Kinematics of Machines

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Module - 3 Lecture - 1

In an earlier lecture, we have already mentioned that there are two types of problems in kinematics namely, kinematic analysis and kinematic synthesis. In kinematic analysis, we determine the relative motion characteristics of a given mechanism. From today's lecture, we shall start the discussion on this topic of kinematic analysis.

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Broadly, we can classify the kinematic analysis problems into three headings namely, displacement analysis, velocity analysis and acceleration analysis. For all these three types of problems, that is displacement analysis, velocity analysis and acceleration analysis, we can use either a graphical method or an analytical method.

In today's lecture, we shall discuss only the graphical method and that too only of displacement analysis. Later on, we shall take up velocity analysis and acceleration analysis. Let us see, what do you mean by a displacement analysis of a mechanism?

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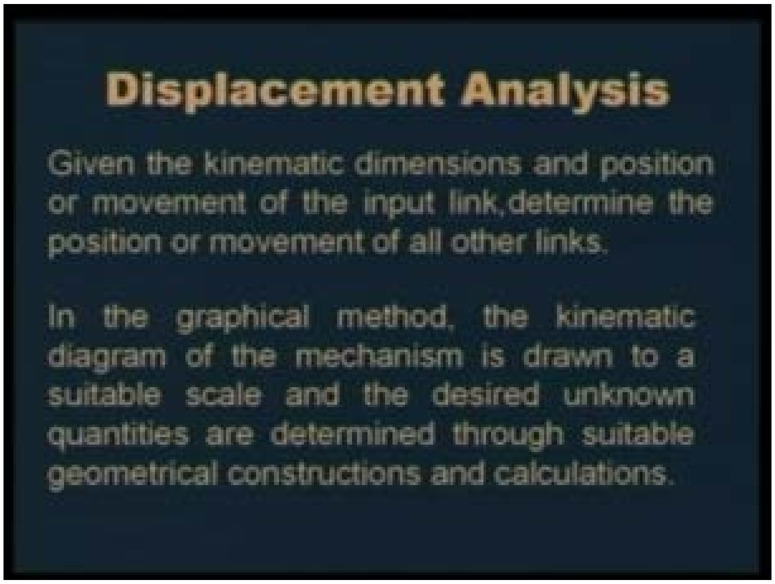

If we have given the kinematic dimensions and the position or the movement of the input link, then we should be able to determine the position or movement of all other links by this displacement analysis. In the graphical method, first we draw the kinematic diagram of the mechanism to a suitable scale and then the desired unknown quantities are determined through suitable geometrical constructions and calculations. We shall demonstrate this graphical method through a series of examples. However, before going into the details of each and every example, let me list the main points that one should remember while using graphical method of displacement analysis.

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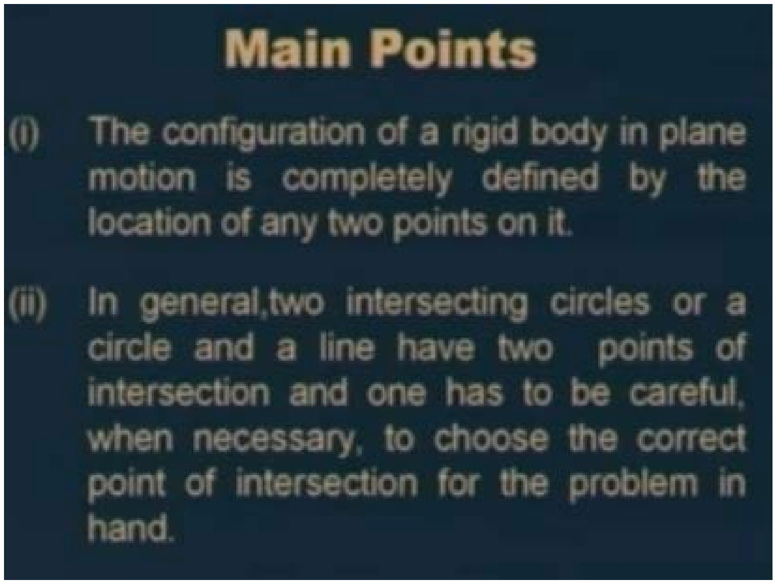

So, the main points are: First of all, we must remember that the configuration of a rigid body in plane motion is completely defined by the location of any two points on it. That means, if we know the location of any two arbitrary points of a rigid body in plane motion, then we know the location of all other points on that rigid body.

The second point is as we will see when we solve the examples that very often, we will need to draw two circles which are intersecting or a line and a circle which are intersecting. We know that such intersection points can be two that means two intersecting circles, in general intersect at two points. Similarly, a line and a circle also intersect generally at two points. However, sometimes it may be necessary to choose the correct point of intersection for the problem in hand. As I said, all these points will be shown very clearly when we solve those particular examples.

