

Microsystem Fabrication with Advanced Manufacturing Techniques
Prof. Shantanu Bhattacharya
Department of Mechanical Engineering
Indian Institute of Technology Kanpur

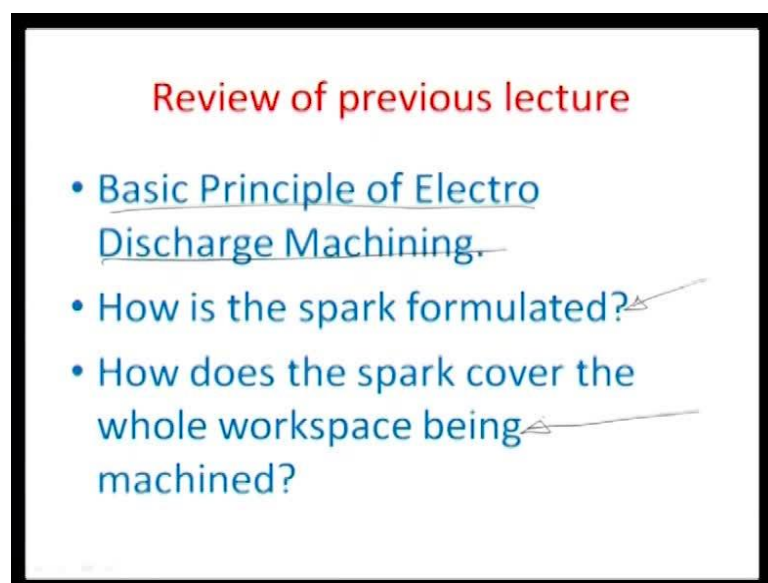
Lecture – 22

(Refer Slide Time: 00:20)



Hello and welcome back to this 22 lecture on Microsystems fabrication by advanced manufacturing processes.

(Refer Slide Time: 00:27)



So, brief recap about what we did in the last lecture, we were talking about process called electro discharge machining. And basically this process is about series of sparks or a series of a small, short time discharges occurring between an anode and cathode which would produce some kind of an impression, on the anode surface.

So, in this particular case the work piece is made the anode and this and the spark discharge is actually the cathode and the basic principle that if the tool surface which is actually the cathode is propagated towards the work piece surface. There is always set of imperfection and discontinuities on both the surfaces. Which result in distance of shortest separation happening between such surfaces between 2 corresponding peaks may be.

Because of that the total electric field which is also the voltage per unit distance if it exceeds by in any chance, the breakdown field of the medium, which is separating them there is a tendency of these stream of electrons to pass by and this electrons actually go from the cathode to the anode surface. There by damaging the anode in terms of local melt pool and a melting surface. So, we talked about how the spark is formulated in such process we also did talk about how the spark covers, the whole works pieces pace even though the spark itself is very small.

So, the work piece surface can be much more about may be couple of 10 of 1000 times larger dimensionally then the spark, which is actually a very small which occupies very small area. So, the whole idea is that, it is about it is a dynamic processes where this spark traverses over different such distances of close proximity is between the work piece on the tool and covers the whole surface in turn and melts the whole surface. So, that there is a uniform machining a layer by layer of a the work piece surface and question.

(Refer Slide Time: 02:39)

Mechanics of EDM processes

- Therefore, a negative of the electrode shape is created over the work-piece electrode. *Amenable to micro-fabrication*
- Generally, the rate of material removal from the cathode is comparatively less than that from the anode due to the following reason:
 1. The momentum with which the stream of electrons strike the anode is much more than that due to the stream of the positive ions impinging on the cathode though the mass of an individual electron is less than that of the positive ions.
 2. The pyrolysis of the dielectric fluid (normally a hydrocarbon) creates a thin film of carbon on the cathode. *Pyrolysis product*
 3. A compressive force is developed on the cathode surface primarily due to cations. *Carbon*

Therefore, normally, the tool is connected to the negative terminal of the dc source.

Today, we will look into a little more depth about the variously shoes regarding EDM process. So, as have already illustrated before and negative of the electrode shape is created over the work piece electrodes. So, this is also the same die sinking operation as happens in any other conventional, non conventional process like ECM. So, here also whatever is the electrode shape negative of that, would be exactly replicated on the work piece surface or work piece electrode which makes it actually a process amenable to micro fabrication.

