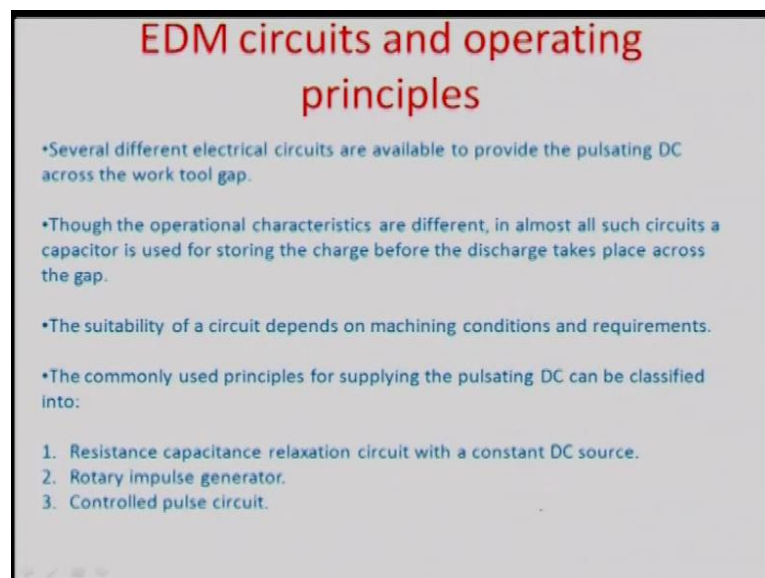
So, it also kind of m e c is the circuit between the tool, and the electrode the work piece to relax a little bit the it is the iron path which has been formulated how soon you can dissuade that how soon you can defuse that into the remaining medium is also of critical

importance you have to ensure that the second spark which formulate should be at a different place it should not be in the same if the conductivity of that particular region is higher, because of a stagnant electro light there is a tendency of spark to get reformulated there itself or sub at least the electron flow to be there itself ok.

And it may not be very efficient material removal process. So, therefore, circulation of dielectric will also that way critical a apart from the fact that the debri removal is also there a you you do not hit upon the same iron column twice, because of force circulation of the dielectric fuel.

So, a there is also something that you have to ensure the voltage across the gap must be kept below a the discharge voltage until deionization is complete, and a this would ensure that the current does not flow in the same iron path which is highly conducting now, and at least that diffusion time for the iron stool go out or emanate out is provided by the system s a therefore, a the time required for a complete deionization depends on the energy released by the by the presiding discharge, and if the discharge were larger in terms of energy a there would be a longer deionization time needed, and with force circulation this will go down further, and further.

(Refer Slide Time: 23:54)



### EDM circuits and operating principles

- Several different electrical circuits are available to provide the pulsating DC across the work tool gap.
- Though the operational characteristics are different, in almost all such circuits a capacitor is used for storing the charge before the discharge takes place across the gap.
- The suitability of a circuit depends on machining conditions and requirements.
- The commonly used principles for supplying the pulsating DC can be classified into:
  1. Resistance capacitance relaxation circuit with a constant DC source.
  2. Rotary impulse generator.
  3. Controlled pulse circuit.

So, that is what the role of a melting temperature would be now we will come to a very different aspect to the problem that we are talking about a case, where we have to keep on feeding voltage in sparks, and somehow frequency at which energy is being apply