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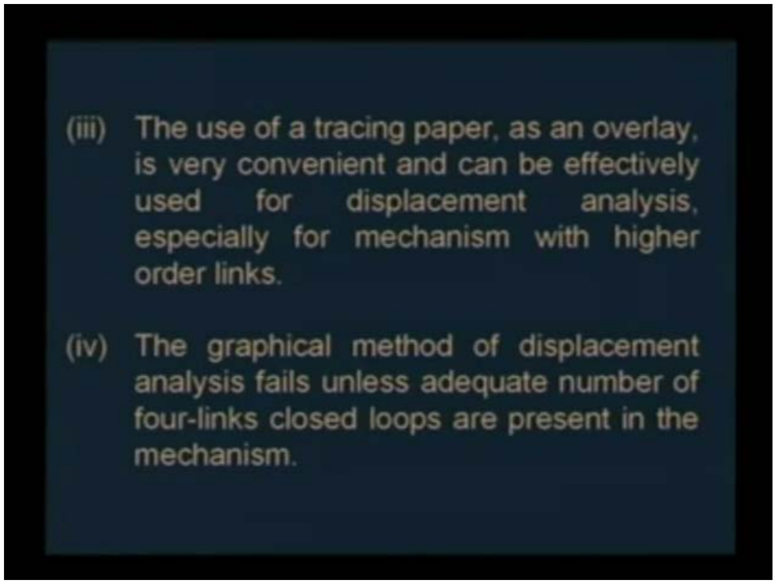

The third problem is sometimes we may use a tracing paper as an overlay and we will see that this will very convenient, especially if there are higher order links present in any particular mechanism and lastly, we must know that the graphical method of displacement analysis cannot be used unless there are adequate number of four-links closed loops in the particular mechanism. Unless, we have adequate number of four-links closed loops, the graphical method of displacement analysis cannot proceed.

So, let me start with the first example, which is a slotted lever quick return mechanism which is used in shipping machines. First, I will show the model, then the kinematic diagram of the same mechanism and then post the question of displacement analysis.

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This is the model of the quick return mechanism that is used in shipping machine. This is the cutting tool. This tool holder moves in a slot in the frame of the machine. This is called the bull gear, which is rotated at a constant speed. Here is a block, which is hinged to the bull gear and this block moves up and down in this slotted lever, the slotted lever is hinged to the frame at this point and the slotted lever is connected to the tool holder by an additional link. So, we have a six-link mechanism where the continuous uniform rotation of the bull gear is converted into the to and fro motion of the cutting tool. The cutting tool during the forward motion is doing useful work and we have to maintain a proper cutting speed. However, during the return stroke of the tool, this is not doing any useful work, so I would like to make the return faster and that is why, it is called quick return mechanism. As we see very clearly, if I rotate the bull gear at almost uniform speed, the forward motion is slow but the return is faster.

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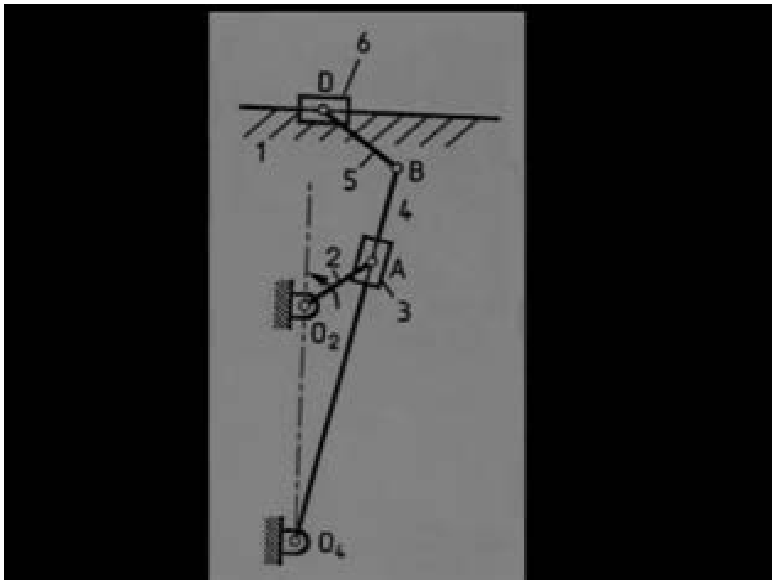

This is the kinematic diagram of the six-link slotted lever mechanism, the model of which we have just seen. Here, this link O_2A, that is link number 2 represents the bull gear, link number 3 is the block which goes up and down in the slotted lever, which is link number 4, link number 5 connects the slotted lever to the tool holder, which is represented by link 6. So, we have a six-link mechanism with a revolute pair at O_2, revolute pair at O_4, revolute pair at A, revolute pair at B, revolute pair at D and there are two prismatic pairs, one between 1 and 6 in this horizontal direction and there is a prismatic pair between link 3 and 4 along the slotted lever. The problem is if link 2 rotates at a constant speed say, omega 2 then we have to find, what we call Q.R.R- Quick Return Ratio.