So, here the whole goal therefore, is to be able to produce the exact negative replica on the tool of the feature of the shape that, you want to imprint on the surface in question at the micron scale. So, generally the rate of material removal from the cathode is comparatively lesser than that from the anode and it is obvious; why this happens 1 of the main reasons why the material removal rate is always higher at the anode this case is this stream of the momentum with which the stream of electrons strike, the anode is much more than that due to the stream of positive ions impinging on the cathode though the mass of an individual electron is less than that of the positive ions.

So, what is the spark, it is essentially a breakdown of the medium. And once the electrons are generated from the cathode side and it comes out of the cathode, they are free electrons. And they would create a lot of collagens between the medium particles which are there in its path that is creating a ions and more electrons. So, impressive

there are 2 different kind of electrons, which are their 1 is the primary electron which being emanated from the cathode an another is actually the secondary electron which comes cause of the collision of those primary electrons with the particles of the media.

So, therefore, the it is an avalanche really the electron side. So, that is the tendency of the electron density to be much higher, in this case then the number of ions per unit volume. So, the volume density of the ions may be lower and particularly, for the cases where the products of such dissociation reactions are multivalent in nature. The electrons would defiantly be much more in numbers then the number of ions which are produced.

So, that is 1 of the principle reasons why the total momentum which is transferred by the electrons on to 1 of the electrode that is the anode surfaces much more, in comparison to the momentum being transferred by the cetains, which are produce in the medium because of such electron season reaction or collegian reactions. So, the ion the ion flux is really lesser and the ion flux, is a principle removal mechanism for the cathode side and the electron flux is the principle removal mechanism for the anode side.

Therefore, the a anode has a much more erosion rate and then actually the cathode or the tools side. The others reasons which are responsible for this a difference in the rate of removals between anode and cathode are principally, the pyrolysis and particularly, the pyrolysis of the dielectric fluid, which is normally hydrocarbon as I have already elastrator many times before that, in the EDM process there is always a medium in between the tool the cathode and the anode and that is typically, 1 with the high dielectric constant and is an insulating medium.

So, the question of breakdown only arises when such high dielectric fluids are circulated and hydrocarbons are the most prominent once, because they them themselves, do not participates in formulating oxides or nitrides are so on, so forth of the material on the surface, it is a high temperature melting process of machining that we are talking about. So, what happens is that this dielectric fluid typically breaks down and because of the break down, because of the season reaction, which is happening in between the were the both the electrodes in a there is always by products in form of hydro carbon which are produced.

As such these are the cations and they rust towards the cathode their positive ions. And they would create a thin film of carbon on the cathode itself. So, therefore, this again

gives a situation, where already there is some kind of layer or a coating, which is present on the cathode side which is generating the electrons due to which again the bombardment of the anions and ions.

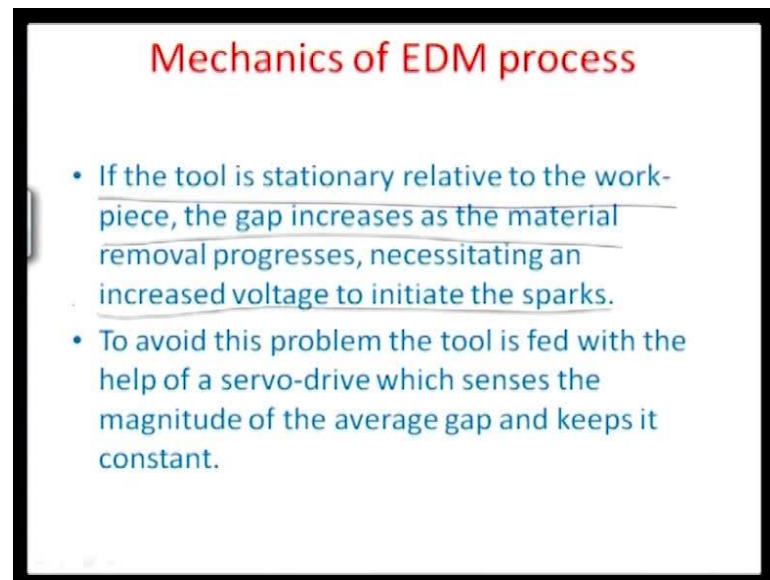
So, a cation is sorry to that to that electrode is kind of shielded, because of this superficial layer which develops sacrificially which develops on the surface of such an electrode material. And there is another reason apart from the pyrolysis and deposition of a film and the electron flux being greater than the ion flux. The third reason is basically, the compressive force that is typically developed on the cathode surface primarily due to the cations.

So, when we are talking about the cations in general they are much more heavier objects, because of the atomic mass of the species which is present. And at least the cations that we are talking about is much more than the electrons in terms of their mass. So, this compressive force typically, that is developed is because of the high momentum of the cations rushing towards the cathode.