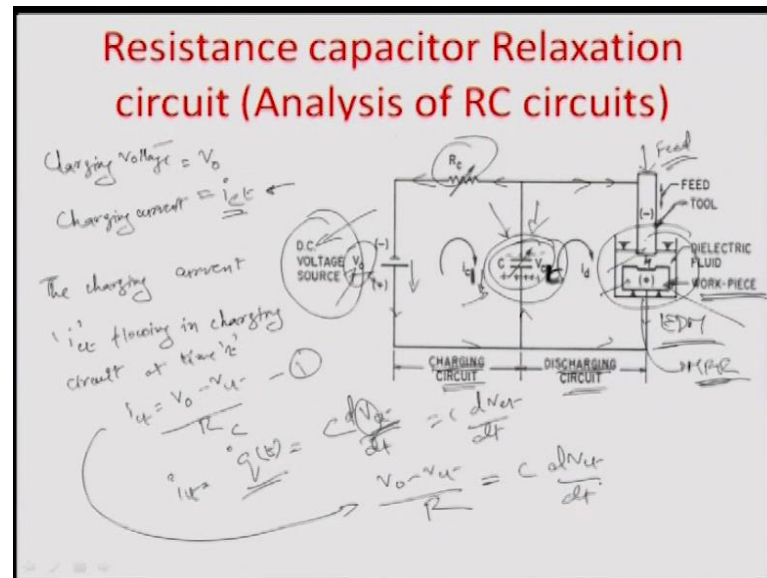
should also be optimized with the frequency at which the spark is formulated otherwise typically we are losing energy if we give a continuous supply of energy, and let the process follow itself. So, the point of time when this spark is formulated should really match with the frequency at which the energy is being supplied. So, the packet comes, and it gets delivered, and then there is another packet which gets delivered.

So, there are several different electrical circuit designs which are available to provide this pulsating dc across the work tool gap, and though the operational characteristics are different in almost all such circuits a capacitor is typically used for storing the charge a capacitor before that is charged expressed in the gaps. So, it is used as a charge storing device it is a device which would give a if we should charge to a certain point of time, and then try to outburst the charge at all at one go as a packet, and it inductively seems reasonable that we should include this in the part of the circuit for talking about equivalence of frequency in which supplying of the energy from a source to the electrode is happening with respect to the way that spark is getting transport between the electrodes

So, of course the suitability of a circuit highly depends on machining conditions on the requirements, and there are three different principles which are used for supplying pulsating dc one is the simple RC relaxation circuit a resistance capacitance relaxation circuit, and the new kind of course modify it, and make it rotary impulse generator or a controlled pulse circuit so on so forth.

So, these are three principles a mechanisms with which we can supply a pulsating dc signal in such cases. So, let us look at all these techniques one by one, and the idea is that from circuit design itself we should be able to predict somehow, what is the spark energy that we are sending, and because spark energy is highly correlated to the volume of material removed per unit time it should also be able to give an estimate of what is the material removed from the basic operating conditions of the circuit. So, it is in a way to a sort of a predictable manner a remove material by fixing the various voltage current relationships that you are providing on one side. So, power is  $P = VI$  as we know in order to remove some material per unit time on the other side.

(Refer Slide Time: 26:56)



So, let actually look at the first basic such circuit there is the resistance capacitance relaxation circuit. So, this estimate. In fact, the a such a circuit. So, there are two parts of the circuits one is off course were the d c voltage source is connected, and this hide of the circuit is called the charging circuits this is a a the common capacitor which is their between the charging, and discharging side meaning there by that in the first co the first instance there is a tendency of the current to charge a the charging part a charge the capacitor a through the charging part of the circuit. So,, because as you know that capacitor is an open circuit d c therefore, typically this side of the capacitor would get positively charge, and this would get negatively charge, and there is a huge charge containment inside this capacitor.

Now, if we assume the other part of the circuit that is that is charging circuit to have the tool electrode combination. So, this is the work piece just here write here this is the tool and. So, this the tool electrode combination, and there is a dielectric fluid in between, and typically this is what the e d m set up would look like this is the e d m part the electro discharge machining part of the, and supposing we have a feed in the down word direction provided this tool. So, that that condition break down happens in there is a spark which is created. So, as the the tool is moving slowly towards the work piece, and the d v by d x is increase in the electric field is increasing there would be a point of time when whatever charge has been stored in the capacitor here, and this particular circuits should typically get discharge, because there is a spark being formulated break down of

the medium which is between the tool, and the electrode between the tool, and the electrode. So, all this charge should be discharge there discharge here through the e d m, and that would. In fact, cause the m r r to happen the material removal to happen, and therefore, this part is called the discharging part of the circuit.