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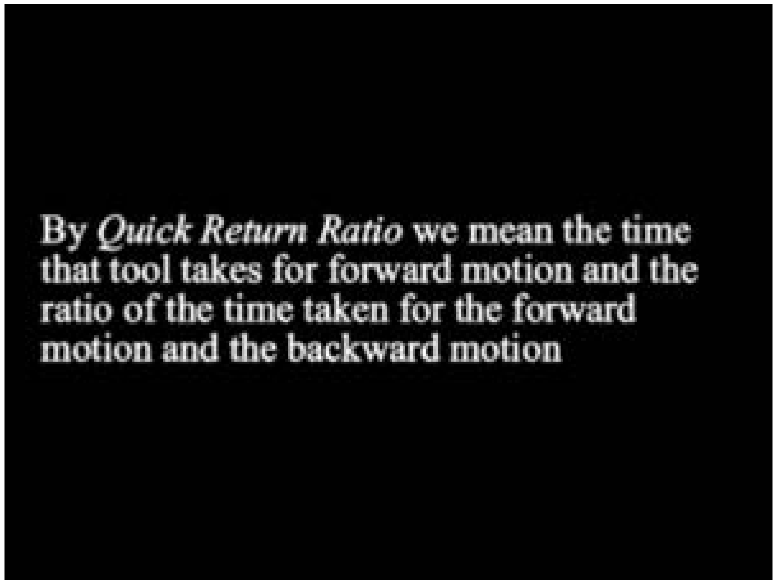

By Quick Return Ratio we mean the time that the tool takes for the forward motion and ratio of the time taken for the forward motion and the backward motion.

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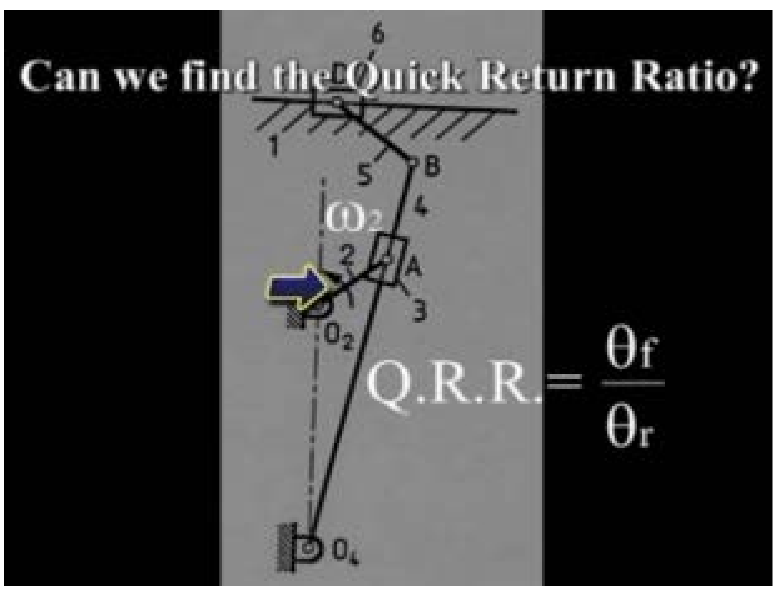

If link 2 is rotating at a constant speed, say omega 2, then the time it takes for the forward motion is theta$_{forward}$ that is the rotation of the link 2 during the forward motion and backward motion is say return, theta$_r$ where theta$_r$ is the rotation of link 2 during the return motion. So the problem is during this mechanism, can we find the quick return ratio? Before I start solving the problem, let me restate the problem for your benefit, which is what we are taking as our example 1.

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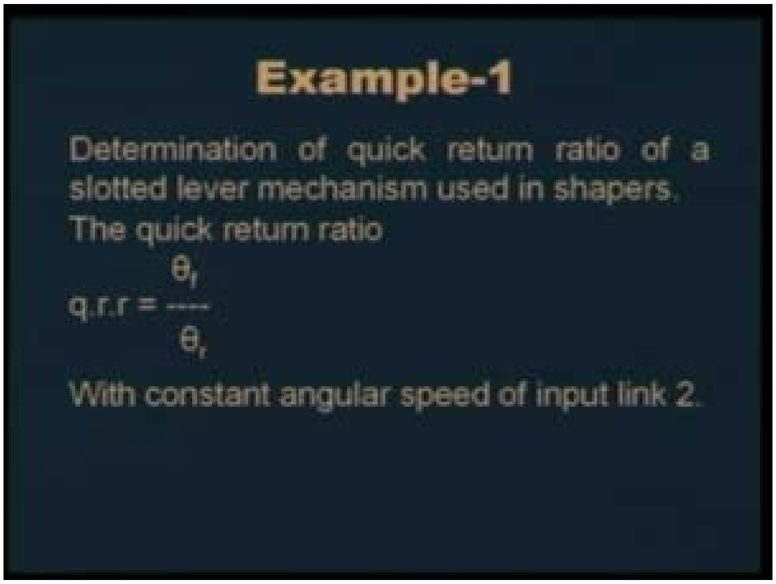

Example 1 is determination of quick return ratio of a slotted lever mechanism used in shapers. It is already been seen that the quick return ratio which is defined as q.r.r is equal to theta$_f$ divided by theta$_r$, where theta$_f$ is the rotation of link 2 during the forward motion and theta$_r$ is the rotation of same link 2, that is the bull gear during the return motion of the cutting tool. Of course, we are assuming that the angular speed of link 2 remains constant.