So, therefore, if there is a high pressure because of the ion flux on the surface in general there is a tendency of the material, which is melted away from the cathode to not go into the material medium the dielectric medium because of that pressure. So, that pressure incidentally, is not very high on the anode surface because the electrode materials are very lighter in their weights and the electron base about approximately 9.1×10^{-31} kg's.

So, therefore, the compressive forces because of the cations is the much more thus creating pressure, thus creating less chances of diffusion of the melt, which is not true in the case of faying. So, these are 3 principle mechanisms mentioned here for which are responsible, for the differential removal rates across both the cathodes and the anode and therefore, it is preferable that the tool be connected to the negative terminal of the DC source. Thus making it a cathode and the work piece is connected to the positive terminal that is making it an anode.

(Refer Slide Time: 09:28)



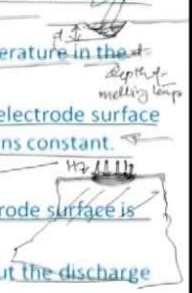
So, now a situation has been obtained where, there is an electric field there is a potential and slowly the field is growing. So, that it exceeds a breakdown field of the media. So, if the tool is stationery relative to the work piece, the gap would increase right as the material removal progresses. For obvious; reason is that there would be a melting wherever, the spark is generated and, because of that there is going to be an erosion the melt is going to come back into the dielectric fluid and the 1 of the surfaces that is in this case the work piece surfaces reseeds away from the tool.

Thus creating more distance and that is creating a situation way are the electric field would no longer be hold to the break down field. And the spark would simply see to exists or asset of spark should simply see to exists. So, therefore, there are 2ways that we can control this situation to make it a continues continued machining process.

(Refer Slide Time: 11:54)

Theoretical MRR values

- The quantity of material removal due to a single discharge can be determined by considering the diameter of the crater and the depth to which the melting temperature is reached.
- We make some assumptions for modeling the temperature in the zone of the spark:
 1. The spark is a uniform circular heat source on the electrode surface and the diameter $= 2a$ of this circular source remains constant.
 2. The electrode surface is a semi infinite region.
 3. Except for the portion of the heat source the electrode surface is insulated.
 4. The rate of heat input remains constant through out the discharge duration.
 5. The properties of the electrode materials do not change with temperature.
 6. The vaporization of the electrode material is neglected.



One is the that we keep on increasing the voltage. So that even if the distances. So, generated are smaller are larger, as the melting happens. So, the gradient of voltaged v by $d \times$ which is actually equal to a the electric field that, gradient always keeps constant to the level of the breakdown field and a condition which is sufficient for thus spark to happen between both the electrodes.

So, therefore, when ways to increase the voltage to initiate the spark and a the other way really to operate on the distance of separation between both the electrodes, by moving the tool closer to the work piece. So, if you have a situation where the tool is going closer to the work piece, then also the distance d would decrease and because of that field would again be higher at a constant voltage value. So, therefore, in order to avoid this problem of increasing the voltage, it is better to fed the tool with the help of a servo drive.

This can be a control system, where in gap is sensed and the magnitude of the average gap, can be controlled and it keeps it constant, in a by sensing the voltage and a equating at every time to the breakdown field a which is needed, a breakdown potential which is needed of the particular media. So, theoretically if we were to estimate the material removal rate values we really need to consider, the heat transfer process between 1 such spark which is happening over a certain surface in the weight keeps the heat or delivers the heat energy to cause the material to melt and go out.

So, let us now see how to estimate the quantity of the material removal due to such single discharge or single spark. And this can be determined by considering the diameter of the crater and the depth to which the melting temperature would reach. So, if a crater like this let say formulated on a surface like this, because of a arc which comes in strikes the surface we really need to estimate, what is the zone which gets effected really is the zone where melting would happen.

So, this mass here, write here is actually a melt pool. And we call this depth b ; the depth of melting temperature. So, we somehow need to estimate, what this depth of melting temperature is for a fix crater diameter and in order to understand this model we have to make some assumptions and we really what we model for is the temperature in the zone of the spark. So, the assumptions that we make for estimating; this as the spark is a uniform circular heat source on the electrode surface and the diameter of this heat source is $2a$, of this circular source and it remains constant throughout the process of discharge.