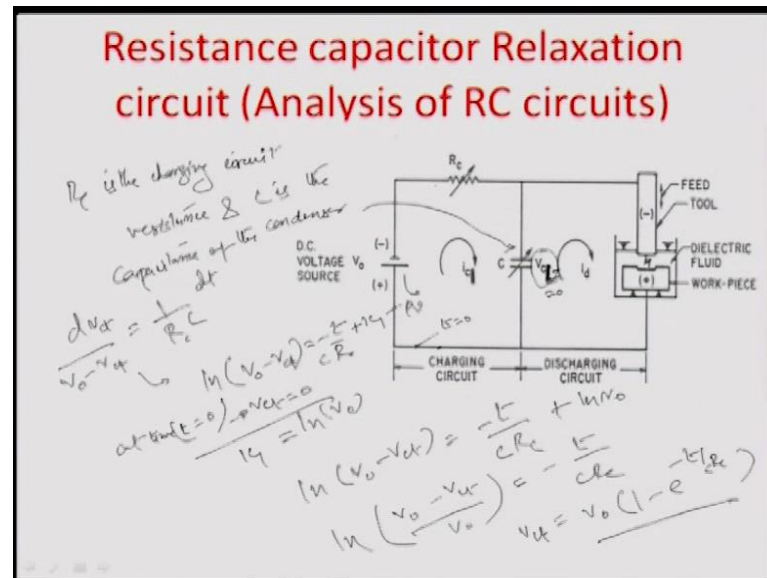
So, you really have a relationship between the way that the capacitor is charged a through this d c voltage, and then the way that the capacitor is discharge a which does not going to the voltage source, but actually goes into the discharging part of the circuit. So, that you have a spark.

So, let us look at some analysis here. So, charging voltage in this particular cases  $v_0$  given by the source voltage here, and let us assume that charge in current in this particular cases some value  $i_c$ . So, we call it  $i_c(t)$ , because there is off course a time component it is a transient process the way that the charge transfers from the voltage source to the capacitor here.

And. So, therefore, the charging current  $i_c(t)$  flowing in charging circuit lets a timing sense  $t$  is off course given by the expression  $v_0 - v_c(t)$  by  $r_c$ , that is assume there is a voltage  $v_c(t)$  being formulated across the capacitor now off course this voltage would been opposition to the voltage which is formulated here. So, a voltage formulated here is let change this direction a this other voltage would also been opposition to this direction.

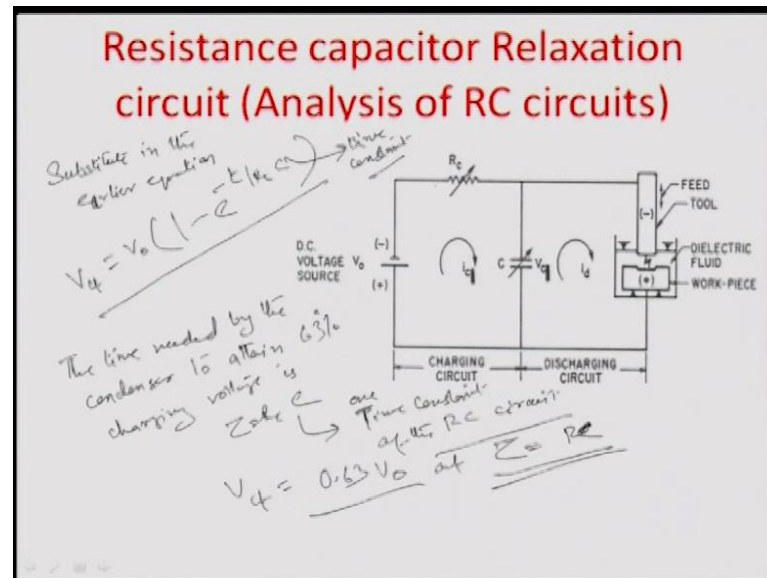
So, this is  $v_c(t)$  this is  $v_0$ . So,  $v_0 - v_c(t)$  is the drop, and the drop is across the a variable resistance a very arc  $r_c$ , and that is what  $i_c(t)$  e s. So, the way if we consider the the way that the capacitance charges that say if  $q$  a  $q(t)$  is the amount of charge that is coming in to the capacitor. So,  $q(t)$  can also be given by the fix capacitance value  $C$  times off  $v_a(t)$  there  $v_c$  is the a a voltage on the charge on the capacitors function of time. So,  $i_c(t)$  is really nothing, but  $q \cdot t$ . So, you can simply have it has a  $v_c(t)$  by  $d t$ , and therefore, a you can say that this  $v_0 - v_c(t)$  by  $r$  which has also been predictive from the a earlier equation one a is actually equal to  $C v_c(t)$  by  $d t$ . So, we can solve this equation.