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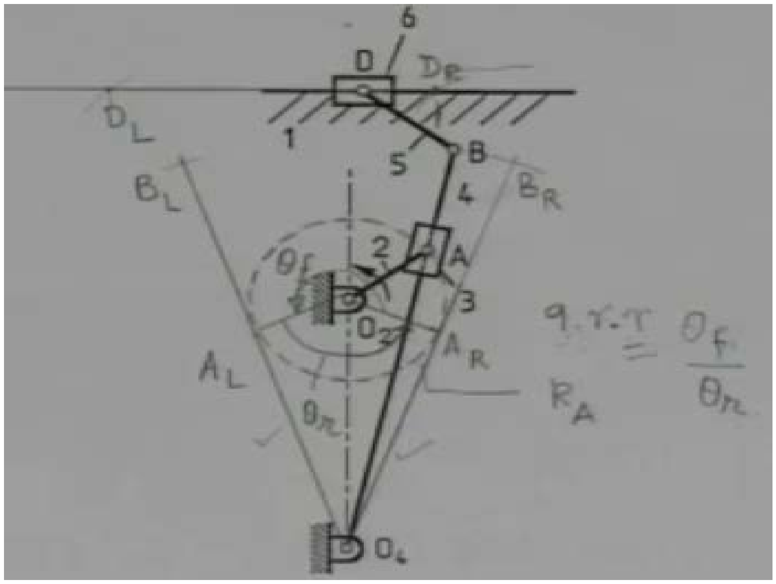

Let us now return to the kinematic diagram of the same slotted lever quick return mechanism in which link 2, that is this line O_2A represents the bull gear of the input link and link 6, that is this block D represents the tool holder of the cutting tool. As this bull gear rotates counterclockwise direction, this tool moves from right to left and left to right. This motion from right to left is the cutting motion and left to right is the return motion. Our objective is to determine the quick return ratio of this particular mechanism and also to determine what the stroke length of the tool is.

To analyze the problem, let me first note that this point A moves along this circle whose centre is at O_2, the radius is O_2A. That is, this circle represents the path of the point A and I call it k_A. To locate the extreme positions of the slotted lever, that is this link 4, I draw two tangents from the point O_4 to this circle k_A. This is the tangent to the right and similarly, this is the tangent to the left. Consequently, the right most position of this slotted lever, that is link 4 is represented by this line and the left most extreme position of the same slotted lever O_4B is represented by this line. When the slotted lever along these two lines, the point A this at this location let me call it A_R and at this position, let me call A_L. Since the distance O_4B does not change, I can also locate right most position of the revolute pair at B by drawing this circular arc with O_4 as centre and O_4B as radius. So, I gave this point which I call B_R, denoting the right most position of B.

The same way on this line O_4 A_L, I locate the left most position of the revolute pair B_L. Now, let us notice that the point D is going along this horizontal straight line and the distance BD does not change. So, when B occupies B_R, let me locate where D_R is. To do that, I draw a circular arc with B_R as centre and B_D as radius intersects this horizontal line. That is the line of the stroke of the cutting tool at this point which I mark as D_R. So D_R indicates the extreme right most position of link 6.

The same way, I can look at D_L on the same horizontal line. This is the line of movement of D and D does not change, B_L is here. So, I draw a circular arc with B_L as centre and B_D as radius to locate the extreme position of the point D which I call D_L. So this distance D_L, D_R represents the stroke of the cutting tool according to the scale of the figure. Now to find out the quick return ratio, we see that the link O_2A rotates from O_2A_R to O_2A_L during the forward movement. So, this is the angle through which link 2 is rotating through its forward movement which I call theta$_f$.

Same way, during the return motion, the point A is going from A_L to A_R, that is link O_2A is going from O_2A_L to O_2A_R and rotating through an angle which I call theta$_r$. So, we can easily determine the quick return ratio, q.r.r as the ratio of the angle theta$_f$ divided by theta$_r$. We have solved the problem. We have determined this given slotted lever quick return mechanism. The quick return ratio is given by theta$_f$ divided by theta$_r$ and the stroke of the tool as D_RD_L for the particular given mechanism. We also note that if the stroke length D_RD_L has to be decreased then I have to change the length O_2A. As we decrease the length O_2A, the distance D_RD_L will change, that is the stroke length of the tool will be decreased. However, as we decrease the radius O_2A then this tangent from this O_4 to this circle k_A, that is these points A_R and A_L move upward and the angle theta$_f$ and theta$_r$ both tend to value 180 degree. Consequently, q.r.r which is theta$_f$ divided by theta$_r$, when both of them tend to the same value 180 degree, q.r.r reduces. That is, this mechanism is okay for producing quick return effect as long the stroke of the tool is sufficiently large, quick return effect continuously decreases as the stroke length of the tool decreases.

We have just now explained the slotted lever quick return mechanism used for shipping machines is not good for smaller stroke length because the quick return ratio tends to become 1. That is, the quick return ratio decreases with decreasing stroke length.