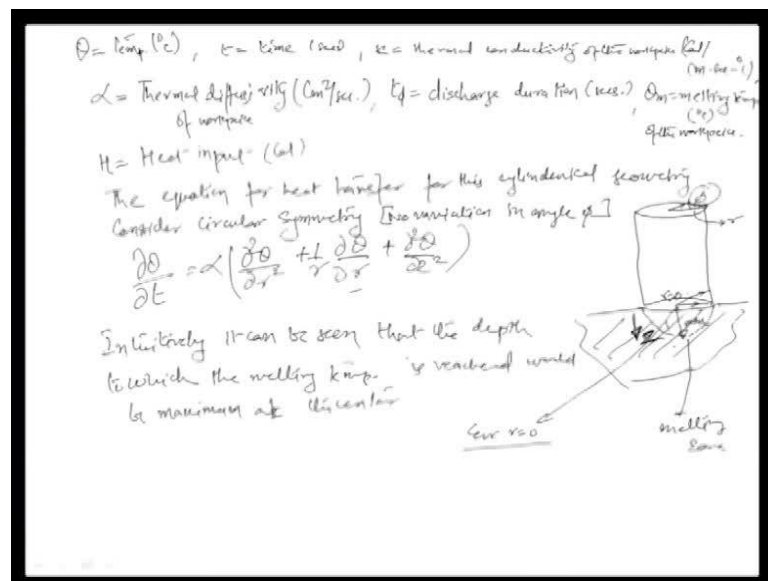
So, therefore, we do not vary the diameter in our model assuming that the approximate 1020 of diameter of striking on the surfaces same. Even though the spark may have some characteristics of variability, but, at least the heat transfer is taking place across a circular area if consider, the spark to be like a cylinder of constant size. We also assume that the electrode surface is a semi infinite region that is making our problem simpler and which is obvious; because the work piece actually in question is can be very large, even though the area or the zone which is effected very small on the surface.

So, it can be treated as a part of the semi infinite surface and we also assume that except for the portion of the heat source, the electrode surface is insulated from all sites. So, the only heat transfer which is going to take place is across this spot and remaining everything else is totally heat insulated. So, there is no heat transfer which takes place away from the spot as it moves. These are something which for simplification of the model you have to assume these.

Also we assume that, the rate of heat input remains constant throughout the discharge duration, which may not be really true because you know the discharge also is quite transient in nature, it is time varying, but then for sake of simplicity we have to assume that the heat input rate here.

Let say h if the heat per in a time heat energy per in a time supplier on to the surface. Or that remains constant throughout the discharge duration. We also assume that the properties of the electrode materials do not change, they not temperature dependent. So, there would not change with temperature. And also that the vaporization of the electrode material is neglected we assume, the electrode is not really effected by the spark of electrons which are sent. So, these are the 6 assumptions that you make in order to estimate this model. And a really the model is quite simple.

(Refer Slide Time: 16:22)



We now, need to assume certain things for example, let θ be temperature in degree celsius of a particular zone in that circular region t be the time in seconds, at which θ is being monitored, k be the thermal conductivity of the anode, which is also the work piece over which this melt phenomena is happening the, this can be assumed to have a unit of calorie per centimeter, second degree celsius.

And let us assume, a thermal diffusivity α , in work piece an assume all know that thermal diffusivity basically, the has the units of centimeters square per second cut also we assumed, T_d to be the discharge duration in seconds. And θ_m to be the melting temperature and degree celsius on these are related to the work piece. So, this for example, is thermal conductivity of the work piece or anode same is 2 for this thermal diffusivity of work piece and this is melting temperature of the work piece.

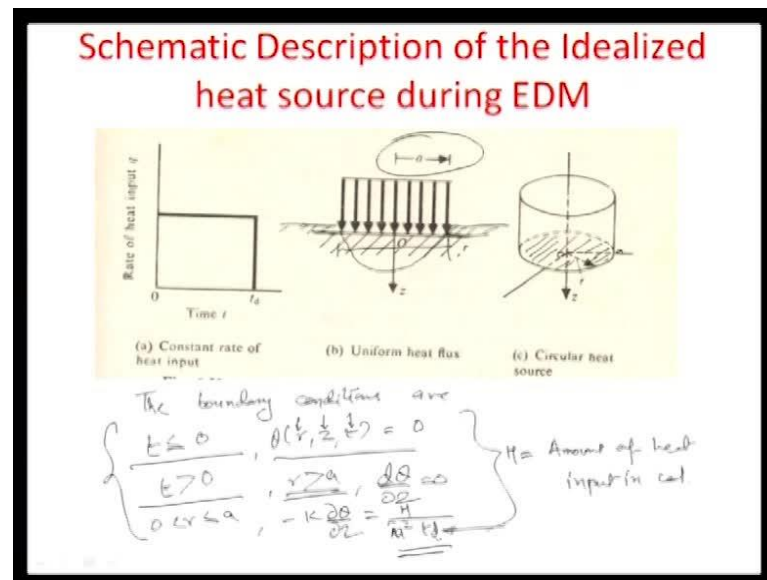
We also assume that is to be the input heat rate in calories. So, input heat which comes to that small area whether, spark contact contacts at the $c b$ in calories unit of energy. And the job now, is simple because we really need to estimate what the equation for heat conduction for the cylindrical geometry is this is geometry that we are considering on a semi infinite work piece surface. So, therefore, if you assume no variation, with the angle ϕ ; ϕ is this angle.