(Refer Slide Time: 32:31)



And here we know that  $R_c$  is the charging circuit resistance, and  $C$  is the capacitance of the condenser in question this condenser. So,  $dV_c/dt$  by  $V_0 - V_c$  is  $1/R_c C$  for you know if you integrated in time you get  $\ln(V_0 - V_c)$  is actually equal to  $-\frac{t}{R_c C} + \ln V_0$  we know that at time  $t$  equal to zero the amount of charge which has been developed here the  $V_c$  is also equal to zero. So,  $V_c$  equal to zero at time  $t$  equal to zero, because hardly in it charge has flown from the voltage source we can assume a switch actually in the circuit, and we can say that the switch close at time  $t$  equal to zero, and a therefore, a we know that  $\ln V_0$  for  $t$  equal to zero  $V_c$  equal to zero becomes  $\ln V_0$ .

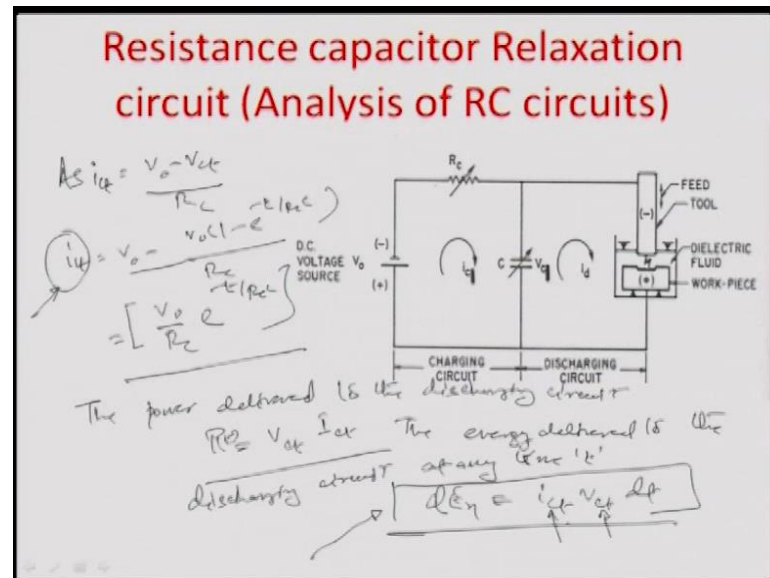
(Refer Slide Time: 34:13)



So, if we substitute in the earlier equation which was actually this right here equation a, and we would get  $\ln \frac{V_0 - V_t}{V_0 - V_c} = -\frac{t}{RC}$  or we can say  $V_0 - V_t = (V_0 - V_c)e^{-t/(RC)}$ . Another words the  $V_c$  value is actually equal to  $V_0(1 - e^{-t/(RC)})$ , that is how the RC circuit typically the derivation is a you have  $V_c$  is actually equal to  $V_0(1 - e^{-t/(RC)})$ . So, this is called the time constant of the RC circuit, and we also know that a the time really which is needed by the condenser to attain about six three percent a charging voltage is equal to  $RC$  right. So, this is one time constant from the definition of an RC circuit. So, in be that this  $V_c$  to is actually zero point six three  $V_0$  at tow equal to  $RC$  to one time constant.