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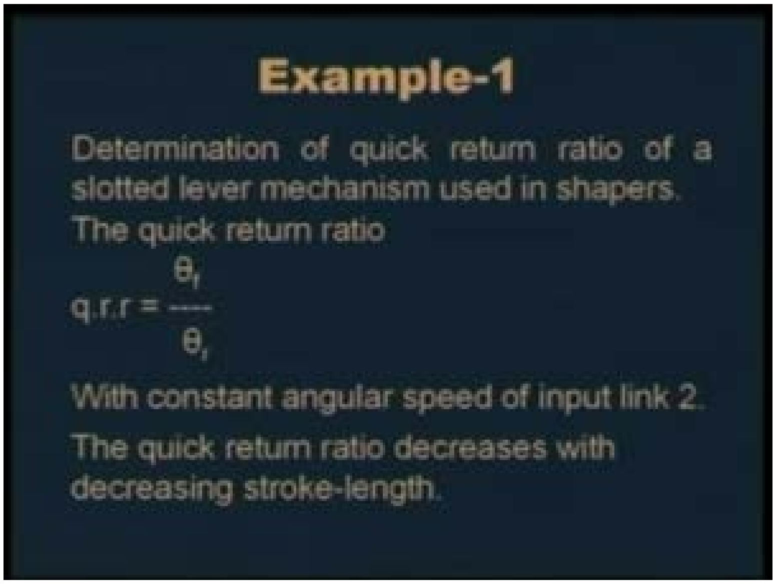

However, for slotting machine where the stroke lengths are normally short, we have to have different types of quick return mechanism where the quick return ratio is the independent of the stroke length. Such a mechanism is called Whitworth quick return mechanism. So we shall first see the model of such a Whitworth quick return mechanism then, pose the same problem as example 2, determine the quick return ratio of this Whitworth quick return mechanism and also see how it is independent of the stroke length of the tool. Before showing the model and solving the problem, let me restate the problem for your benefit.

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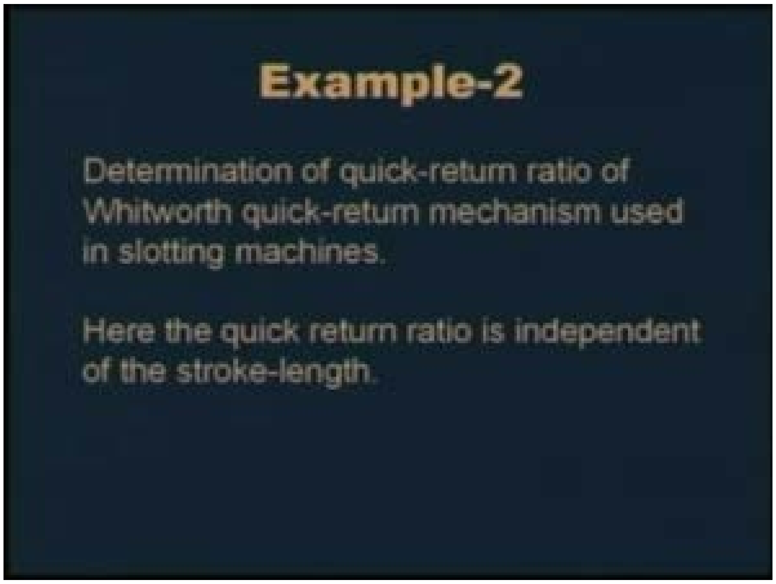

This is our example 2, that is determination of quick return ratio of Whitworth quick return mechanism used in slotting machines. As I said earlier, we will also show that in this mechanism the quick return ratio will be independent of the stroke length.

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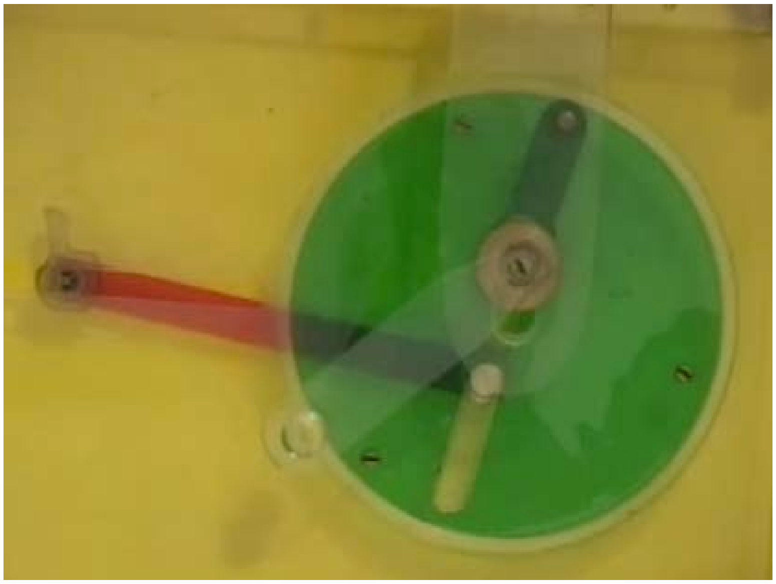

This is the model of Whitworth quick return mechanism. Here, the cutting tool is this one which is moving in this horizontal slot. So, if we rotate the input link at a constant speed, this is obvious that the forward and return motion of the tool is taking at different time. It is going much faster in this direction and coming back slower in this direction. So, if we consider this as the cutting tool, which is cutting in that direction, then we should rotate it in the opposite direction. It is cutting slowly and returning much faster. We shall see the kinematic diagram of this Whitworth quick return mechanism and analyze its quick return ratio.