So, we can consider circular symmetry then the way that temperature behaves in time as represented as thermal diffusivity α times of d , variation of the second derivative of θ with to the temperature the radius radial distance plus 1 by $d\theta$ by dr plus $d\theta$ to $d\theta$ by dz 2 and z is basically the variation in this direction. This is the z direction, this really is the r direction and this exactly is the angular variation. So, the way that temperature; let say this is the melting zone that is formulated.

So, the way that the temperature vary is in this melting zone is really a function of the rate of variation of temperature with respect to z ; z being this depth direction and it is also a function of the rate of variation of temperature with respect to the radial direction r from the assuming a circular symmetry of this particular heat source. So, intuitively it can be seen that the depth to which, the melting temperature is reached would be maximum at the center.

So, that is considering the reproduces the crater. This theory is the maximum an as it moves away from the let us say r equal to 0 region to the outside, the temperature varies and also the melting temperature depth varies based on that. So, our interest principally lies in this region corresponding to r equal to 0. Because this gives you the maximum depth up to which the melting zone reaches, that is giving you and estimate of the crater volume that you are able to remove using that maximum depth and using a circular diameter of equal to $2r$ on the surface of the work.

(Refer Slide Time: 23:33)



So, let us now assume boundary conditions. So, in our particular case in this example, we can assume, the following more boundary conditions to saw this particular equation. So, the various boundary conditions are that corresponding to time, before the time of observation that is at point of time 0 in our reference frame we do not consider in any temperature around the region. So, in fact, theta which is the function now of the radius the depth and the time t is consider to be equal to room temperature.

This case we consider, room temperature to be in our reference frame 0 temperature, we then assume that a time t greater than 0 as the process has a started for any r, which is greater than a; a being the total radius of the spark as you can see here. This is the zone which is heat effected you assume this other to be fully insulated region. So, you assume that for r greater than a there is no variation of theta such in thus heat direction right.

So, basically the crater formulates like this. And here there is no melting point, which has been reached similarly, here in this zone there is no melting point which has been reached and finally, we have for the region r between 0 and a. So, typically means; the bulk of the circular area thus spark between 0 and a here assume that, the heat transfer is basically by the equation $k \frac{\partial \theta}{\partial z}$ is equal to the total amount of per unit area per unit time.

So, is basically the heat rate coming into in the area per unit time. So, is the flux rate is square time of discharge t b. We have already assumed h to be the amount of heat input

in calories. So, with these 3 boundary conditions we kind of limit ourselves to a very simplified you know solution of the equation, which has been given in the earliest tab, were these space time relationship of temperature in the melting zone has been sort of co related and the final solution which is arrived at from this equation oh which, we can actually do by either variable separation method or by assuming a combined parameter method.

(Refer Slide Time: 26:49)

Solution of the equation

Variation of θ in z direction ($r=0$)

$$\theta(0, z, t) = \frac{1}{2\pi k a t d} \int_0^{\infty} J_0(\zeta a) J_1(\zeta a) [e^{-\zeta^2 a^2 t d} - e^{-\zeta^2 z^2}] \frac{d\zeta}{\zeta}$$

ζ is a dummy variable

$\theta = \text{temp}$ & depth in melting zone
 $\rightarrow = \theta_m \rightarrow$ find out a plot for ζ values

So, we have the variation of theta in z temperature theta in z direction, who then assumption that we do everything at r equal to 0 that depth is the maximum as represented by theta at 0 for different z values, for the whole duration of the spark discharge t d to be given by 1 by 2 pie times of k thermal conductivity a is the diameter times of h and divided by time of discharge t d integral 0 to infinity j 0 zeta a j 1 zeta a times of e to the power of minus zeta z some error function of the cooption z by twice rout of alpha t d minus zeta rout of alpha t d dwells zeta by zeta.