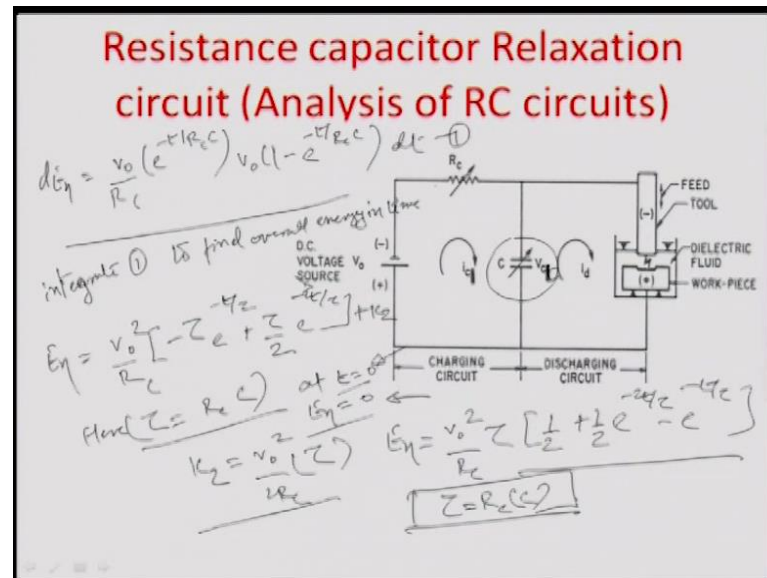
(Refer Slide Time: 36:39)



As  $i_{ct}$  is actually  $V_0 - V_{ct}$  by  $R_c$  how we can find this out. So, we already have the value of  $V_{ct}$  from the previous equation as  $V_0(1 - e^{-t/RC})$  to the power of minus  $t$  by  $R_c C$  by  $R_c$ , and therefore, how we can write this down as  $V_0$  by  $R_c e^{-t/RC}$  to the power of minus  $t$  by  $R_c C$ . So, that is the value of the current in time.

So, the power delivered to the discharging circuit is basically  $V_{ct} i_{ct}$  right power as you know  $V_{ct}$ . So, we can write this as  $V_{ct} i_{ct}$ . So, this is the function of time right. And so the amount of total energy delivered to the discharging circuit at any time  $t$  is given by  $dE_T$  equals  $i_{ct} V_{ct} dt$  that is the amount of energy which is charged at a certain instant of time.

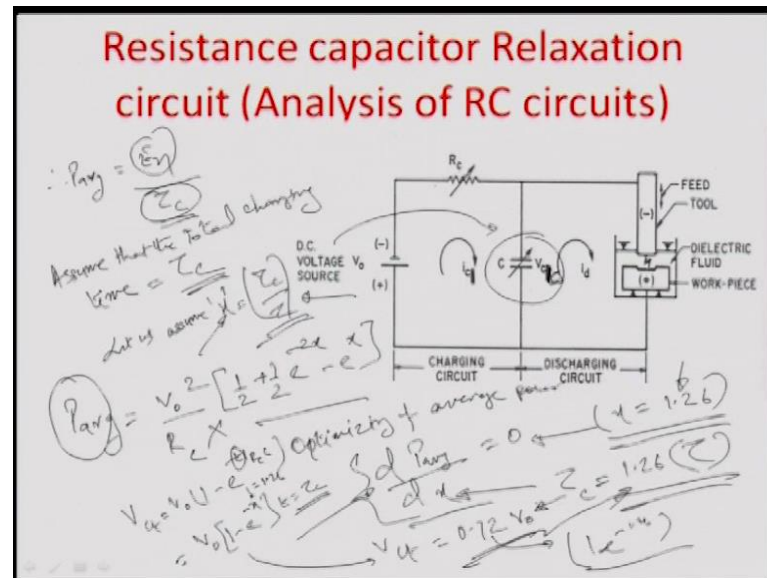
(Refer Slide Time: 38:38)