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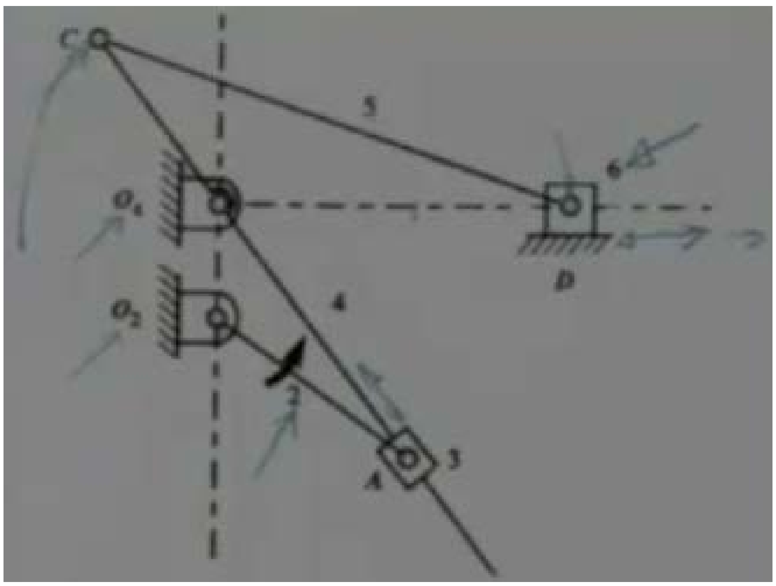

This is the kinematic diagram of that Whitworth quick return mechanism. As we see, this is also a six-link mechanism with five revolute pairs and two prismatic pairs. This link 6 is the tool holder and link 2 is the input link which rotates at constant angular speed, link 3 is the block which goes along link 4 at this prismatic pair, link 4 is hinged to the fixed link at O_4, link 2 is hinged to the fixed link at O_2, link 4 or 5 are connected by this revolute pair at C. Link 5 and 6 are connected by this revolute pair and link 6 has a prismatic pair with fixed link 1 along this horizontal direction. For the given kinematic dimensions, our objective is to determine the quick return ratio. If the link 2, that is input link rotates at constant angular speed.

First thing to note that the stroke of the tool that is, this 6 is entirely decided by the length O_4C. When D goes to the right most position in this direction, C comes on this line, O_4C

and CD becomes collinear. Similarly, for the left most position of this link 6, again O_4C and CD becomes collinear but C comes on this side on the line O_4D and the stroke length of the tool is obviously equal to twice of O_4C because the maximum distance is O_4C plus CD and the minimum distance is CD minus O_4C. So, the total movement of the tool is given by twice O_4C, that is the stroke length is changed by changing the length O_4C, no change is made in the length O_2A, whereas, we shall see that the quick return ratio will be entirely decided by the link length O_2A, the position of O_2 and the position of this horizontal line.

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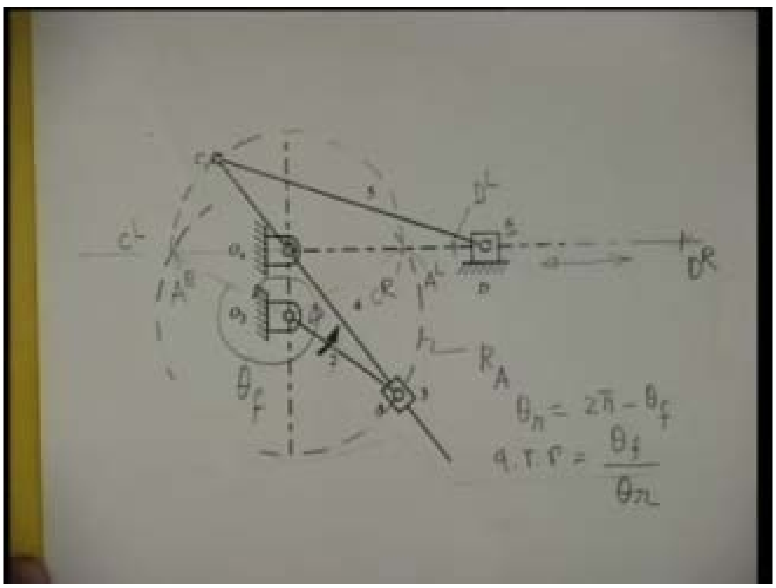

This is the kinematic diagram of the same Whitworth quick return mechanism which we have just seen. This link O_2A is the input link number 2 and this link 6 is the output link, that is the tool holder. Due to continuous uniform rotation of this input link 2, this tool holder 6 oscillates along this horizontal line.

The point to note is this point A moves on a circle with O_2 as centre and O_2A as the radius. This is the path of the point A, let me call it k_A. Again, we should note that the point A, O_4 and C always lie on the same straight line and O_4 is never moving. So, when the point A comes here, that is the point of intersection k_A and the line of reciprocation of the tool, let me call this point of intersection is A to the power L. Corresponding to this position of A, since A, O_4 and C are always on one line, C will also be on this horizontal line and at a distance O_4C from O_4, because this is also a link length which is not changing. Corresponding position of C, as C moves on this circle with O_4 as centre and O_4C as radius. So, I will call this point C to the power L that is, the left most position of C. Correspondingly, D will move here such that C to the power L, D to the power L is CD because this link length is also not changing. So, this is the left most position of the tool holder. Let me call it D to the power L.