So, intact this is what the estimation for temperature would be at point r equal to 0, at the center of the spot in question zeta here, is just a dummy variable is a variable of a integration that we are concerned with. And this actually can be a combined parameter you k now. So, it can behaving a containing the radius, it can be containing the time or it can also be containing the z value.

So, with this combine parameter method, you can actually get this estimate at the point the center of the spark are equal to 0 corresponding, to all z s an all the depths which are obtain and all the time of discharge which is treated to be constant i our assumption and this particular case for the spark. So, having said that we now have a relationship between the temperature and the depth in the melting zone. So, if this is equated to the melting point of the material, you can find out a float for the z values from this relationship.

(Refer Slide Time: 30:24)

Solution of the equation

If 'z' is the depth to which the melting temp. is reached,

$$\theta_m = \frac{2h\sqrt{\alpha t d}}{\pi K a^2 t d} \left[\operatorname{ierfc}\left(\frac{z}{2\sqrt{\alpha t d}}\right) - \operatorname{ierfc}\left(\frac{\sqrt{z^2 + a^2}}{2\sqrt{\alpha t d}}\right) \right]$$

where $\operatorname{ierfc}(k_1) = \frac{1}{\sqrt{\pi}} e^{-k_1^2} - k_1 \operatorname{erfc}(k_1)$

$$\operatorname{erfc}(k_1) = 1 - \operatorname{erf}(k_1)$$

$$\operatorname{erf}(k_1) = \frac{2}{\sqrt{\pi}} \int_0^{k_1} e^{-x^2} dx$$

So, let us see that if z is the depth to which the melting temperature is reached, the question can be simplified as theta m equal to twice h root of alpha t d by pie thermal conductivity a square t d times of the error function 3 of the coefficient z divided by twice route alpha t d minus error function of third kind i e r f c of the co option route of square plus a square divided by twice rout alpha t d. And the way that this error function of third kind is defined is that this i e r f c of the variable zeta is defined as 1 by route of pi e to the power of minus zeta square minus zeta e r f c zeta.

Where e r f c game second error function defined as 1 minus e r f zeta a near of zeta as we all know is the numerical integral 2 by rout pi 0 to zeta e to the power of minus x square d x. So, therefore, we have a very concrete way to interrelate the depth of melting temperature with depth z from this particular equation. So, once the value of this depth of

melting temperature is arrived at the question is that is supposing we define the amount of heat energy in a little different manner.

Particularly, you have to accommodate for the fact that the material is melting. And there is some kind of heat which cause in to the mix without really getting register on the temperature scale and. So, that amount of heat is also known as the latent heat. So, if we somehow assume that heat to we not at recording recorded in terms of the temperature rise then that heat needs to be take an off from the final heat equation. So, we that kind of an assumption the amount of heat would actually a we used for the temperature raise whether, in the molten state or in the solid state the material, would be quite different by neglecting the late and heat of melting this particular case.

(Refer Slide Time: 34:35)

Mechanics of EDM

•To take care of the latent heat of the molten material, the actual heat input rate can be found out by subtracting the heat used to melt the material from the total heat supplied by the spark.

•Thus the rate of heat input per unit area per unit time is given by the following equation:

$$H_{\text{total}} = H_m \rho (\pi r^2 z)$$

dependent

$$\frac{H_{\text{total}}}{\pi r^2 t d}$$

$H_{\text{total}} =$ Total heat energy (Cal)

$H_m =$ Latent heat (Cal/gram)

$\rho =$ density of the material

$2r \rightarrow$ diameter of the workpiece

So, we have 2 modify the equation quite a bit and the rate of heat input per unit area, per unit time would then be given by the equation the total heat h_{total} , which comes from the spark in to the work piece minus the latent heat per gram of the material which is h_m assume to be h_m in this particular case, times of density, times of the volume the material which is remove which is again the volume.

We assume this volume to be cylindrical for our estimation times $\pi r^2 z$ divided by off course, because it is a heat flux per unit area per unit time $\pi r^2 t d$. So, h_{total} here is actually total heat energy in calories. We have already define this earlier our earlier assumption. And h_m is latent heat in calorie per gram row is the density of the

molten material an off course $2a$ is crater diameter. So, typically we are assuming as you have seen here cylindrical crater although in actuality, it is actually a hemispherical or aqua spherical crater.

(Refer Slide Time: 36:25)

Mechanics of EDM

where, $2a$ = diameter of the crater = $k W^{\eta_1} (td)^{\eta_2}$ cm (empirical)

W = Total pulse energy (Joules), td = Time of discharge (sec)

η_1, η_2 and k are all constants

are all dependent on the tool & dielectric medium.