So, let us now actually calculate the amount of energy which is needed by substituting the values of the  $i$ ,  $c$ ,  $t$ ,  $v$  etcetera in this particular equation and  $n$ . Therefore, can be written as the value of  $v$ ,  $c$ ,  $d$ ,  $i$ ,  $c$ ,  $d$ ,  $v$  zero by  $r$ ,  $c$ ,  $e$  to the power of minus  $t$  by  $c$ ,  $r$ ,  $c$  times of  $v$ ,  $c$ ,  $d$  which is  $v$  zero one minus  $e$  to the power of minus  $t$  by  $c$ ,  $r$ ,  $c$ ,  $d$ ,  $t$  that is what  $d$ ,  $e$ ,  $n$ ,  $s$ . So, if integrate equation one here to find the overall power with time, we have over all energy, I am sorry in time we have  $e$ ,  $n$  equals  $v$  zero square by  $r$ ,  $c$  times of minus  $t$  to the power of minus  $t$  by  $t$  plus  $t$  to the power of minus two  $t$  by  $t$  plus  $k$  two here the  $t$  is a time constant which is  $r$ ,  $c$ ,  $c$ ? You can assume to be  $r$ ,  $c$ ,  $c$ . So, at time  $t$  equal to zero we know that the total amount of energy which has been delivered to the capacitor is also zero the process as not yet started on the charging such circuit which is you have you assume this kind of a made similar assumptions in earlier expressions.

So, if you put the value of  $t$  equal to zero as  $\ln 0$  here a  $k_2$  comes out to be equal to  $v_0^2$  by  $2RC$  times  $\ln 0$  put in the value of  $t$  equal zero, and this particular expression, and therefore, the overall  $\ln$  total energy delivered in time as the process of charging of this capacitor  $C$  happens is given by  $v_0^2$  by  $RC$  times  $\ln 0$  times of half plus half  $e$  to the power of minus twice  $t$  by  $\ln 0$  minus  $e$  to the power of minus  $t$  by  $\ln 0$  where  $\ln 0$  is actually equal to  $RC$  time of  $C$ .

(Refer Slide Time: 41:19)



So, the average power can be obtained by making a ratio of the total energy during the total charging times. So, if you assume that the total charging time is equal to  $t_c$  power should be the total energy per unit time time is  $t_c$ . So,  $E$  by  $t_c$ . So, let us assume parameter  $x$  here is equal to charging time per unit time constant. So, another words how many time constants contribute to the charging time is a the new parameter  $x$  that we have introduced. So, if in terms of  $x$  if you want to represent power average the average power can be represented as  $V_0^2$  square by  $R_c$  times of  $x$  times of half plus half  $e$  to the power of minus twice  $x$  minus  $e$  to the power of  $x$   $x$  is again this how many times the time constant is really the total charging time.

So, we can actually optimize this power, because that is the whole goal a behind the charging discharging. So, optimizing of average power is carried out by a different shading this with respect to the parameter  $x$ . So, we will really have to see at what point a or at what  $x$  value or at corresponding to what the ratio between the total charging time, and the time constant tow a would the optimum power be delivered. So, it gives you an idea of the frequents right, because one charging time is indicating of or its indicate if of one charging discharging cycle.