Exactly the same way, when the point A occupies this point, which is the intersection of k_A and the line of reciprocation of the tool through O_4, let me call this point of intersection as A to the power of R. Since C is moving on this circle and A, O_4 and C must be on line, if I draw this circle, that is the path of C, when it intersects this line of reciprocation, I will call that the intersection as C to the power of R. Again, the distance CD is unique. So, from C to the power of R, if I draw an arc with CD as the radius, I get the right most position of D, which I call D to the power of R. Thus, this tool that is this link 6 moves from D to the power L to D to the power R that is, the stroke length which is exactly equal to two times O_4C.

However, during this movement from D to the power L to D to the power of R, the point A goes from A to the power L to A to the power of R, from D to the power of R to D to the power L, the point A goes from A to the power of R to A to the power L. As we see the O_2A, this link is rotating with constant input speed. So from right to left, the rotation is this angle, which I call $theta_f$ and from left to right it rotates only through this angle which is 2 pi minus $theta_f$. That is, during the return stroke $theta_r$ is 2 pi minus $theta_f$ and the quick return ratio is $theta_f$ divided by $theta_r$. As we see, the stroke length can be changed by changing the length O_4C which has no role to play so far as these two lines are concerned, that is O_2A to the power L and O_2A to the power of R that is decided by the intersection of the circle k_A and the line of reciprocation of this point D. So, the quick return ratio remains same even if we change the link length O_4C which causes a change in the stroke length. So, this is the quick return ratio of the Whitworth quick return mechanism which is independent of the stroke length.

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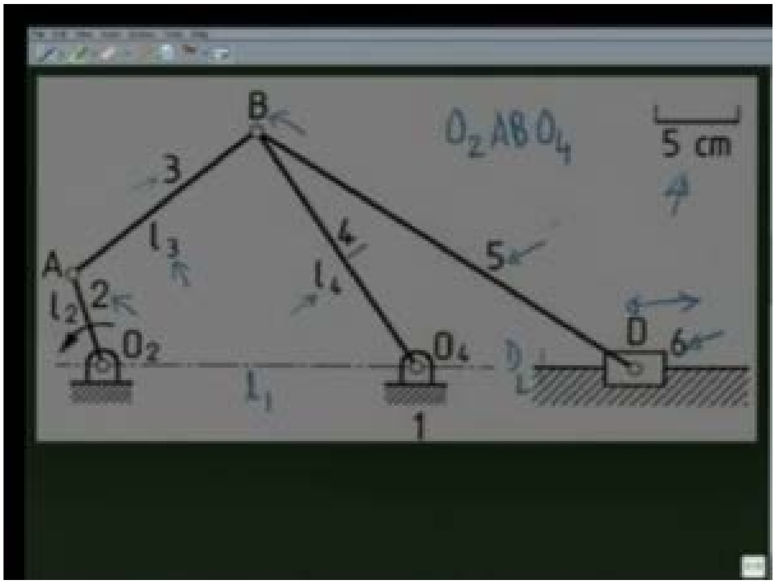

As our next example, let us consider another six-link mechanism which is shown in this figure. Here, as we see there is a four-bar mechanism O_2ABO_4. Link 4 of this four-link mechanism, that is this link is connected to another link 5 at this compound hinge B where 3 links are connected namely, 3, 4 and 5. Link 5 is connected to this slider which is link 6 and slider has a prismatic pair with the fixed link such that the point D moves in the horizontal direction. The question is if this link O_2A, that is link number 2 rotates completely, what is the stroke length of the slider at D? The scale of the diagram has been shown here that this distance is equal 5 cm.

For this problem, first we have to see that the four-link mechanism O_2ABO_4 happens to be a crank rocker because the maximum link length O_2A for which I may call l_1 as l_{max} and l_2 as l_{min}, l_{max} plus l_{min} is less than $l_3$plus l_4, that is the other two link lengths. So, this is the Grashof linkage with O_2A as the shortest link. Consequently, this link O_2A will rotate completely. We have to find out what is the maximum right most position of this point D and what is the left most position of this point D such that I can determine D_L and D_R which will give the stroke length of the slider 6. Before we solve this problem, let me write out this problem for your benefit.

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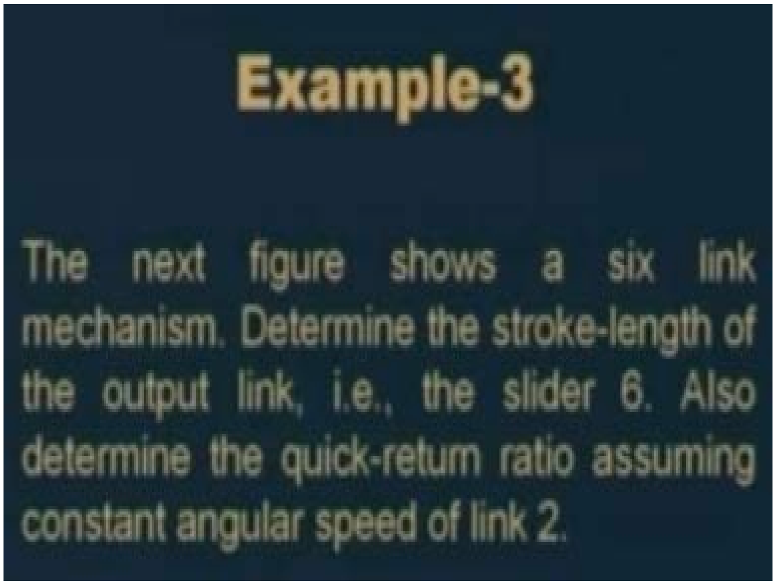

This is our third example and example 3 is the figure shows a six link mechanism. Determine the stroke-length of the output link, that is the slider 6. Also, determine the quick return ratio assuming the constant angular speed of link 2.