The melting length depth is $1/2$ of the crater diameter = $2a$, at $x=0$, $Z_{max} = h_c$ = crater depth (cm)

$Z_{max} \propto 2$

$V_c = \frac{\pi}{6} h_c (3a^2 + h_c^2) h_c \propto 2$

$\rightarrow \text{cm}^3$

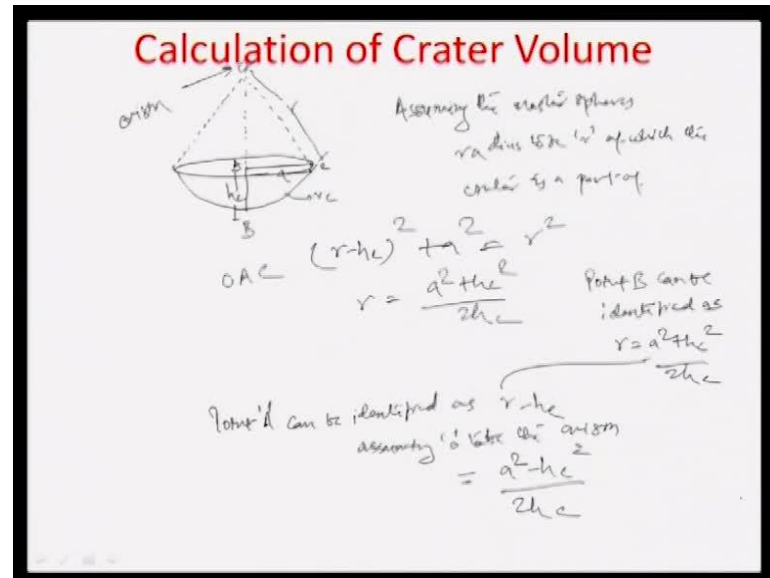
So, were this $2a$ the spark diameter or maybe you can call it the diameter of the crater assuming it to be the same as the spark diameter. Is actually given by an empirical relationship k times, of w to the power of η_1 times of td into the power of η_2 and this is actually is in centimeters. So, this purely empirical from experimental experimentally of thinned results W here is the total pulse energy. And normally this energy is the given injuries you already know that td is the time in this time of discharge in seconds.

The co options η_1 and η_2 and k are all constants. And they really depend on the property of both the electrodes on the medium. So, the k the constant here, η_1 on the first co efficient the power co efficient η_1 of the total pulse energy w and the power co efficient η_2 of the total time of discharge, they all depend on the tool electrode or work piece material as well as the dielectric medium which is between them. So, they are all dependent, they are essentially constant and are all dependent on the tool work piece and dielectric medium.

We found out last end that the amount of heat of latent you know nature is estimated by assuming a cylindrical size of a cavity although it is actually a quart spherical size of the

cavity. So, let us actually also try to estimate the volume of such a crater. So, that a more accurate form can emanate out of the total heat; which is needed for the depth of melting temperature analysis.

(Refer Slide Time: 40:45)



So, here let say there if the melting temperature depth is z . And the crater diameter is $2a$ and at r equal to 0; that means, radius equal to 0. The maximum melting temperature depth z maximum is given by h_c , which is actually the crater depth for all practical purposes centimeters. And the z max off course is proportional to z h_c is proportional to z .

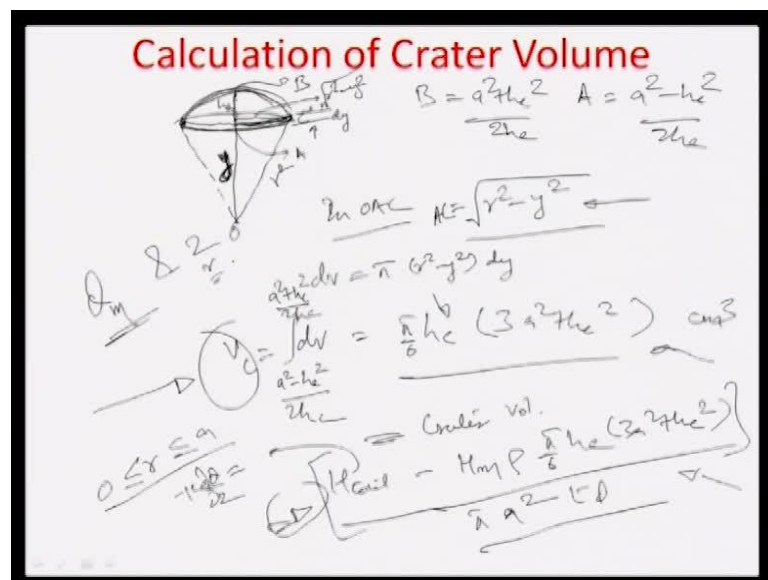
So, the total of volume of the material v_c which removes are which is actually contained, in the melt pool of this particular crater size of maximum depth h_c and the total diameter $2a$ is given by very simplistic expression. And will try to arrive at it from just by using a elemental analysis of the of the sphere. So, it is π by $6 h_c^3 a^2$ plus h_c^4 square in centimeter cube.