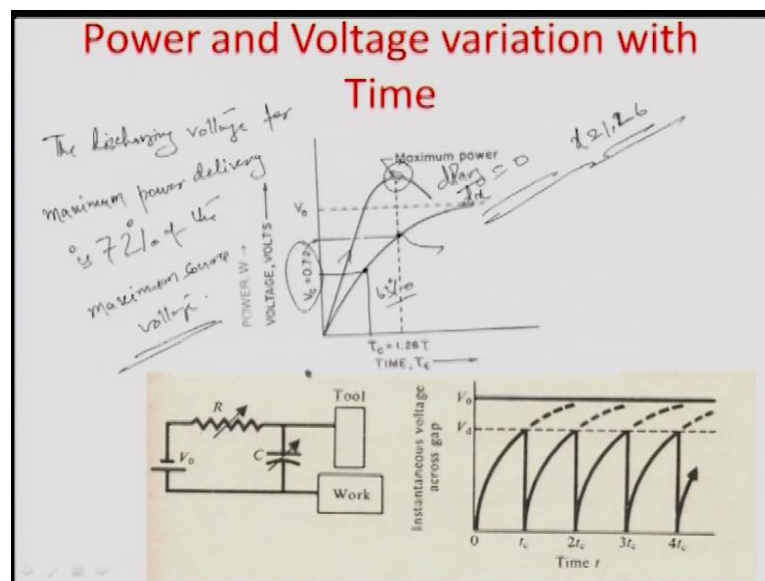
So, if the total charging time is more than that; that means, there is more than one charging discharging cycle which is there, and therefore, a  $dP_{avg}/dx = 0$  would give you that condition, and a it has been obtained that this is only possible

for a value of  $x$  equal to one point two six a. So, therefore, the charging time is one point two six times the time constant of the capacitor a this corresponds to the optimum power delivery a from the voltage source to this charging capacitor  $c$  in question. So, if you put the value of this a one point two six a we can find out what really the  $v_c t$  a, and of what percentage of the voltage rise we need in the capacitor for this optimum best performance a at  $x$  equal to one point two six. So, this can be represented as  $v_0 (1 - e^{-x})$  to the power of  $x$  by  $r c c$ .

We are considering this  $t$  to be equal to  $t_c$  charging time therefore,  $v_0 (1 - e^{-x})$  to the power of  $x$  if you put the value of  $x$  equal to one point two six here  $v_c t$  would come out to be equal to zero point seven two  $v_0$  zero zero point seven two is one  $e$  to the power of one minus  $e$  to the power of minus one point two six ok.

Another word it means that a the total amount of charge of the capacitor in time is actually seventy two percent of the output power. So, that is about the time that you have to let the charge to rise in the capacitor before the discharge would happen, and that kind of also determines the frequency at which the capacitor should charge, and discharge for optimum power delivery from the power source on one side to the discharging or to the electrode a discharge machining circuit on other side.

(Refer Slide Time: 45:50)



So, that is what a this optimum point of seventy two percent a comes out to be. So, this is kind of represented in this a figure here. So, you can see that actually this operating point

is a corresponding to about a seventy two percent of  $v_0$  the time constant of the capacitor may be somewhere here which is sixty three percent of  $v_0$ , and it is going up of the operating point here for the power graph to rise to its maximum this is the power condition a corresponding to  $dP_{average} / dx = 0$   $x = 1.6$ .

So, in other words a we really can say that the amount of discharging voltage for maximum power delivery is seventy two percent of the maximum source voltage a supply voltage. So, this is one aspect of the system that we have consider today, and then we will consider the discharging part were whatever has been now supply to the capacitor would further be discharge into the medium in term of spark, and from that we can have a co relation between the the material removal rate which is, because of the discharging process, and the source voltage that you are using with the i c t. So, you have a control on the operational parameters to estimate predictably what would be the material removal rate. So, we will complete this in the next class.

Thank you.