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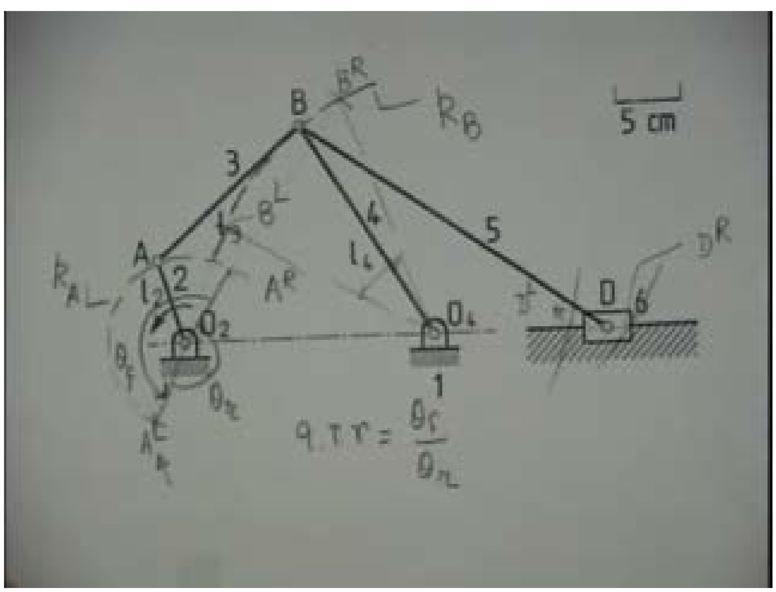

Let us solve this example 3 and the kinematic diagram of that mechanism is shown here again. The problem is first to find out the stroke length of this link 6, that is the slider at

D. To find the extreme right position of the point D, first we have to find out what is the extreme right position of this point B? Due to this length O_4B that is link 4, B moves on a circle with O_4 as centre and O_4B as radius. This circle represents the path of B. Let me call it k_B. The extreme right position of the point B will be taken up. As we have discussed earlier, when link 2, that is O_2A and link 3, that is AB become collinear. So the extreme position of B let me call it B to the power of R, when O_2B to the power of R is equal to O_2A plus AB. So, I take O_2A plus AB and from O_2, I mark that distance at B arc.

Let me repeat. B moves on this circle and at the extreme position, O_2A and AB become collinear. So, O_2B to the power of R becomes O_2A plus AB. Similarly, for the extreme left position, again link 2 and link 3 become collinear but O_2A comes here. Let me call it A to the power L. This A to the power L B to the power L is equal to AB and this point becomes B to the power of L. That is, O_2B to the power L becomes the difference of the link lengths AB minus O_2A. So, I take the difference of AB and O_2A and take that from O_2 and mark it on k_B. The link 4 oscillates from O_2B to the power of R to O_2B to the power L. So, this is a crank rocker mechanism and this is the rocking movement of the link 4. BD is of fixed length and B moves on this horizontal line. From this B to the power of R, if I mark this is BD. From B to the power of R, I mark this circle and wherever it intersects the horizontal line through D that determines the extreme position of D, I call it D to the power of R.

Similarly, from B to the power L, again taking the same length BD, I draw a circular arc and wherever it intersects this horizontal line through D that determines the extreme left position of D, that is D to the power L. So, this distance D to the power of L D to the power of R determines the stroke length of this slider 6. Now, to determine the quick return ratio that is the time taken from left to right and right to left, I can find out assuming, of course that link 2 rotates at uniform speed. For B to the power of R, the corresponding point of A and this is the circle on which A moves with O_2 as centre and O_2A as radius which we call k_A. For the extreme right position A comes here, let me call it A to the power of R and for the extreme left position, that is B to the power L, this point which we have already mark as A to the power L. As the link rotates uniformly from right to left, the rotation of O_2A is given by this angle, that is the angle between O_2A to the power of R and O_2A to the power L, from left to right, the rotation of same link 2 is given by this angle, that is 2 pi minus this. So if I call from right to left, that is the forward motion is theta_f and return motion is theta_r. Of course, theta_r is nothing but 2 pi minus theta_f and q.r.r-the quick return ratio of this mechanism, we obtain as theta_f by theta_r.

Let me summarize, what we have learned today. We have done the graphical method of displacement analysis and have discussed three different examples of six link mechanism to show how graphically, we can determine the quick return ratio or stroke length once the kinematic dimensions of those mechanisms are given. We started with a slotted lever quick return mechanism used in shipper machines where we saw that the quick return ratio depends on the stroke length.

Then we discussed the Whitworth quick return mechanism where the quick return effect is independent of the stroke length and in the third example, we have seen how from a Grashof Crank Rocker linkage. We can again get a quick return mechanism by using two more extra links and converting it to a six link mechanism. We have also seen that whenever we need the point of intersection of a straight line and a circle, we have to choose a correct point of intersection.

In our next lecture, we shall discuss a little more involved and difficult problems on displacement analysis.