Let see, how supposing we have a crater like this of circular nature. And the crater has maximum depth at sea at the center. And the radius of the crater is a we need to calculate what the volume v_c of this particular crater is. So, let us extent this sent her all the way up to the center of the sphere of which this crater is the part of an we can always arrive at that, this is this being h_c and this being a we have this a , this b and this some point c here.

So, we have that if we assume, the total radius to be r here of the particular sphere assuming the master spheres radius to be r of which the crater is the part of by using pythagoras theorem in the triangle oac , we have $r^2 - h^2 = a^2$ which essentially means that r is equal to $\sqrt{a^2 + h^2}$.

So, the point b on the same scale, can be identified as r which is actually given by this $\sqrt{a^2 + h^2}$. And point a on the same scale can be identified as $\sqrt{r^2 - h^2}$ assuming o be the origin. This is the origin and. So, therefore, this can be a square minus h^2 by twice h as you already know is given by this $a^2 + h^2$.

(Refer Slide Time: 44:10)



So, then it just spoils down to a simple problem of integration that is assume, this crater to be present like this just to steady the sphere of side down this gay this write here is the full depth of the crater h . And it is a part of this is sphere here with center o and basically, we can actually consider a very small element like a cylinder in this particular area and try to see what is the volume of this and integrate such cylinders starting from the point a to the point b ok.

And we already know that: b is defined by r which is $\sqrt{a^2 + h^2}$ plus by twice h a is: defined as $\sqrt{r^2 - h^2}$. So, then it becomes a very simple problem. If this a distance from the center were y . So, you would be left with the radius here, as $\sqrt{r^2 - y^2}$ whole under the root, which would be true

because in triangle o a c you have to applied pythagoras theorem to obtain this expression here and therefore, if the radius of this cylinder here.

This is small cylinder here, is given by this $r^2 - y^2$ whole under the route, the total volume dV that the cylinder would have is $\pi (r^2 - y^2) dy$ and. Let us assume that, this is small length here is dy times of dy and integral dV is really for particularly, for the crater is from r varying between point a that is $h^2 - a^2$ minus or $a^2 - h^2$ square by twice h to the point b $a^2 + h^2$ square by twice h of dV .

So, this comes to be $\pi \left[\frac{h^3}{3} - \frac{a^3}{3} + \frac{h^3}{3} + \frac{a^3}{3} \right]$ square plus $h^2 - a^2$ centimeter cube. So, we obtain that the amount of crater volume, in this particular case comes out to be equal to is $\pi \left[\frac{h^3}{3} - \frac{a^3}{3} + \frac{h^3}{3} + \frac{a^3}{3} \right]$ square plus $h^2 - a^2$. And this volume can be effectively used for predicting the amount of heat lost, because of melting of the material in terms of latent heat of a melting and the net heat which is available now, is the total heat h_{total} minus h_m times of row the density of the material times of this crater volume.

So, it is $h_{total} - h_m$ times of row times of $\pi \left[\frac{h^3}{3} - \frac{a^3}{3} + \frac{h^3}{3} + \frac{a^3}{3} \right]$ square plus $h^2 - a^2$. And this you can off course make per unit area per unit time, because it is a heat flux essentially. So, that is what you can use as a boundary condition for solving for the zone between 0 and a of r were minus $k \frac{dT}{dz}$ by dz is actually equal to this volume. So, we now being able to predict very accurately, the relationship between the depth of melting temperature and this plot of the crater really, which is the various z s at different values of r ; from the center of the particular crater.

So, we to a towards the end of this lecture. And I would just like to recall that, we have being today able to theoretically sort of predict the whole zone of melting, in for a once spark a just to recall there are several such sparks, which occur at a certain frequency and 1 way of estimating the material may be really to look at the spark frequency. And seeing for 1 particular discharge duration, how much material gets removed in terms of a crater.

So, you can get a ball park figure of the estimate of the MRR or material removal rate they are. So, in the next lecture, we would like to extend this further and try to predictively answer some of the question us to what d MRR would be from various methods.

Thank you.